An Update on Game Tree Research

Akihiro Kishimoto and Martin Mueller

Tutorial 3: Alpha-Beta Search and Enhancements

Presenter:
Akihiro Kishimoto, IBM Research - Ireland

Outline of this Talk

- Techniques to play games with alpha-beta algorithm
    - Alpha-beta search and its variants
    - Search enhancements
    - Search extension and reduction
    - Evaluation and machine learning
    - Parallelism

Alpha-Beta Algorithm

- Unnecessary to visit every node to compute the true minimax score
    - E.g. max(20,min(5,*X*))=20, because min(5,*X*)<=5 always holds
    - Idea: Omit calculating *X*
- Idea: keep upper and lower bounds (α,β) on the true minimax score
- Prune a position if its score *v* falls outside the window
    - If *v* < α we will avoid it, we have a better-or-equal alternative
    - If *v* >= β opponent will avoid it, they have a better alternative

How Does Alpha-Beta Work? (1 / 2)

- Let *v* be score of node, *v1, v2, ...,vk* scores of children
- By definition: in MAX node, *v* = max(*v1, v2,..,vk*)
- By definition: in MIN node, *v* = min(*v1, v2, ..., vk*)
- Fully evaluated moves establish lower bound
    - E.g., if *v1*=5, max(5,*v2,...,vk*)>=5
- Other moves of score <= 5 do not help us, can be pruned

How Does Alpha-Beta Work? (2 / 2)

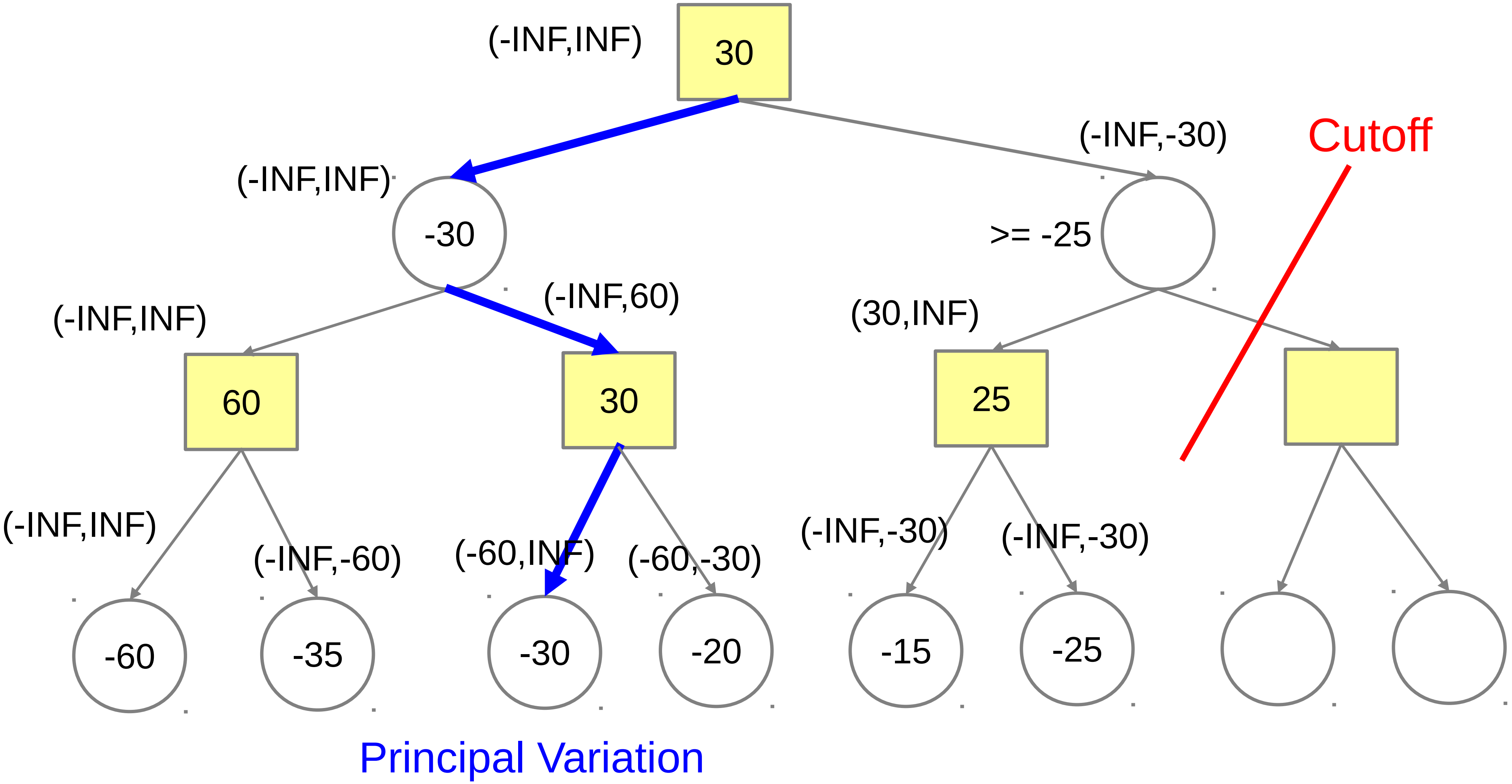
- Similar reasoning at MIN node – move establishes upper bound
    - E.g., $v=2$, $v=\min(2,v2,...,vk)<=2$
- If a move leads to position that is too bad for one of the players, then cut.

Alpha-Beta Algorithm – Pseudo Code

```
int AlphaBeta(GameState state, int alpha, int beta, int depth) {
    if (state.IsTerminal() or depth == 0)
        return state.StaticallyEvaluate()
    score = -INF;
    foreach legal move m from state
        state.Execute(m)
        score  = max(score,-AlphaBeta(state, -beta, -alpha, depth-1))
        alpha = max(score,alpha)
        state.Undo()
        if (alpha >= beta) // Cut-off
            return alpha
    return score
}
```

This is a negamax formulation.
Initial call: AlphaBeta(root, -INF, INF, depth_to_search)

Example of Alpha-Beta Algorithm



Principal Variation (PV)

- Sequence where both sides play a strongest move
- All nodes along PV have the same value as the root
- Neither player can improve upon PV moves
- There may be many different PV if players have equally good move choices
- The term PV is typically used for the *first* sequence discovered. Others are cut off by pruning

Properties of Alpha-Beta

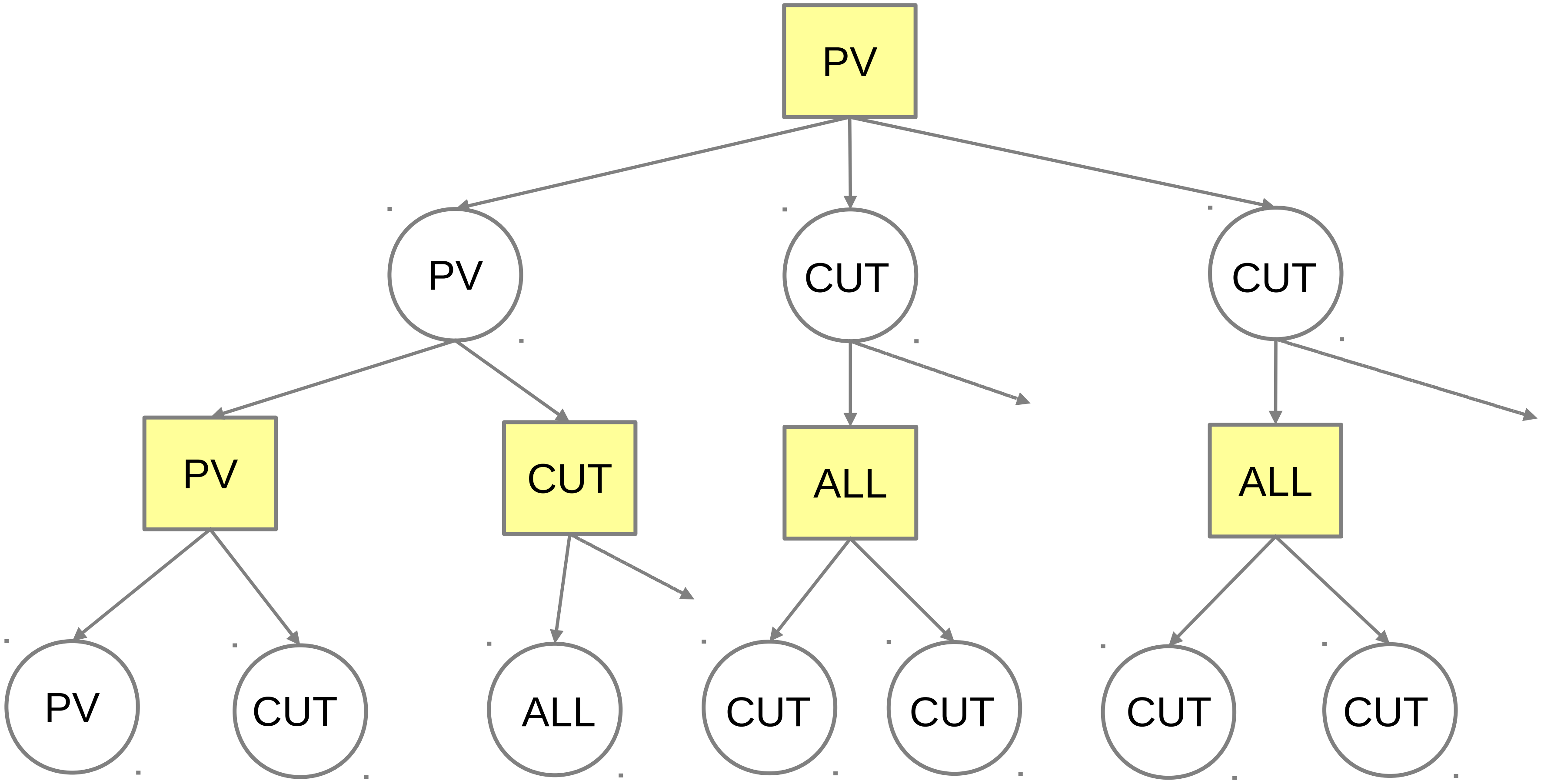
- Number of nodes examined
    - Best case: $b^{\lceil d/2 \rceil} + b^{\lfloor d/2 \rfloor} - 1$ (see minimal tree, next slide)
    - Basic minimax: $O(b^d)$

      b: branching factor, *d*: depth
- Assuming score *v* is obtained after alpha-beta searches with window (α, β) at node *n*, real score *sc* is:
    - If *v* <= α: fail low, *sc* <= *v*,
    - if α < v < β: exact, *sc* = *v*, and
    - if β <= v: fail high, *sc* >= *v*

We will keep using this property in this lecture

Minimal Tree

Tree generated by alpha-beta with perfect ordering
- 3 types of nodes (PV, CUT, and ALL)



Reducing the Search Window

- Classical alpha-beta starts with window (-INF,INF)
- Cutoffs happen only after first move has been searched
- What if we have a “good guess” where the minimax value will be?
    - E.g., “Aspiration window” in chess: take score from last move, (-one-pawn, +one-pawn) or so
- Gamble: can reduce search effort, but can fail

Other Alpha-Beta Based Algorithms

- Idea: smaller windows cause more cutoffs
- *Null window* (α,α+1) – equivalent to Boolean search
    - Answer question whether v <= α or v > α
- With good move ordering, score of first move will allow to cut all other branches
- Change search strategy. Speculative, but remain exact by re-search if needed
- Scout by Judea Pearl, NegaScout by Reinefeld: use null window searches to try to cut all moves but the first
- PVS – principal variation search, equivalent to NegaScout

PVS/NegaScout

[Marsland & Campbell, 1982] [Reinefeld, 1983]

- Idea: search first move fully to establish a lower bound *v*
- Null window search to try to prove that other moves have score <= *v*
- If fail high, re-search to establish exact score of new, better move
- With good move ordering, re-search rarely needed. Savings from using null window outweigh cost of re-search

NegaScout Pseudo-Code

```
int NegaScout(GameState state, int alpha, int beta, int depth) {
  if (state.IsTerminal() || depth = 0)
    return state.Evaluate()
  b = beta
  bestScore = -INF
  foreach legal move mi i=1,2,.. from state
    State.Execute(mi)
    int score = -NegaScout(state, -b, -alpha, depth – 1)
    if (score > alpha && score < beta && i > 1) // re-search
      score = -NegaScout(state, -beta, -score, depth – 1)
    bestScore = max(bestScore,score)
    alpha = max(alpha, score)
    state.Undo()
    if (alpha >= beta)
      return alpha
    b = alpha + 1
  return bestScore
}
```

Note for experts: A condition to reduce re-search overhead is removed here. See [Reinefeld, 1983][Plaat,1996] for details

Search Enhancements

- Basic alpha-beta is simple but limited
- Need many enhancements to create high-performance game-playing programs
- General (game-independent, algorithm-independent) and specific
- Depends on many things: size, structure of search tree, availability of domain knowledge, speed versus quality tradeoff, parallel versus sequential
- Look at some of the most important ones in practice

Enhancements to Alpha-Beta

There are several types of enhancements

- Exact (guarantee minimax value) versus inexact
- Improve move ordering (reduce tree size)
- Improve search behavior
- Improve search space (pruning)

Iterative Deepening

- Series of depth-limited searches d = (0), 1, 2, 3,....
- Advantages
    - Anytime algorithm – first iterations are very fast
    - If branching factor is big, small overhead – last search dominates
    - With transposition table (explain later), store best move from previous iteration to improve move ordering
    - In practice, usually searches less than without iterative deepening
- Some game programs increase d in steps of 2
    - E.g. odd/even fluctuations in evaluation, small branching factor

Iterative Deepening and Time Control

- With fixed time limit, last iteration must usually be aborted
- Always store best move from recent completed iteration
- Try to predict if another iteration can be completed
- Can use incomplete last iteration if at least one move searched (however, the first move is by far the slowest)

Transposition Table (1 / 3)

- Idea: Cache and reuse information about previous search by using hash table
- Avoid searching the same subtree twice
- Get best move information from earlier, shallower searches
- Essential in DAGs where many paths to same node exist
    - Discuss issues in solving games/game positions
- Help significantly even in trees e.g. with iterative deepening
- Replace existing results with new ones if TT is filled up

Transposition Table (2 / 3)

- Typical TT Content
    - Hash code of state (usually not one-on-one, but astronomically small error of different states with identical hash code)

    See http://chessprogramming.wikispaces.com/Zobrist+Hashing

    - Evaluation
    - Flags – exact value, upper bound, lower bound
    - Search depth
    - Best move in previous iteration

Transposition Table (3 / 3)

- When *n* is examined with (α,β), retrieve information TT
- Do not examine *n* further if TT information indicates
    - Node n is examined *deep enough and*
    - TT contains exact value for *n*, or
    - Upperbound in TT <= α, or
    - Lowerbound in TT >= β
- Try best move in TT first if n needs to be examined
    - Best move is often stored in previous iterations
    - Usually causes more cutoffs than without iterative deepening even if search space is tree
- Save evaluation value, search depth, best move etc in TT after *n* is examined

Move Ordering

- Good move ordering is essential for efficient search
- Iterative deepening is effective
- Often use game-specific ordering heuristics e.g. mate threats
- More general: use game-specific evaluation function

History Heuristic
[Schaeffer 1983, 1989]

- Improve move ordering without game-specific knowledge
- Give bonus for moves that lead to cutoff such as
  - history_table[color][move] += d^2
  - history_table[color][move] += 2^d (d: remaining depth)
- Prefer those moves at other places in the search
- Will see later in MCTS – all-moves-as-first heuristic, RAVE
- History heuristic might not be as effective as it used to be but is effectively combined with late move reduction (later)
  - E.g. Chess program Stockfish gives a penalty for “quiet moves” that do not cause cut-offs

Performance Comparison of Alpha-Beta Enhancements

C.f. Figure 8 in

[Marsland, 1986]

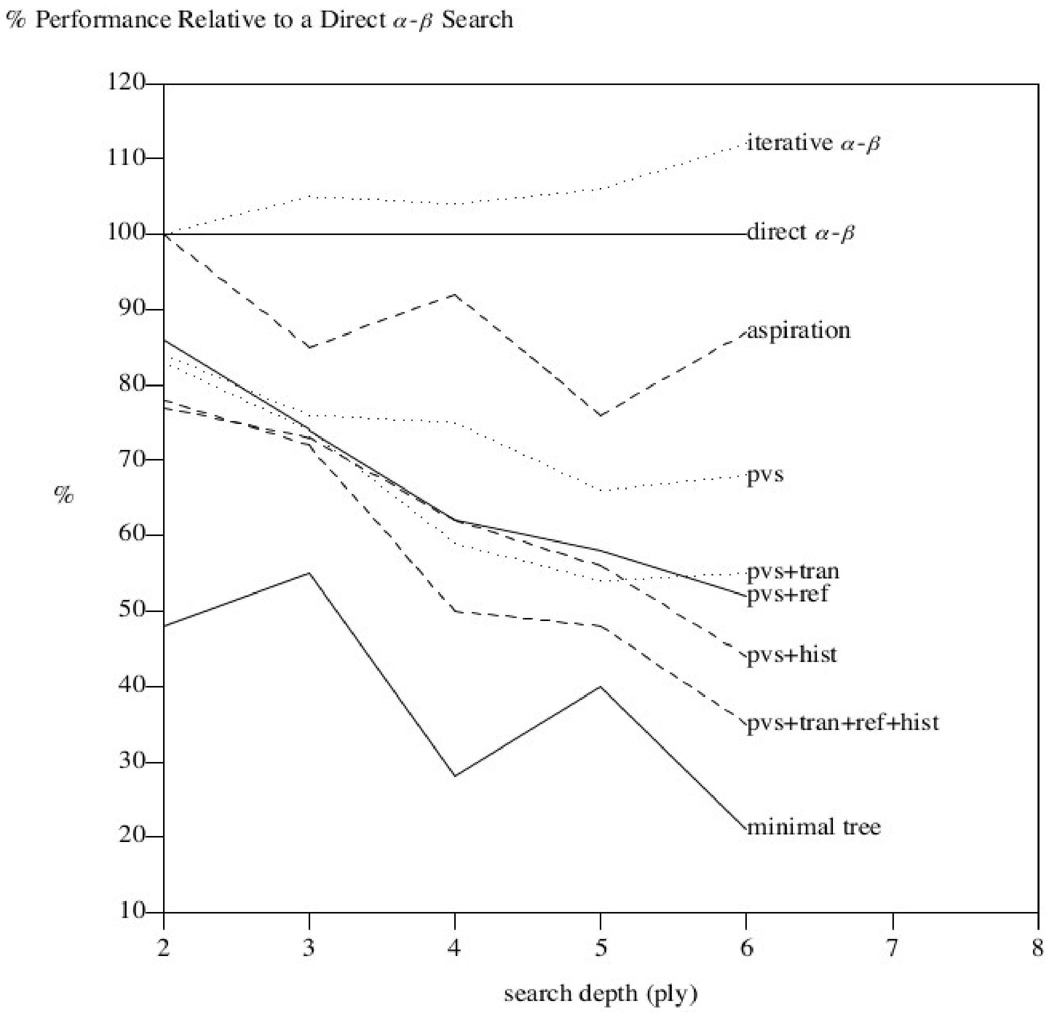


Figure 8: Time Comparison of Alpha-Beta Enhancements

MTD(f) [Plaat et al, 1996]

- PVS, NegaScout: full window search for move 1, null window searches for moves 2, 3, …
- Idea: Only null window searches $(\gamma,\gamma+1)$ that can check either score $<=\gamma$ or $>\gamma$. Compute minimal value by series of null window searches.
- Start with score in a previous iteration, then go up or down
- Perform better than PVS/NegaScout by a factor of 10%
- PVS/NegaScout are still used in practice because of instability of MTD(f)'s behavior

Search Extensions, Reductions, and Selective Search

- Ideas: Search promising moves deeper, unpromising ones less deep
- Avoid “horizon effect”
    - E.g. extend search for check, piece capture in chess
- Shape the search tree
- Both exact and heuristic methods
- Try to perform safe form of pruning in recent approaches
- Look at some of most important approaches

Example of Search Extensions and Reductions

- Quiescence search
- Null move pruning
- Futility pruning
- Late move reduction
- ProbCut
- Realization probability search
- Singular extension

Quiescence Search

- Hard to evaluate chaotic, unstable positions at leaf nodes
    - E.g., King in check, hanging pieces
- Idea: evaluate only “stable” positions
- Replace static evaluation by a small “quiescence search”
- Evaluate leaf nodes (stable positions) generated by quiescence search
- Highly restricted move generation – just resolve instability
    - E.g., generate check, piece exchange, and pass in chess/shogi

Null Move Pruning (1 / 2) [Beal, 1990][Donninger, 1993]

- Almost all searched paths contain at least one terrible move
- Idea: cut-off those subtrees quicker
- Null move: if we pass and can still get a search cut, then prune

Null Move Pruning (2 / 2)

- Assume *n* is examined with window (α, β) with depth *d*
  - Pass and reduce depth to *d-R* where *R* is a tuned value (large when remaining depth is large)
  - Perform null window search to check if returned score >= β or not (from current player's viewpoint)
  - If score >= β, perform cutoff – indication that opponent may have made a terrible move and *n* is unlikely to be in PV line
  - Otherwise, perform normal search
- Scenarios where null move pruning shouldn't be applied
  - E.g., positions in check, chess endgames (avoid Zugzwang)

Futility Pruning and its Extension [Schaeffer,1986][Heinz, 1998]

- Idea: discard moves that are unlikely to become best
- Performed at nodes close to leaf nodes e.g. remaining depth = 1 or 2
- Assume n is examined with window (α, β) with depth d
    - Prepare evaluation function eval0(m) that roughly calculates the score for move m and margin F – use larger F for deeper search
    - If eval0(m)+F <= α, prune m because m has almost no chance to be a good move
    - Otherwise, perform normal search
- Do not apply futility-pruning for tactical moves because they usually have high errors in eval0

Late Move Reduction (LMR)

- See http://chessprogramming.wikispaces.com/Late+Move+Reductions
- Similar to history pruning, history reductions, null window search for realization probability search
- Idea: in likely fail low nodes, reduce search depth of low-ranked moves
- Popular in some strong chess/shogi programs
- Assume n is examined with window (α, β)
    - Perform null window search with reduced depth to check if score <= α for move m ranked low in move ordering
    - If score <= α, cutoff, otherwise perform normal search

ProbCut [Buro 1995,2000]

- Observation: in many games, with good evaluation, search outcomes are highly correlated between different depths
- Reduce search depth for moves that are probably bad
- Yields more time to search more promising moves deeper
- Assume *n* is about to be examined with window (α, β)
  - Perform shallower search for move m and obtain score *sc*
  - Check if $a \times sc + b - \beta >= \Phi^{-1}(p) \times \sigma$, which indicates the real score for move m is >= β with probability *p*
  - Check analogously if real score for m is <= α with probability p
  - Up to two null window searches are performed

Search Performance of Pruning Techniques

C.f. Figure 5 in

[Hoki et al, 2012]

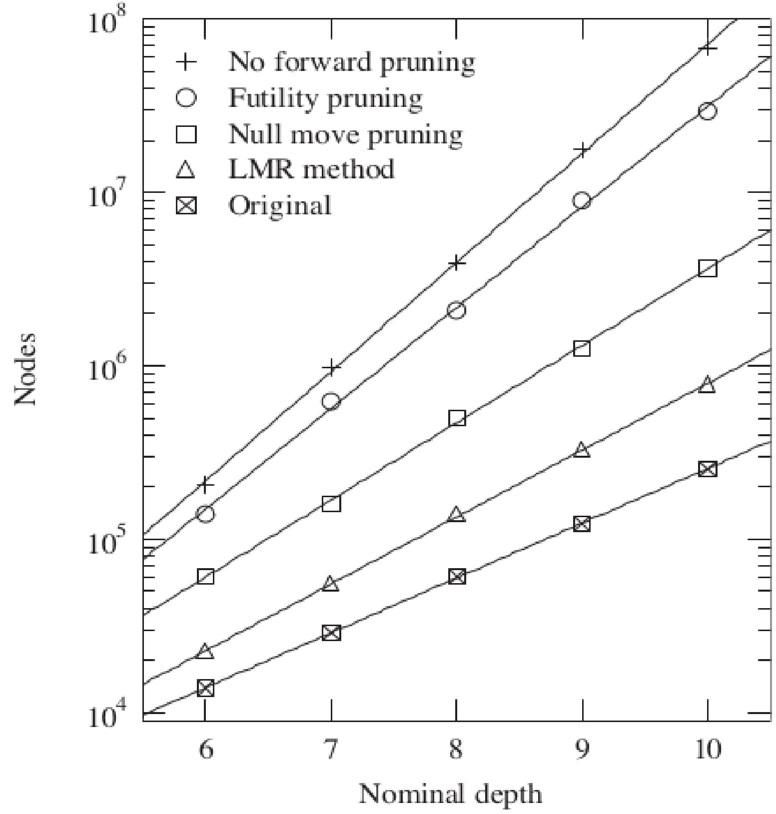


Fig. 5. The search–depth dependency of the number of nodes searched in chess. Crafty is used as a base program of this experiment.

Realization Probability Search [Tsuruoka et al, 2002]

- One example of fractional search depth extensions and reductions
- Define move categories, assign a fractional depth to each category
- Set fractional depth by estimating probability that next move is in specific category from master game records
- Need to avoid horizon effect caused by moves with large fractional depth
    - Perform null window search to check if score sc > current best score
    - Perform full window search with small fractional depth (i.e. deeper search) if sc > current best score

Singular Extension [Anantharaman et al, 1990]

- Observation: One move (singular move) that is much better than the others may have some pitfalls
- Idea: Extend the search for a singular move at (expected) PV and CUT nodes
- Idea can be extended to binary, trinary [Campbell et al, 2002]
- Whether a move is singular or not cannot be known beforehand
- Perform null window searches for non-singular moves with reduced search depths + lowered window values

Evaluation Functions

- Returns heuristic value that indicates probability of winning
- A lot of domain knowledge is added
    - E.g. piece values, material balance, mobility etc in chess
- Trade-off between knowledge and speed
- Most features are linear combination
    - eval(n) = W1 x F1(n) + W2 x F2(n) + … + Wk x Fk(n)

      W1,...,Wk are parameters and F1,..Fk are features
- Parameter tuning – by hand or machine learning
- This tutorial deals with one recent successful approach to tune parameters in shogi
- See references for other approaches e.g., [Buro, 1998]

Minimax Tree Optimization (MMTO) [Hoki and Kaneko, 2014]

- Earlier version known as “Bonanza method” [Hoki, 2006]
- Successful for tuning evaluation function with 40 million parameters in shogi
- All of strong computer shogi programs incorporate machine learning approaches influenced by this approach
- Assumption: grandmasters play good moves
- Idea: Prepare many game records of grandmasters and learn to increase the number of moves that match between alpha-beta and grandmasters

MMTO (Cont'd)

1. Find best ***w*** to maximize $J_{MMTO}^{P}=(w)=J(P,w)+J_C(w)+J_R(w)$

where $J(P,w)=\sum_{p\in P}\sum_{m\in M_p} T\left(s(p.d_p,w)-s(p.m,w)\right)$

$T(x)$: Sigmoid function

$s(p.m,w)$: minimax value for move m at position *p* identified by alpha-beta (use score at PV leaf in practice)

d_p : move played by grandmaster at position *p*

Mp : set of legal moves except d_p at position *p*

$J_C(w)$: constraint term

$J_R(w)$: l_1-regularization term

P : Set of positions

2. Use grid-adjacent update $w_i(t+1)=w_i(t)-h\cdot sgn\left(\frac{\partial J_{MMTO}^{P}(w(t))}{\partial w_i}\right)$

Other Issues on Alpha-Beta in Practice

- In some games, specialized search is invoked by main alpha-beta (previous lecture)
- E.g., in shogi, main alpha-beta cannot often find long sequence to mate player even with search extensions
- Specialized search called tsume-shogi solver with limited time/node expansions is used to avoid loss that results from main alpha-beta failing to find mating sequence
- Tsume-shogi solver cannot always be invoked because of its high overhead
- Typical computer shogi programs invoke tsume-shogi solver only at important lines
    - E.g., PV line, move that improves α value of window (α,β)

Parallel Alpha-Beta

- Known to be notoriously difficult to achieve reasonable parallel performance
- Parallel alpha-beta suffers from performance degradation caused by several types of overhead
    - Search overhead: extra nodes examined only by parallel alpha-beta
    - Synchronization overhead: idle time for other processors to finish work
    - Communication overhead: communication latency in the network
    - Load balance: metric on how evenly work is distributed

Young Brothers Wait Concept (YBWC) [Feldmann, 1993]

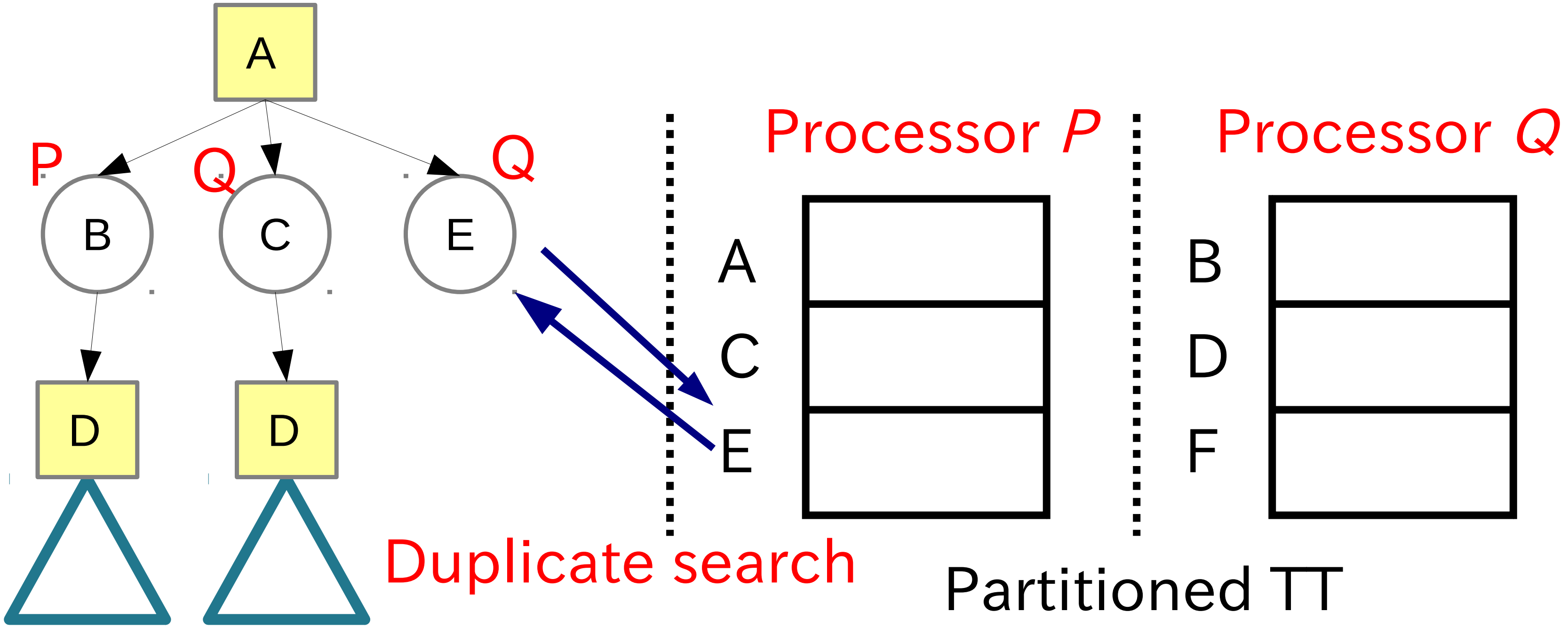
- Generalization to PVSplit [Marsland & Popowich, 1985] and many variants exist
- Observation: High-performance alpha-beta achieves good move ordering
  - First move to try has a high probability of causing cutoffs/narrowing windows at PV nodes
- Idea: recursively apply the rule that the “left-most” branch at a node must be examined before the others are examined
- Achieves reasonable parallelism with small search overhead
- Global synchronization point at each iteration – work starvation in the beginning and end of iterations

Issues in Distributed Memory Environments

- High-performance alpha-beta uses transposition tables
- Search space of many games are DAG or DCG
- Identical states can be reached via different paths
- Sequential alpha-beta effectively uses information saved in transposition table
- Shared-memory parallel alpha-beta can still share TT among threads
- How to effectively share TT in distributed memory environments?
- See approaches e.g. [Brockington & Schaeffer,2000][Feldmann, 1993][Romein, 2001][Kishimoto & Schaeffer, 2002]

Partitioned Transposition Table [Feldmann,1993]

- Each processor preserves part of TT disjointly
- Distribute work and use *work stealing* for load balance
- Ask corresponding processor for TT information
- Incur communication & synchronization overhead for TT accesses, and additional search overhead for DAG

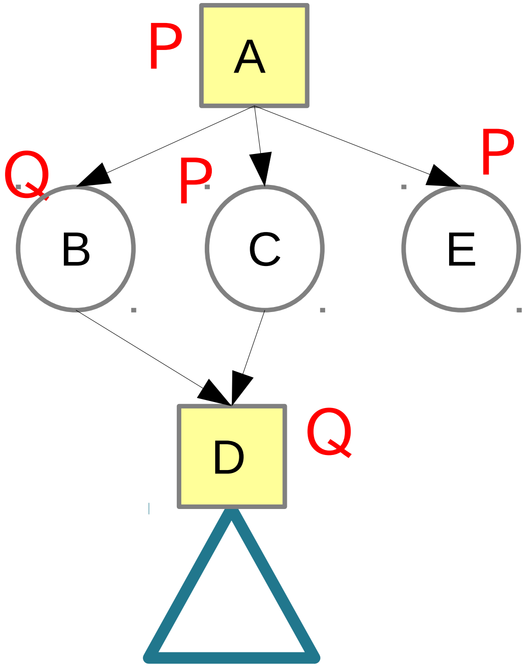


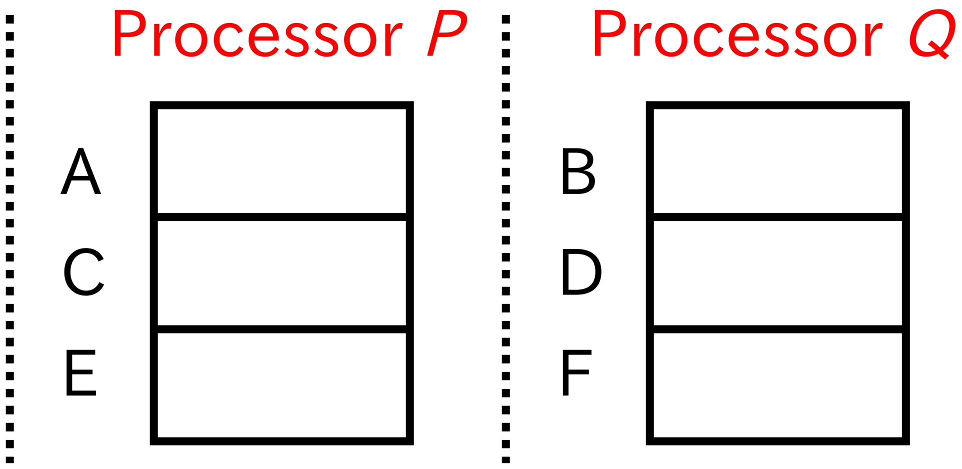
Duplicate search

Partitioned TT

TDSAB
[Kishimoto & Schaeffer, 2002]

- Apply Transposition-table driven scheduling (TDS) [Romein et al, 1999] to alpha-beta
- Can remove synchronization overhead to access TT and some search overhead for DAG
- See MCTS part as successful example of TDS





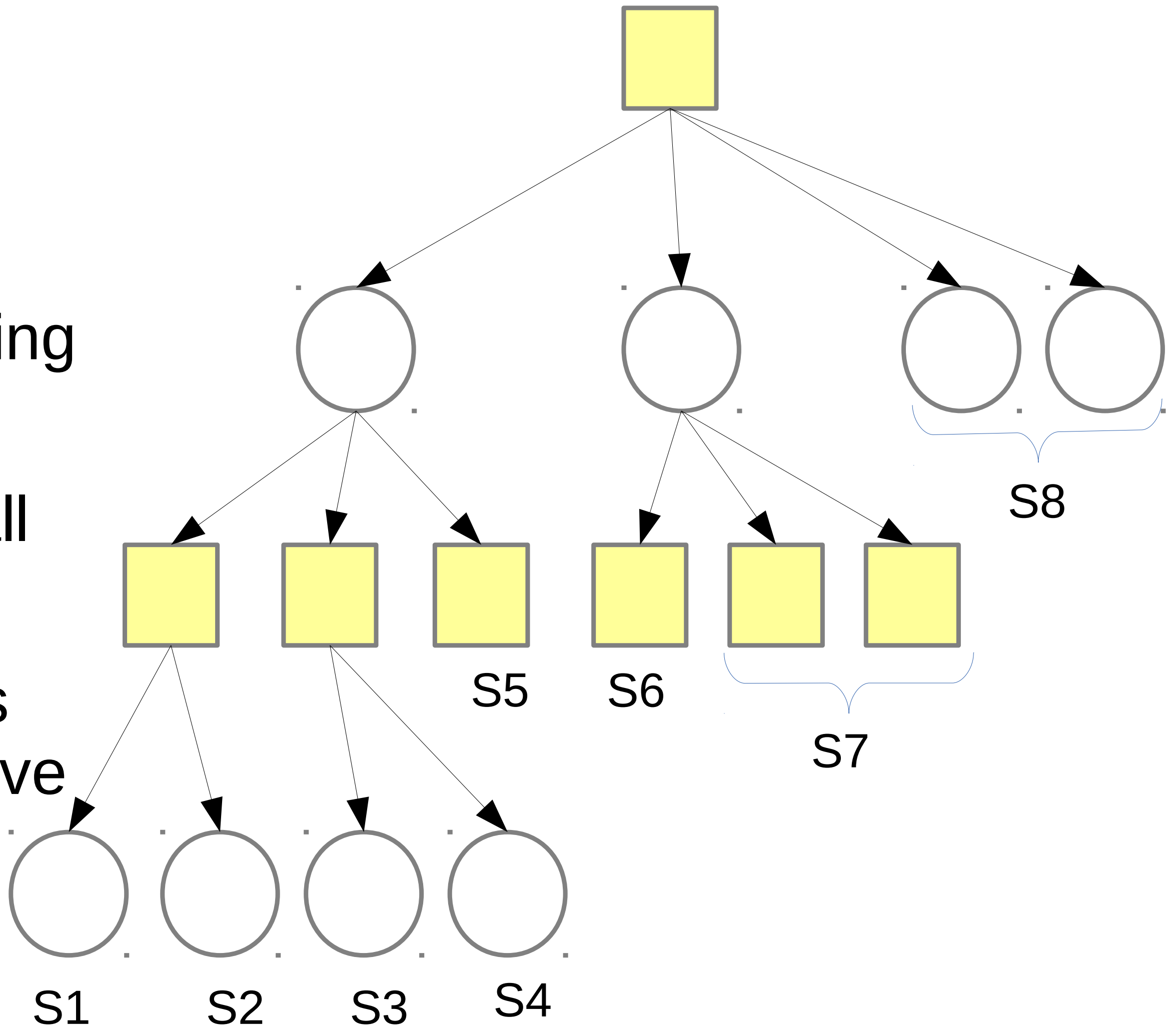
Partitioned TT

Massively Parallel Alpha-Beta in GPSShogi [Kaneko & Tanaka 2012,2013]

- Very recent method that might be less efficient but is much simpler than previous approaches
- Won against Miura (professional 8-dan player) with 679 computers (> 2700 cores, mostly iMac 2.5GHz)
- Uses one master and many slaves
  - Master manages a tree from root and generates work assigned to slaves
  - Slave independently examines states assigned by master
  - Master updates its tree when slave reports new scores

Master's Algorithm in GPSShogi

- Assign more slaves to promising subtrees
- Perform quick alpha-beta search to select k promising children (e.g., 1 sec)
- Repeat recursively until all slaves have work
- Effectively reuse master's tree when opponent's move matches predicted move [Himstedit 2012]



Comments on Alpha-Beta (1 / 2)

- Time: node evaluation, execute/undo moves, alpha-beta logic – low overhead
- Memory: depth-first search, need only path from root to current node – very low overhead
- Memory(2): can take advantage of extra use of transposition table
- Very good overall

Comments on Alpha-Beta (2 / 2)

- Evaluation function: must be reasonably accurate, trade-off between speed and accuracy
- Solving games/game positions
    - Fixed-depth search nature is a problem even with search extensions+fractional depth
    - Rules of repetition depends on rules, e.g. draw in chess, illegal in Go
    - Repetitions must be handled correctly
    - Practical “solutions” ignore history – leads to graph history interaction problem
    - Issues about repetitions are handled in the lectures in the afternoon

Conclusions

- Gave an overview of alpha-beta algorithms and enhancements
    - Alpha-beta variants
    - Search enhancements
    - Search extension and reductions
    - Evaluation function and machine learning
    - Parallel alpha-beta