From Deep Blue to Monte Carlo: An Update on Game Tree Research

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AAAI-14 Tutorial 5:
Monte Carlo Tree Search

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Tutorial 5 – MCTS - Contents

Part 1:

- Limitations of alphabeta and PNS
- Simulations as evaluation replacement
- Bandits, UCB and UCT
- Monte Carlo Tree Search (MCTS)

Tutorial 5 – MCTS - Contents

Part 2:

- MCTS enhancements: RAVE and prior knowledge
- Parallel MCTS
- Applications
- Research challenges, ongoing work

Go: a Failure for Alphabeta

- Game of Go
- Decades of Research on knowledge-based and alphabeta approaches
- Level weak to intermediate
- Alphabeta works much less well than in many other games
- Why?

Problems for Alphabeta in Go

- Reason usually given: Depth and width of game tree
  - 250 moves on average
  - game length > 200 moves
- **Real reason: Lack** of good **evaluation function**
  - Too subtle to model: very similar looking positions can have completely different outcome
  - Material is mostly irrelevant
    - Stones can remain on the board long after they "die"
  - Finding safe stones and estimating territories is hard

Monte Carlo Methods to the Rescue!

- Hugely successful
  - Backgammon (Tesauro 1995)
  - Go (many)
  - Amazons, Havannah, Lines of Action, ...
- Application to deterministic games pretty recent (less than 10 years)
- Explosion in interest, applications far beyond games
  - Planning, motion planning, optimization, finance, energy management,…

Brief History of Monte Carlo Methods

1940's – now	Popular in Physics, Economics, … to simulate complex systems
1990	(Abramson 1990) expected-outcome
1993	Brügmann, *Gobble*
2003 – 05	Bouzy, Monte Carlo experiments
2006	Coulom, *Crazy Stone*, **MCTS**
2006	(Kocsis & Szepesvari2006) **UCT**
2007 – now	*MoGo*, *Zen*, *Fuego*, many others
2012 – now	MCTS survey paper (Browne et al 2012); huge number of applications

Idea: Monte Carlo Simulation

- No evaluation function? No problem!
- Simulate rest of game using random moves (easy)
- Score the game at the end (easy)
- Use that as evaluation (hmm, **but**...)

The GIGO Principle

- **G**arbage **I**n, **G**arbage **O**ut
- Even the best algorithms do not work if the input data is bad
- How can we gain any information from playing random games?

Well, it Works!

- For many games, anyway
  - Go, NoGo, Lines of Action, Amazons, Konane, DisKonnect,…,…,…
- Even random moves often preserve *some* difference between a good position and a bad one
- The rest is statistics...
- ...well, not quite.

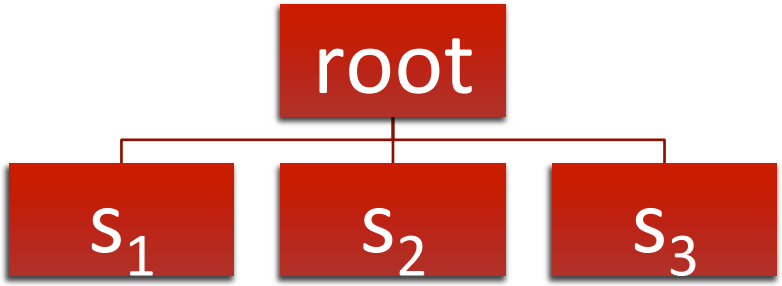
(Very) Basic Monte Carlo Search

- Play lots of random games
  - start with each possible legal move
- Keep winning statistics
  - Separately for each startingmove
- Keep going as long as you have time, then…
- Play move with best winning percentage

Simulation Example in NoGo

- Demo using *GoGui* and *BobNoGo* program
- Random legal moves
- End of game when *ToPlay* has no move (loss)
- Evaluate:
  +1 for win for current player
  0 for loss

Example – Basic Monte Carlo Search

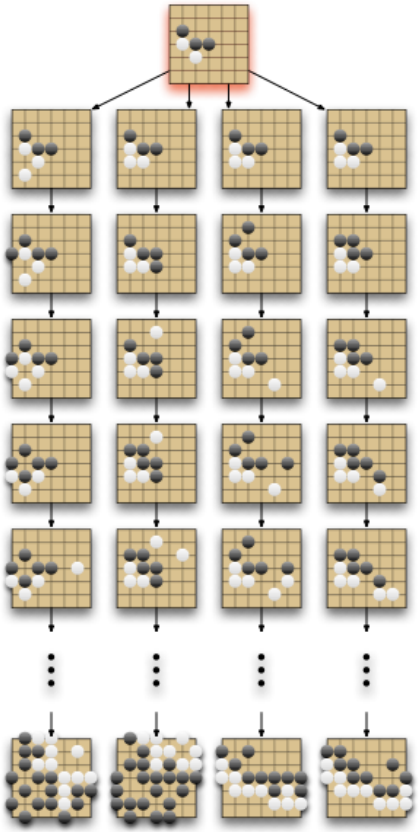


1 ply tree

root = current position

s_1 = state after move m_1

s_2 = …



state s_i

$V(m_i) = 2/4 = 0.5$

Simulations

1 1 0 0 *Outcomes*

Example for NoGo

- Demo for NoGo
- 1 ply search plus random simulations
- Show winning percentages for different first moves

Evaluation

- Surprisingly good e.g. in Go - much better than random or simple knowledge-based players
- Still limited
- Prefers moves that work “on average”
- Often these moves fail against the best response
- Likes “silly threats”

Improving the Monte Carlo Approach

- Add a game tree search (Monte Carlo Tree Search)
  - Major new game tree search algorithm
- Improved, better-than-random simulations
  - Mostly game-specific
- Add statistics over move quality
  - RAVE, AMAF
- Add knowledge in the game tree
  - human knowledge
  - machine-learnt knowledge

Add game tree search (Monte Carlo Tree Search)

- Naïve approach and why it fails
- Bandits and Bandit algorithms
  - Regret, exploration-exploitation, UCB algorithm
- Monte Carlo Tree Search
  - UCT algorithm

Naïve Approach

- Use simulations directly as an evaluation function for αβ
- Problems
  - Single simulation is very noisy, only 0/1 signal
  - running many simulations for one evaluation is very slow
  - Example:
    - typical speed of chess programs **1 million** eval/second
    - Go: 1 million moves/second, 400 moves/simulation, 100 simulations/eval = **25** eval/second
- Result: Monte Carlo was ignored for over 10 years in Go

Monte Carlo Tree Search

- Idea: use results of simulations to guide growth of the game tree
- **Exploitation**: focus on promising moves
- **Exploration**: focus on moves where uncertainty about evaluation is high
- Two contradictory goals?
  - Theory of *bandits* can help

Bandits

- Multi-armed bandits (slot machines in Casino)
- Assumptions:
  - Choice of several *arms*
  - each arm pull is independent of other pulls
  - Each arm has *fixed, unknown average payoff*
- Which arm has the best average payoff?
- Want to minimize *regret* = loss from playing non-optimal arm

Example (1)

- Three arms A, B, C
- Each pull of one arm is either
  - a win (payoff 1) or
  - a loss (payoff 0)
- Probability of win for each arm is fixed but *unknown*:
  - p(A wins) = 60%
  - p(B wins) = 55%
  - p(C wins) = 40%
- A is best arm (but we don't know that)

Example (2)

- How to find out which arm is best?
- The only thing we can do is play them
- Example:
  - Play A, win
  - Play B, loss
  - Play C, win
  - Play A, loss
  - Play B, loss
- Which arm is best ?????
- Play each arm many times
  - the empirical payoff will approach the (unknown) true payoff
- It is expensive to play bad arms too often
- How to choose which arm to pull in each round?

Applying the Bandit Model to Games

- Bandit arm ≈ move in game
- Payoff ≈ quality of move
- Regret ≈ difference to best move

Explore and Exploit with Bandits

- *Explore* all arms, but also:
- *Exploit*: play promising arms more often
- Minimize *regret* from playing poor arms

Formal Setting for Bandits

- One specific setting, more general ones exist
- K arms (actions, possible moves) named 1, 2, ..., K
- $t \geq 1$ *time steps*
- X_i random variable, payoff of arm i
  - Assumed *independent of time* here
  - Later: discussion of *drift* over time, i.e. with trees
- Assume $X_i \in [0...1]$ e.g. 0 = loss, 1 = win
- $\mu_i = E[X_i]$ expected payoff of arm i
- r_t *reward* at time t
  - realization of random variable X_i from playing arm i at time t

Formalization Example

- Same example as with A, B, C before, but use formal notation
- K=3 .. 3 arms, arm 1 = A, arm 2 = B, arm 3 = C
- X_1 = random variable – pull arm 1
  - X_1 = 1 with probability 0.6
  - X_1 = 0 with probability 1 - 0.6 = 0.4
  - similar for X_2, X_3
  - $\mu_1 = E[X_1] = 0.6$, $\mu_2 = E[X_2] = 0.55$, $\mu_3 = E[X_3] = 0.4$
- Each r_t is either 0 or 1, with probability given by the arm which was pulled.
  - Example: $r_1 = 0$, $r_2 = 0$, $r_3 = 1$, $r_4 = 1$, $r_5 = 0$, $r_6 = 1$, …

Formal Setting for Bandits (2)

- *Policy*: Strategy for choosing arm to play at time t
  - given arm selections and outcomes of previous trials at times 1, ..., $t - 1$.
- $I_t \in \{1,...,K\}$ .. arm selected at time t
- $T_i(t) = \sum_{s=1}^{t} \mathbb{I}(I_s = i)$
  .. total number of times arm i was played from time 1, ..., t

Example

- Example: $I_1 = 2, I_2 = 3, I_3 = 2, I_4 = 3, I_5 = 2, I_6 = 2$
- $T_1(6) = 0, T_2(6) = 4, T_3(6) = 2$
- Simple policies:
  - Uniform - play a least-played arm, break ties randomly
  - Greedy - play an arm with highest empirical playoff
  - Question – what is a *smart* strategy?

Formal Setting for Bandits (3)

- Best possible payoff: $\mu^* = max_{1 \leq i \leq K} \mu_i$
- Expected payoff after *n* steps: $\sum_{i=1}^{K} \mu_i \mathbb{E}[T_i(n)]$
- *Regret* after *n* steps is the difference:

$$n\mu^* - \sum_{i=1}^{K} \mu_i \mathbb{E}[T_i(n)]$$

- Minimize regret: minimize $T_i(n)$ for the non-optimal moves, especially the worst ones

Example, continued

- $\mu_1 = 0.6$, $\mu_2 = 0.55$, $\mu_3 = 0.4$
- $\mu^* = 0.6$
- With our fixed exploration policy from before:
  - $E[T_1(6)] = 0$, $E[T_2(6)] = 4$, $E[T_3(6)] = 2$
  - expected payoff $\mu_1 * 0 + \mu_2 * 4 + \mu_3 * 2 = 3.0$
  - expected payoff if always plays arm 1: $\mu^* * 6 = 3.6$
  - Regret = 3.6 – 3.0 = 0.6
- Important: regret of a policy is expected regret
  - Will be achieved in the limit, as average of many repetitions of this experiment
  - In any single experiment with six rounds, the payoff can be anything from 0 to 6, with varying probabilities

Formal Setting for Bandits (4)

- (Auer et al 2002)
- Statistics on each arm so far
- $\bar{x}_i$ average reward from arm *i* so far
- n_i number of times arm *i* played so far (same meaning as $T_i(t)$ above)
- *n* total number of trials so far

UCB1 Formula (Auer et al 2002)

- Name UCB stands for Upper Confidence Bound
- Policy:

1. First, try each arm once
2. Then, at each time step:
   - choose arm *i* that maximizes the *UCB1 formula* for the upper confidence bound:

$$\bar{x}_i + \sqrt{\frac{2\ln(n)}{n_i}}$$

UCB Demystified - Formula

$$\bar{x}_i + \sqrt{\frac{2\ln(n)}{n_i}}$$

- Exploitation: higher observed reward $\bar{x}_i$ is better
- Expect "true value" μ_i to be in some *confidence interval* around $\bar{x}_i$.
- "Optimism in face of uncertainty":
  choose move for which the upper bound of confidence interval is highest

UCB Demystified – Exploration Term

$$\bar{x}_i + \sqrt{\frac{2\ln(n)}{n_i}}$$

- Interval is large when number of trials n_i is small. Interval shrinks in proportion to $\sqrt{n_i}$
- High uncertainty about move
  - large exploration term in UCB formula
  - move is explored
- $\sqrt{\ln(n)}$ term, intuition:
  explore children more if parent is important (has many simulations)

Theoretical Properties of UCB1

- Main question: rate of convergence to optimal arm
- Huge amount of literature on different bandit algorithms and their properties
- Typical goal: regret O(log n) for n trials
- For many kinds of problems, cannot do better asymptotically (Lai and Robbins 1985)
- UCB1 is a simple algorithm that achieves this asymptotic bound for many input distributions

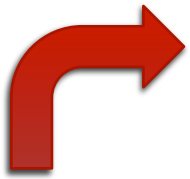
Is UCB What we Really Want???

- No.
- UCB minimizes *cumulative* regret
- Regret is accumulated over all trials
- In games, we only care about the final move choice
  - We do not care about simulating bad moves
- *Simple regret*: loss of our final move choice, compared to best move
  - Better measure, but theory is much less developed for trees

The case of Trees: From UCB to UCT

- UCB makes a single decision
- What about sequences of decisions (e.g. planning, games)?
- Answer: use a lookahead tree (as in games)
- Scenarios
  - Single-agent (planning, all actions controlled)
  - **Adversarial** (as in games, or worst-case analysis)
  - Probabilistic (average case, "neutral" environment)

Our Focus



Monte Carlo Planning - UCT

- Main ideas:
- Build lookahead tree (e.g. game tree)
- Use rollouts (simulations) to generate rewards
- Apply UCB – like formula in interior nodes of tree
  - choose “optimistically” where to expand next

Generic Monte Carlo Planning Algorithm

MonteCarloPlanning(state)
repeat search(state, 0) until Timeout
return bestAction(state,0)

search(state, depth)
if Terminal(state) then return 0
if Leaf(state, depth) then return Evaluate(state)
action := selectAction(state, depth)
(nextstate, reward) := simulate (state, action)
q := reward + γ search(nextstate, depth + 1)
UpdateValue(state, action, q, depth)
return q

- Reinforcement-learning-like framework (Kocsis and Szepesvari 2006)
- Rewards at every time step
  - future rewards discounted by factor γ
- Apply to games:
  - 0/1 reward, only at end of game
  - γ = 1 (no discount)

Generic Monte Carlo Tree Search

- *Select* leaf node L in game tree
- *Expand* children of L
- *Simulate* a randomized game from (new) leaf node
- *Update* (or backpropagate) statistics on path to root

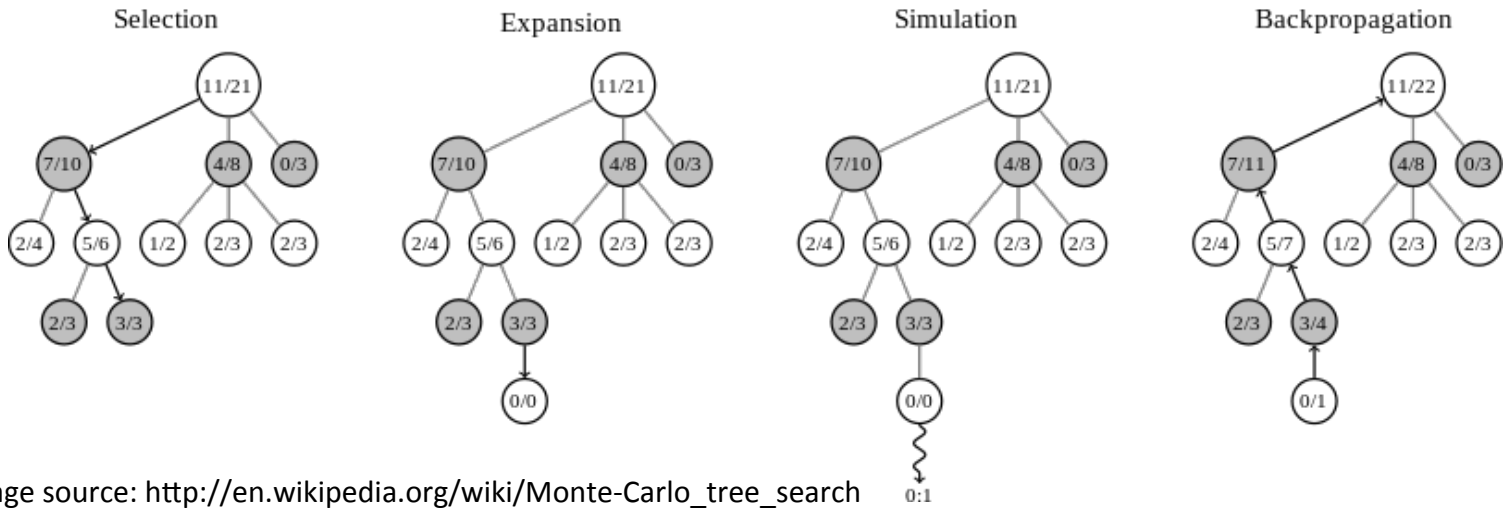


Image source: http://en.wikipedia.org/wiki/Monte-Carlo_tree_search

Drift

- In basic bandit framework, we assumed that payoff for each arm comes from a *fixed* (stationary) distribution
- If distribution changes over time, UCB will still converge under some relatively weak conditions
- In UCT, the tree changes over time
  - payoffs of choices within tree also change
  - Example: better move is discovered for one of the players

Convergence Property of UCT

- Very informal presentation here.
  See (K+S 2006), Section 2.4 for precise statements.
- Assumptions:
  1. average payoffs converge for each arm *I*
  2. "tail inequalities": probability of being "far off" is very small
- Under those conditions:
  probability of selecting a suboptimal move approaches zero in the limit

Towards Practice: UCB1-tuned

- Finite-time Analysis of the Multiarmed Bandit Problem (Auer et al 2002)
- UCB1 formula simply assumes variance decreases with 1/sqrt of number of trials n_i
- UCB1-tuned idea: take *measured variance* of each arm (move choice) into account
- Compute upper confidence bound using that measured variance
  - Can be better in practice
- We will see many more extensions to UCB ideas

MoGo – First UCT Go Program

- Original MoGo technical report (Gelly et al 2006)
- Modify UCB1-tuned, add two parameters:
  - *First-play urgency* - value for unplayed move
  - *exploration constant c* (called *p* in first paper) - controls rate of exploration
    p = 1.2 found best empirically for early MoGo

$$\bar{X}_j + p\sqrt{\frac{\log n}{T_j(n)}\min\{1/4, V_j(n_j)\}}$$

Formula from original MoGo report

Move Selection for UCT

- Scenario:
  - run UCT as long as we can
  - run simulations, grow tree
- When out of time, which move to play?
  - Highest mean
  - Highest UCB
  - **Most-simulated move**
    - later refinement: most wins

Summary – MCTS So Far

- UCB, UCT are very important algorithms in both theory and practice
- Well founded, convergence guarantees under relatively weak conditions
- Basis for extremely successful programs for games and many other applications

MCTS Enhancements

- Improved simulations
  - Mostly game-specific
  - We will discuss it later
- Improved in-tree child selection
  - General approaches
  - Review – the history heuristic
  - AMAF and RAVE
- Prior knowledge for initializing nodes in tree

Improved In-Tree Child Selection

- Plain UCT: in-tree child selection by UCB formula
  - Components: exploitation term (mean) and exploration term
- Enhancements: modify formula, add other terms
  - Collect other kinds of statistics – AMAF, RAVE
  - Prior knowledge – game specific evaluation terms
- Two main approaches
  - Add another term
  - “Equivalent experience” – translate knowledge into (virtual, fake) simulation wins or losses

Review - History Heuristic

- Game-independent enhancement for alphabeta
- Goal: improve move ordering
  (Schaeffer 1983, 1989)
- Give bonus for moves that lead to cutoff
  Prefer those moves at other places in the search
- Similar ideas in MCTS:
  - all-moves-as-first (AMAF) heuristic, RAVE

Assumptions of History Heuristic

- Abstract concept of *move*
  - Not just a single edge in the game graph
  - identify *class of all moves* e.g. “Black F3” - place stone of given color on given square
- History heuristic: quality of such moves is correlated
  - tries to exploit that correlation
  - Special case of reasoning by similarity: in similar state, the same action may also be good
    - Classical: if move often lead to a beta cut in search, try it again, might lead to similar cutoff in similar position.
    - MCTS: if move helped to win previous simulations, then give it a bonus for its evaluation - will lead to more exploration of the move

All Moves As First (AMAF) Heuristic

- (Brügmann 1993)
- Plain Monte Carlo search:
  - no game tree, only simulations, winrate statistics for each first move
- AMAF idea: bonus for *all* moves in a winning simulation, not just the first.
  - Treat all moves like the first
  - Statistics in *global table, separate* from winrate
- Main advantage: statistics accumulate much faster
- Disadvantage: some moves good only if played right now - they will get a very bad AMAF score.

RAVE - Rapid Action Value Estimate

- Idea (Gelly and Silver 2007): compute separate AMAF statistics in *each node* of the MCTS tree
- After each simulation, update the RAVE scores of all ancestors that are in the tree
- Each move *i* in the tree now also has a RAVE score:
  - number of simulations $n_{i,RAVE}$
  - number of wins $v_{i,RAVE}$
  - *RAVE value* $x_{i,RAVE} = v_{i,RAVE}/n_{i,RAVE}$

RAVE Illustration

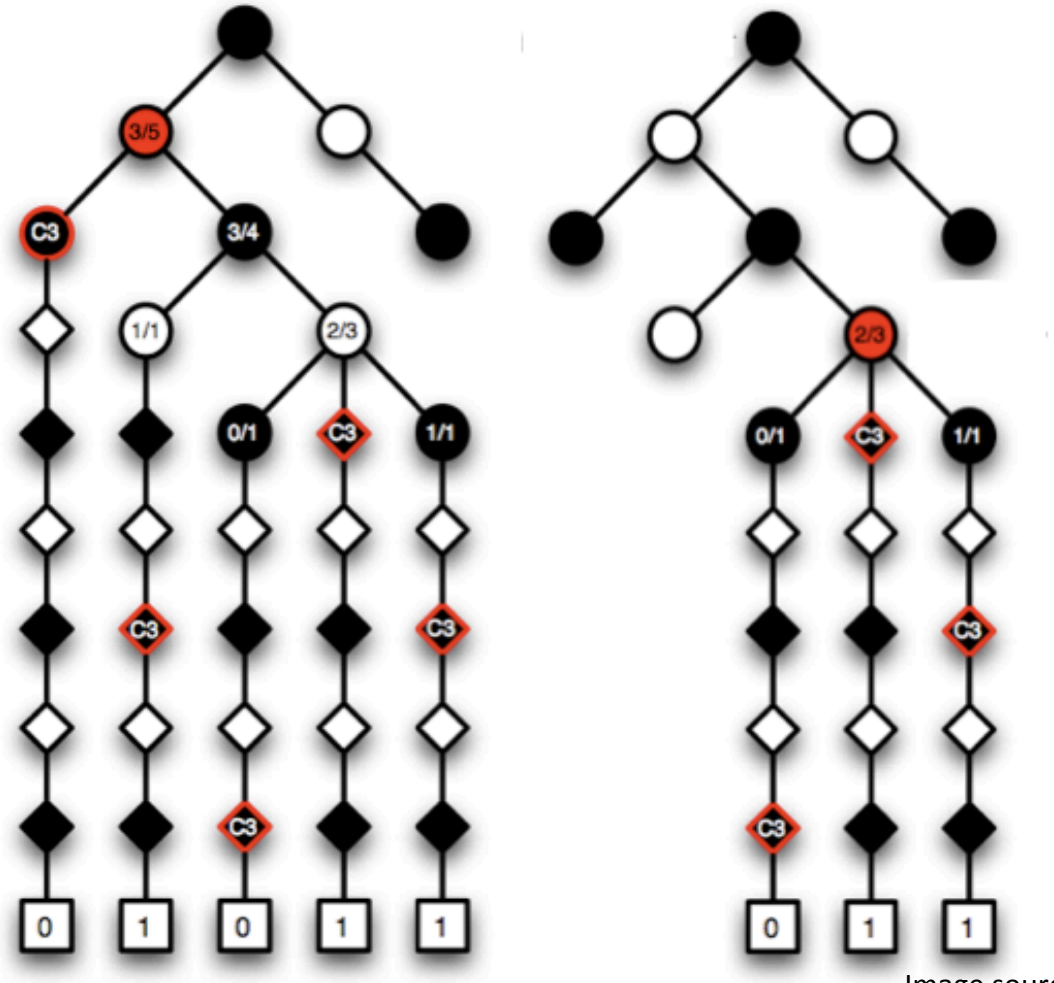


Image source: (Silver 2009)

Adding RAVE to the UCB Formula

- Basic idea: replace mean value x_i with weighted combination of mean value and RAVE value

$$\beta\, x_i + (1 - \beta)\, x_{i,RAVE}$$

- How to choose β?
  Not constant, depends on all statistics
- Try to find best combined estimator given x_i and $x_{i,RAVE}$

Adding RAVE (2)

- Original method in MoGo (Gelly and Silver 2007):
  - *equivalence parameter k* = number of simulations when mean and RAVE have equal weight
    - When $n_i = k$, then $\beta = 0.5$
  - Results were quite stable for wide range of *k=50…10000*
- Formula

$$\beta(s,a) = \sqrt{\frac{k}{3n(s)+k}}$$

Adding RAVE (3)

- (Silver 2009, Chapter 8.4.3)
  - Assume independence of estimates
    - not true in real life, but useful assumption
  - Can compute optimal choice in closed form (!)
  - Estimated by machine learning, or trial and error

Adding RAVE (4) – Fuego Program

- General scheme to combine different estimators
  - Combining mean and RAVE is special case
    - Very similar to Silver's scheme
- General scheme: each estimator has:
  1. *initial slope*
  2. *final asymptotic value*
  - Details: http: //fuego.sourceforge.net/fuego-doc-1.1/ smartgame-doc/sguctsearchweights.html

Using Prior Knowledge

- (Gelly and Silver 2007)
- Most nodes in the game tree are leaf nodes (exponential growth)
- Almost no statistics for leaf nodes - only simulated once
- Use domain-specific knowledge to initialize nodes
  - “equivalent experience” - a number of wins and losses
  - additive term (Rosin 2011)
- Similar to heuristic initialization in proof-number search

Types of Prior Knowledge

- (Silver 2009) machine-learned 3x3 pattern values
- Later Mogo and Fuego: hand-crafted features
- Crazy Stone: many features, weights trained by Minorization-Maximization (MM) algorithm (Coulom 2007)
- Fuego today:
  - large number of simple features
  - weights and interaction weights trained by *Latent Feature Ranking* (Wistuba et al 2013)

Example – Pattern Features (Coulom)

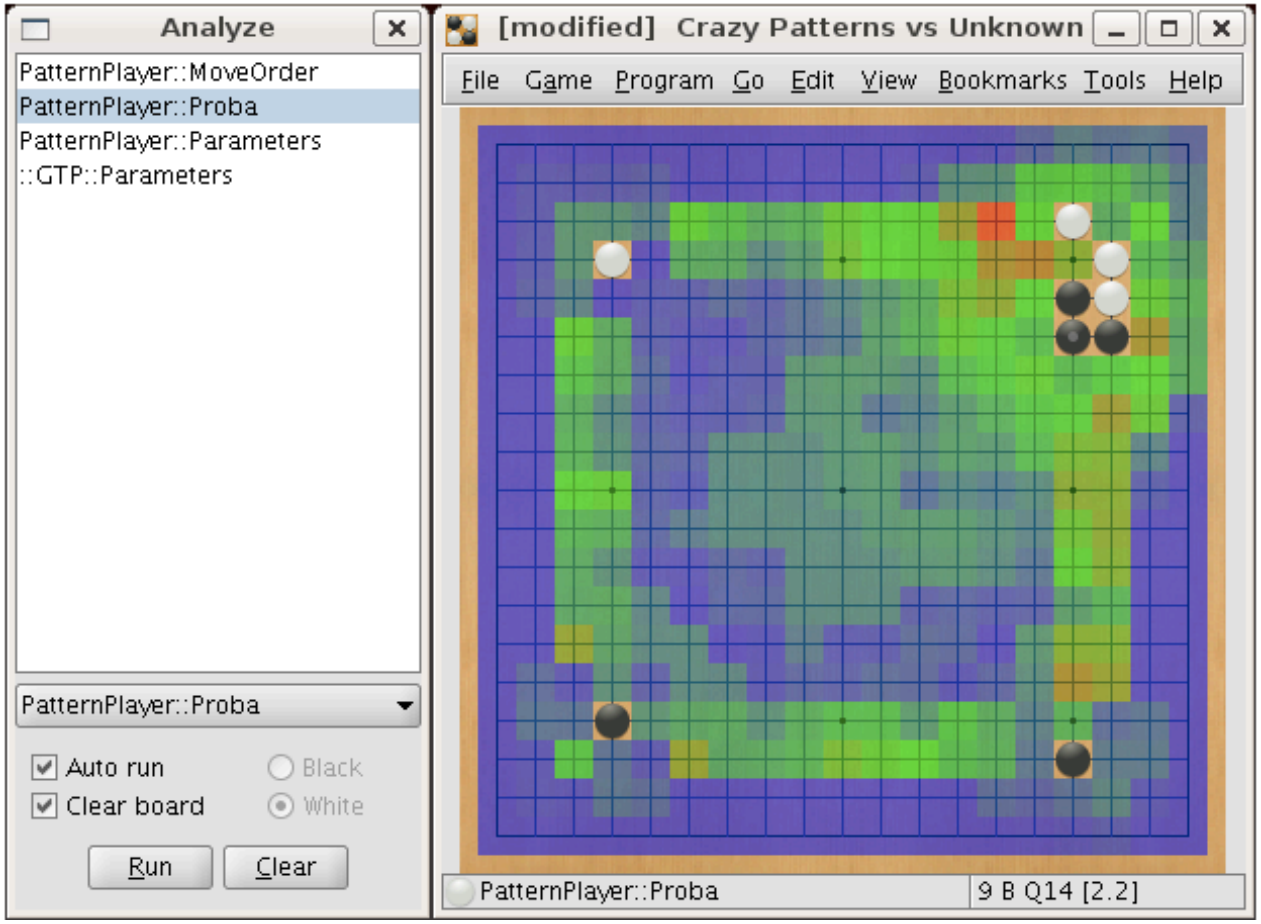


Image source: Remi Coulom

Improving Simulations

- Goal: strong correlation between initial position and result of simulation
- Preserve wins and losses
- How?
  - Avoid blunders
  - “Stabilize” position
    - Go: prefer local replies
    - Go: urgent pattern replies

Improving Simulations (2)

- Game-independent techniques
  - If there is an immediate win,
    then take it (1 ply win check)
  - Avoid immediate losses in simulation
    (1 ply mate check)
  - Avoid moves that give opponent an immediate win
    (2 play mate check)
  - Last Good Reply – next slide

Last Good Reply

- Last Good Reply (Drake 2009),
  Last Good Reply with Forgetting (Baier et al 2010)
- Idea: after winning simulation, store (opponent move, our answer) move pairs
  - Try same reply in future simulations
  - Forgetting: delete move pair if it fails
- Evaluation: worked well for Go program with simpler playout policy (Orego)
  - Trouble reproducing success with stronger Go programs
- Simple form of adaptive simulations

Hybrid Approaches

- Combine MCTS with “older” ideas from the alphabeta world
- Examples
  - Prove wins/losses
  - Use evaluation function
  - Hybrid search strategy MCTS+alphabeta

Hybrids: MCTS + Game Solver

- Recognize leaf nodes that are wins/losses
- Backup in minimax/proof tree fashion
- Problem: how to adapt child selection if some children are proven wins or losses?
  - At least, don't expand those anymore
- Useful in many games, e.g. Hex, Lines of Action, NoGo, Havannah, Konane,…

Hybrids: MCTS + Evaluation

- Use evaluation function
  - Standard MCTS plays until end of game
  - Some games have reasonable and fast evaluation functions, but can still profit from exploration
    - Examples: Amazons, Lines of Action
- Hybrid approach (Lorentz 2008, Winands et al 2010)
  - run short simulation for fixed number of moves (e.g. 5-6 in Amazons)
  - call static evaluation at end, use as simulation result

Hybrids: MCTS + Minimax

- 1-2 ply lookahead in playouts (discussed before)
  - Require strong evaluation function
- (Baier and Winands 2013) add minimax with no evaluation function to MCTS
  - Playouts
    - Avoid forced losses
  - Selection/Expansion
    - Find shallow wins/losses

Towards a Tournament-Level Program

- Early search termination – best move cannot change
- Pondering – think in opponent's time
- Time control – how much time to spend for each move
- Reuse sub-tree from previous search
- Multithreading (see later)
- Code optimization
- Testing, testing, testing,…

Machine Learning for MCTS

- Learn better knowledge
  - Patterns, features (discussed before)
- Learn better simulation policies
  - Simulation balancing (Silver and Tesauro 2009)
  - Simulation balancing in practice (Huang et al 2011)
- Adapt simulations online
  - Dyna2, RLGo (Silver et al 2012)
  - Nested Rollout Policy Adaptation (Rosin 2011)
  - Last Good Reply (discussed before)
  - Use RAVE (Rimmel et al 2011)

Parallel MCTS

- MCTS scales well with more computation
- Currently, hardware is moving quickly towards more parallelism
- MCTS simulations are “embarassingly parallel”
- Growing the tree is a sequential algorithm
  - How to parallelize it?

Parallel MCTS - Approaches

- root parallelism
- shared memory
- distributed memory

- New algorithm: depth-first UCT (Yoshizoe et al 2011)
  - Avoid bottleneck of updates to the root

Root Parallelism

- (Cazenave and Jouandeau 2007, Soejima et al. 2010)
- Run *n* independent MCTS searches on *n* nodes
- Add up the top-level statistics
- Easiest to implement, but limited
- Majority vote may be better

Shared Memory Parallelism

- *n* cores together build one tree in shared memory
- How to synchronize access? Need to write results (changes to statistics for mean and RAVE), add nodes, and read statistics for in-tree move selection
- Simplest approach: lock tree during each change
- Better: lock-free hash table (Coulom2008) or tree (Enzenberger and Müller 2010)
- Possible to use spinlock

Limits to Parallelism

- Loss of information from running *n* simulations in parallel as opposed to sequentially
- Experiment (Segal 2010)
  - run single-threaded
  - delay tree updates by $n - 1$ simulations
- Best-case experiment for behavior of parallel MCTS
- Predicts upper limit of strength over 4000 Elo above single-threaded performance

Virtual Loss

- Record simulation as a loss at start
  - Leads to more variety in UCT-like child selection
- Change to a win if outcome is a win
- Crucial technique for scaling
- With virtual loss, scales well up to 64 threads
- Can also use *virtual wins*

Fuego Virtual Loss Experiment

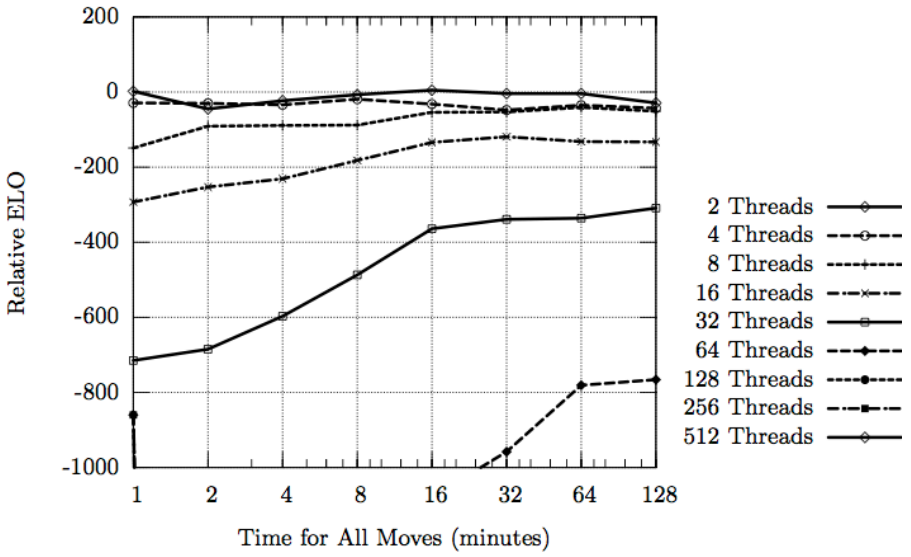


Fig. 2. Self-play of N threads against a uni-processor with equal total computation.

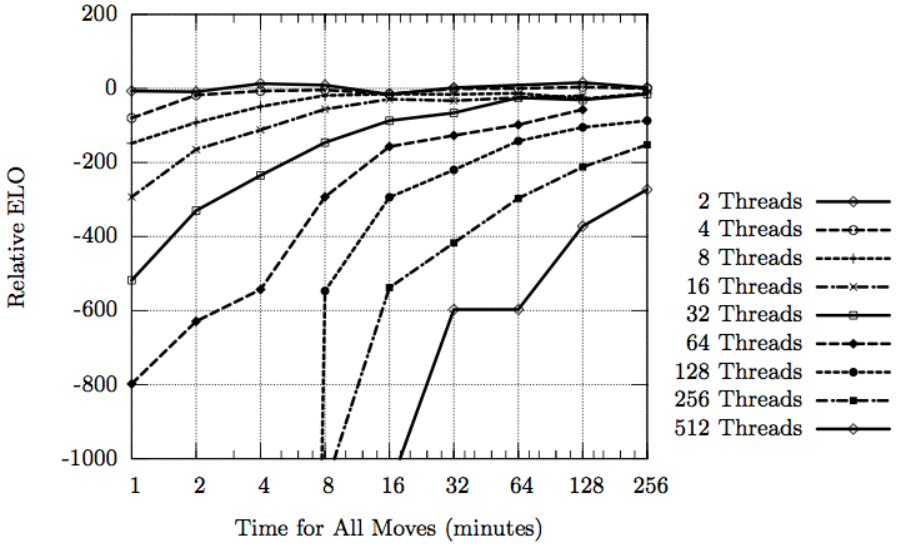


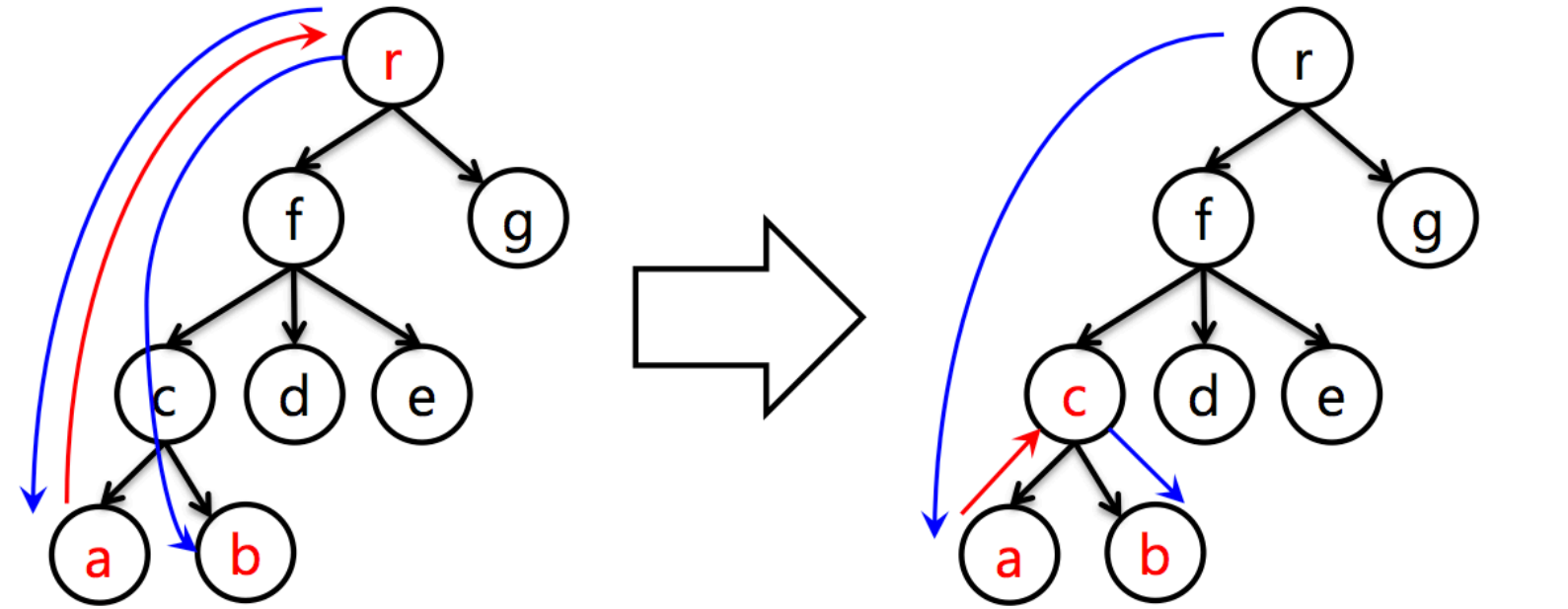
Fig. 4. Self-play of N threads against a uni-processor and virtual loss enabled.

Image source: (Segal 2010)

Distributed Memory Parallelism

- Many copies of MCTS engine, one on each compute node
- Communicate by message passing (MPI)
- MoGo model:
  - synchronize a few times per second
  - synchronize only “heavy” nodes which have many simulations
- Performance depends on
  - hardware for communication
  - shape of tree
  - game-specific properties, length of playouts

Normal UCT vs. Depth-first UCT



Normal UCT

always return to root

Depth First UCT

returns only if needed

Image source: K. Yoshizoe

Depth-first UCT

- Bottleneck of updates to “heavy” nodes including root
- Depth-first reformulation of UCT
  - stay in subtree while best-child selection is unlikely to change
    - about 1 - 2% wrong child selections
  - Delay updates further up the tree
  - Similar idea as df-pn
  - Unlike df-pn, sometimes the 3rd-best (or worse) child can become best

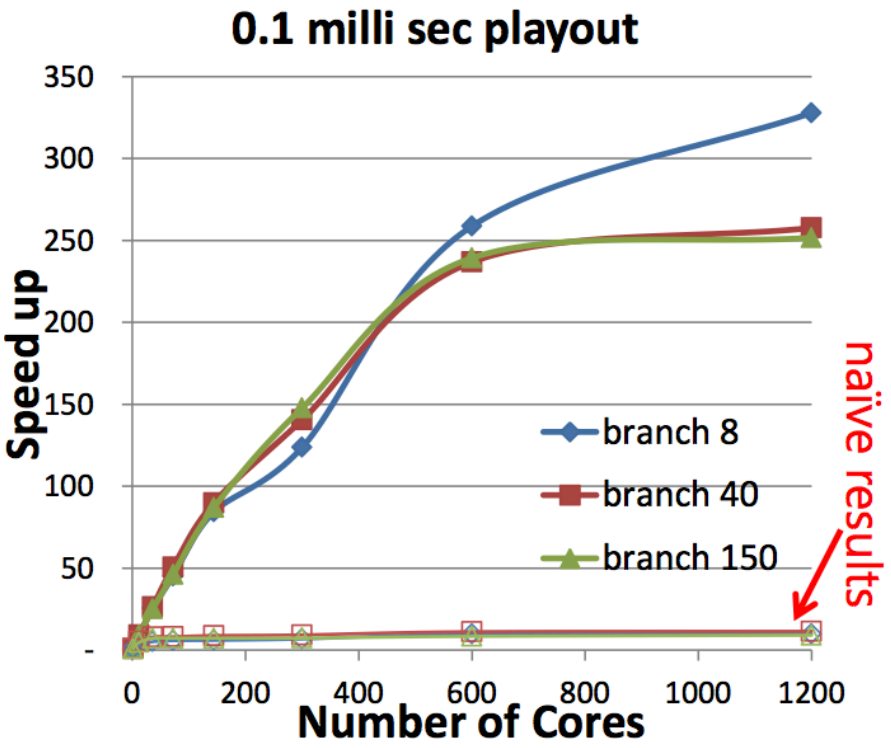
Distributed Memory: TDS

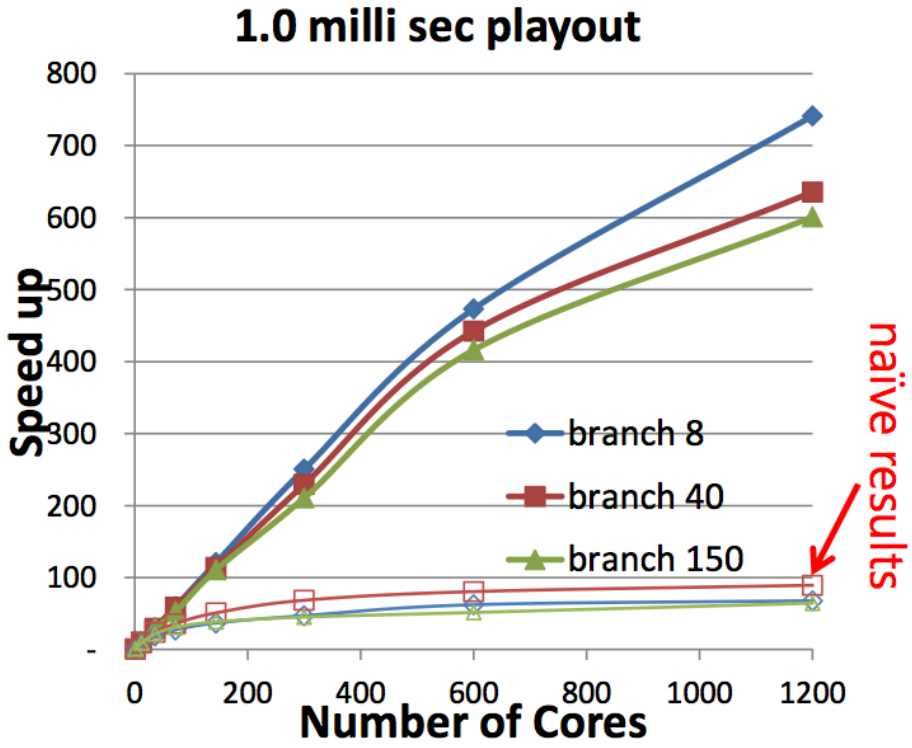
- TDS – Transposition Table Driven Scheduling (Romein et al 1999)
- Single global hash table
  - Each node in tree owned by one processor
  - Work is sent to the processor that owns the node
  - In single-agent search, achieved almost perfect speedup on mid-size parallel machines

TDS-df-UCT

- Use TDS approach to implement df-UCT on (massively) parallel machines
  - TSUBAME2 (17984 cores)
  - SGI UV-1000 (2048 cores)
- Implemented artificial game (P-game) and Go (MP-Fuego program)
  - In P-game: measure effect of playout speed (artificial slowdown for fake simulations)

TDS-df-UCT Speedup - 1200 Cores

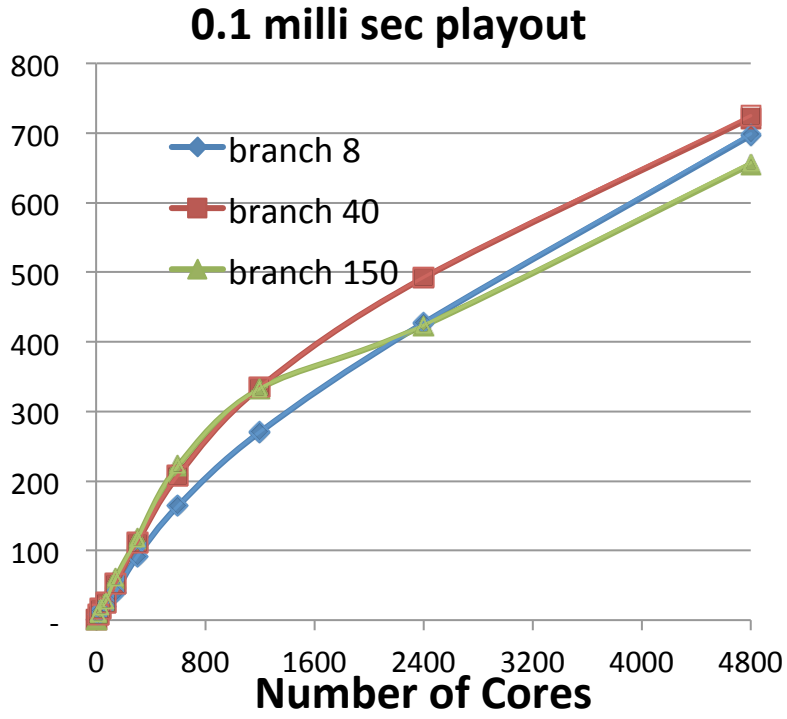


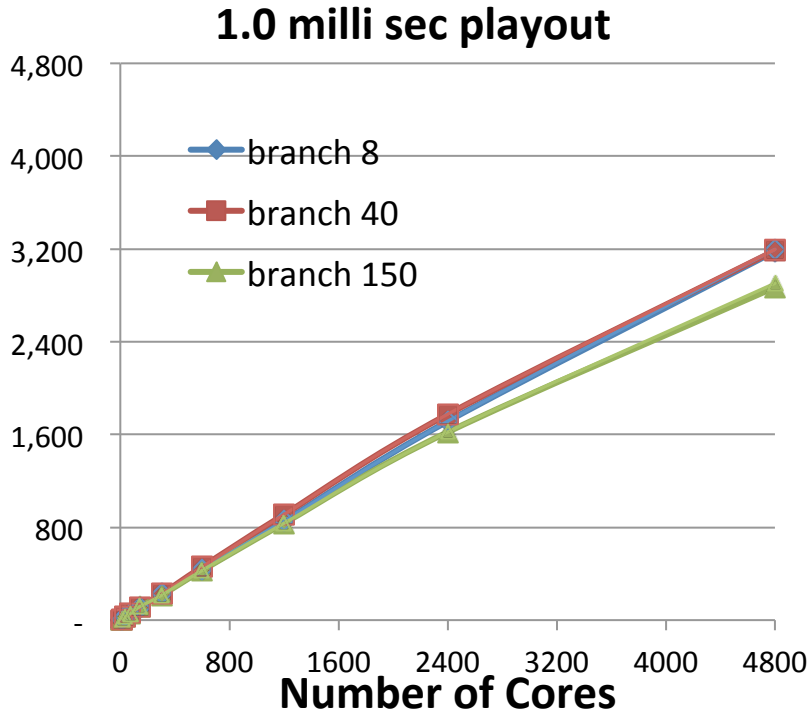


330 fold speedup for 0.1 ms playout
740 fold speedup for 1.0 ms playout

Image source: K. Yoshizoe

P-game 4,800 Cores





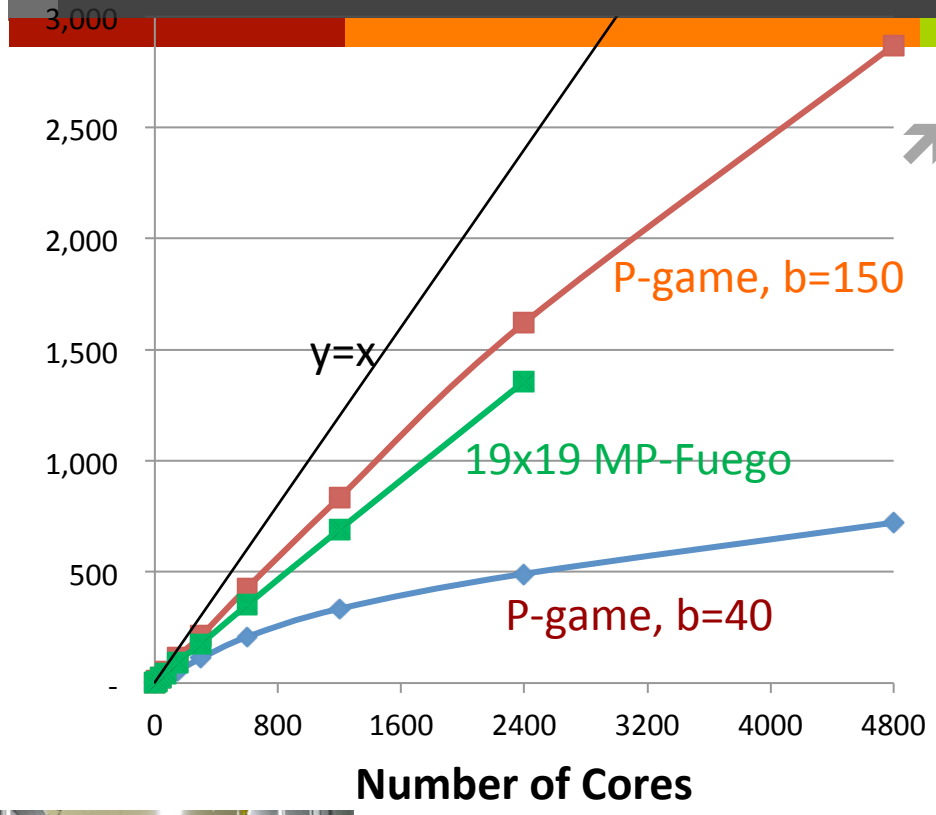
job number
= cores x 10

700-fold for 0.1 ms playout
3,200-fold for 1.0 ms playout

Image source: K. Yoshizoe

TDS-df-UCT = TDS + depth first UCT

Speedup including Go



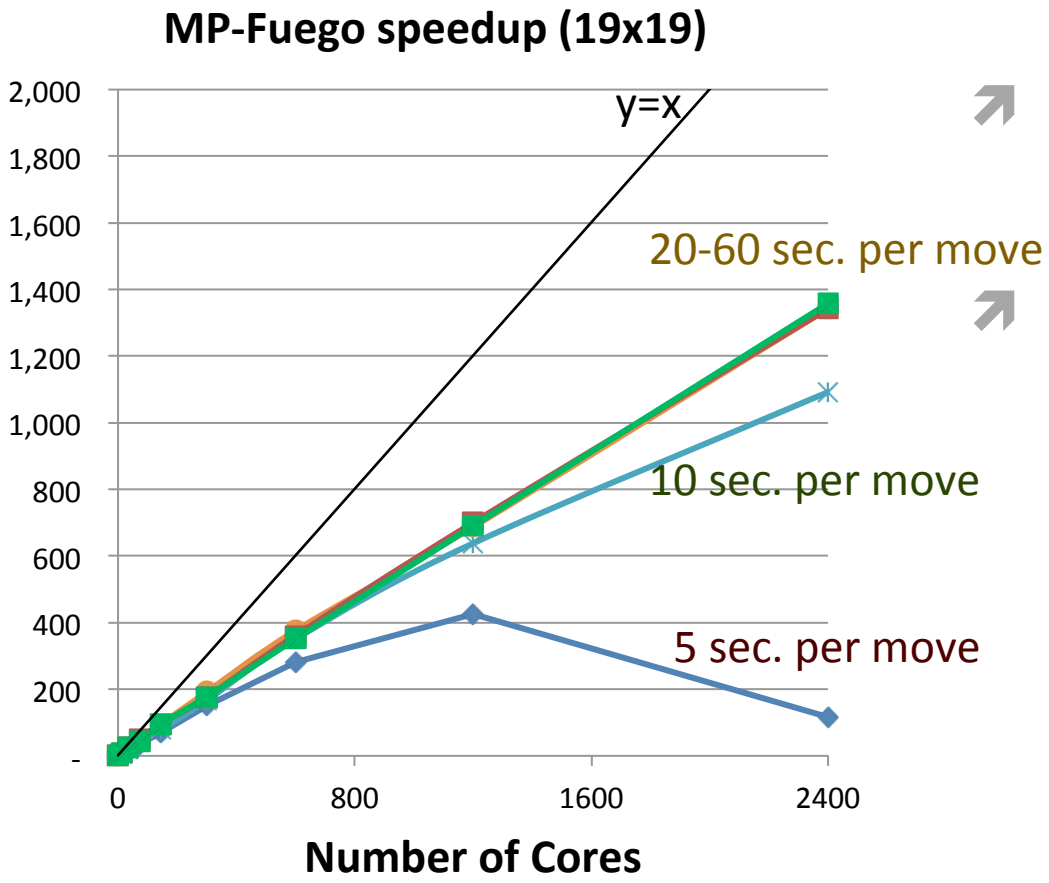
- MP-Fuego
  - 2 playouts at leaf
  - (approx. 0.8 ms playout)
  - 5 jobs/core

Hardware1: TSUBAME2 supercomputer

Hardware2: SGI UV1000 (Hungabee)

Image source: K. Yoshizoe

Search Time and Speedup



- Short thinking time = slower speedup
- One major difficulty in massive parallel search

Image source: K. Yoshizoe

Summary – MCTS Tutorial so far…

- Reviewed algorithms, enhancements, applications
  - Bandits
  - Simulations
  - Monte Carlo Tree Search
  - AMAF, RAVE, adding knowledge
  - Hybrid algorithms
  - Parallel algorithms
- Still to come: impact of MCTS, research topics

Impact - Applications of MCTS

- Classical Board Games
  - Go, Hex
  - Amazons
  - Lines of Action, Arimaa, Havannah, NoGo, Konane,…
- Multi-player games, card games, RTS, video games
- Probabilistic Planning, MDP, POMDP
- Optimization, energy management, scheduling, distributed constraint satisfaction, library performance tuning, …

Impact – Strengths of MCTS

- Very general algorithm for decision making
- Works with very little domain-specific knowledge
  - Need a simulator of the domain
- Can take advantage of knowledge when present
- Successful parallelizations for both shared memory and massively parallel distributed systems

Current Topics in MCTS

- Recent progress, Limitations, random half-baked ideas, challenges for future work,...
- Dynamically adaptive simulations
- Integrating local search and analysis
- Improve in-tree child selection
- Parallel search
  - Extra simulations should never hurt
  - Sequential halving and SHOT

Dynamically Adaptive Simulations

- Idea: adapt simulations to specific current context
  - Very appealing idea, only modest results so far
  - Biasing using RAVE (Rimmel et al 2010) – small improvement
  - Last Good Reply (with Forgetting) (Drake 2009, Baier et al 2010)

Integrating Local Search and Analysis

- Mainly For Go
  - Players do much local analysis
  - Much of the work on simulation policies and knowledge is about local replies
- Combinatorial Game Theory has many theoretical concepts
- Tactical alphabeta search (Fuego, unpublished)
- Life and death solvers

Improve In-tree Child Selection

- Intuition: want to maximize if we're certain, average if uncertain
- Is there a better formula than average weighted by number of simulations? (My intuition: there has to be…)
- Part of the benefits of iterative widening may be that the max is over fewer sibling nodes – measure that
  - Restrict averaging to top *n* nodes

Extra Simulations Should Never Hurt

- Ideally, adding more search should never make an algorithm weaker
- For example, if you search nodes that could be pruned in alphabeta, it just becomes slower, but produces the same result
- Unfortunately it is not true for MCTS
- Because of averaging, adding more simulations to bad moves hurts performance - it is worse than doing nothing!

Extra Simulations Should Never Hurt (2)

- Challenge: design a MCTS algorithm that is robust against extra search at the “wrong” nodes
- This would be great for parallel search
- A rough idea: keep two counters in each node - total simulations, and “useful” simulations
- Use only the “useful” simulations for child selections
- Could also “disable” old, obsolete simulations?

Sequential Halving, SHOT

- Early MC algorithm: successive elimination of empirically worst move (Bouzy 2005)
- Sequential halving (Karnin et al 2013):
  - Rounds of uniform sampling
  - keep top half of all moves for next round
- SHOT (Cazenave 2014)
  - Sequential halving applied to trees
  - Like UCT, uses bandit algorithm to control tree growth
  - Promising results for NoGo
  - Promising for parallel search