Lecture 20: Graphs

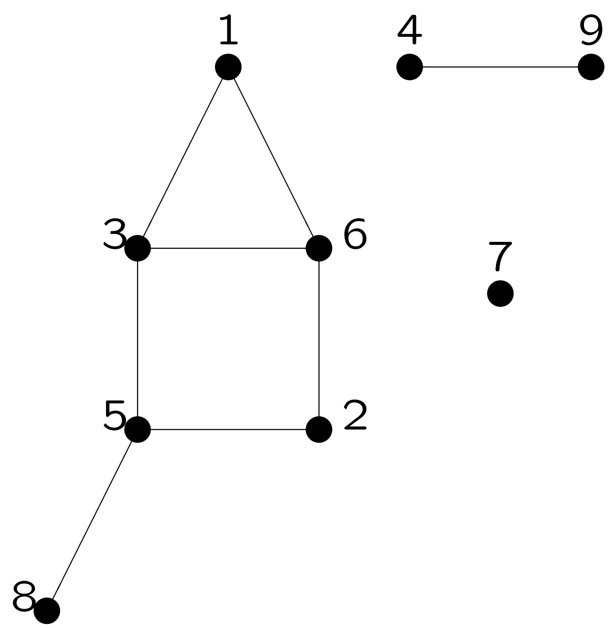
Agenda:

- Basic definitions
- Typically
  - connected component
  - biconnected component

Reading:

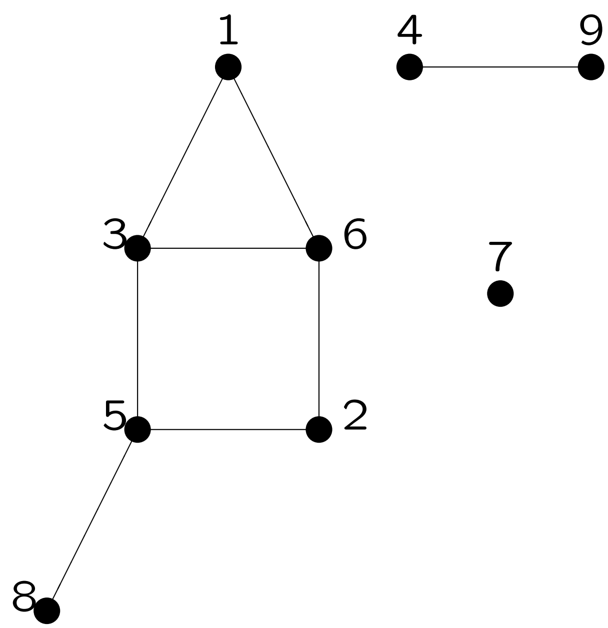
- Textbook pages 1080 – 1084, 527 – 531, 558 – 559

An example:



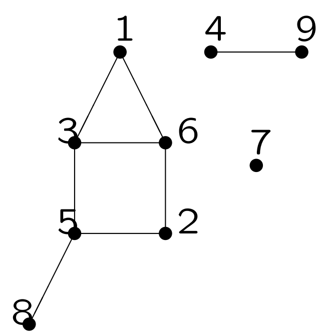
1:	3	6	
2:	5	6	
3:	1	5	6
4:	9		
5:	2	3	8
6:	1	2	3
7:			
8:	5		
9:	4		

An example:



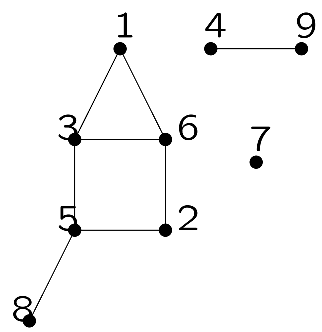
	1	2	3	4	5	6	7	8	9
1			*			*			
2					*	*			
3	*				*	*			
4									*
5		*	*					*	
6	*	*	*						
7									
8					*				
9				*					

Definitions:



- (simple, undirected) graph $G = (V, E)$
  - vertex set V
  - edge set E
    * an edge e is a pair of vertices v_1 and v_2
    * unordered — undirected
    * $v_1 \neq v_2$ — simple
- $V = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
  $E = \{\{1, 3\}, \{1, 6\}, \{2, 5\}, \{2, 6\}, \{3, 5\}, \{3, 6\}, \{4, 9\}, \{5, 8\}\}$
- Notions:
  - adjacent (vertex – vertex, edge – edge)
    e.g., 1 and 3 are adjacent; $(1, 3)$ and $(3, 5)$ are adjacent
  - incident (vertex – edge)
    e.g., 1 is incident with $(1, 3)$

Graph notions:



- Computer representations:
  - adjacency lists
  - adjacency matrix
- Neighborhood of a vertex
- Degree of a vertex — size of its neighborhood
- Path (vertex – vertex), simple path
  e.g., $\langle 1, 3, 5, 2, 6, 3, 5, 8\rangle$ and $\langle 1, 3, 5, 2, 6\rangle$ are paths
  the latter is a simple path
- Connected (every pair of vertices is connected via a path)
- Subgraph $G' = (V', E')$ of $G = (V, E)$
  - it is a graph
  - $V' \subseteq V$
  - $E' \subseteq E$
- Connected component (maximal connected subgraph)

Binary equivalence relation:

- A relation $\sim$ involving two elements (in a set A)
  for example, "$\leq$" relation for real numbers
- Reflexive: $a \sim a$ for any $a \in A$
- Symmetric: $a_1 \sim a_2$ iff $a_2 \sim a_1$
- Transitive: $a_1 \sim a_2$ and $a_2 \sim a_3$ imply $a_1 \sim a_3$
- Binary equivalence relation:
  reflexive + symmetric + transitive
  e.g., "$=$" relation for real numbers
- Equivalence class of a

  the subset of elements b such that $a \sim b$

  Therefore, the equivalence class of a contains b implies it is also the equivalence class of b ...
- The equivalence classes form a partition of A
  - union to A
  - disjoint

Connected component:

- A binary equivalence relation $\sim$ on vertex set V

  $v_1 \sim v_2$ iff "there is a path connecting v_1 and v_2"

- The connected component containing vertex v is the equivalence class of v:
  - the connected components form a partition of G, such that
  - no edge crossing the components

Biconnected component:

- Simple path connecting v_1 and v_2

  — all vertices in the path are distinct

- Two paths connecting v_1 and v_2 are vertex-disjoint

  — share no common internal vertex

- Biconnected graph
  - $|V| > 2$
  - connected
  - every pair of vertices are connected via two vertex-disjoint (simple) paths

- Notes:
  - don't bother the case $|V| \leq 2$
  - connectivity does NOT implies biconnectivity
  - articulation vertex — cut vertex

    !!! its removal disconnects G
  - bridge — cut edge

    !!! its removal disconnects G

- Biconnected component — maximal biconnected subgraph
  - a partition of E (not necessarily a partition of V)

Future subjects:

- How to compute the connected components?
  — using data structure Disjoint Sets (next lecture)
- How to compute the biconnected components?
  — using graph traversal Depth-First-Search

Future graph definitions:

- Not necessarily simple — multiple edges and loops exist
- Directed — edge ordered
- Hypergraph — an edge might contain more then 2 vertices

Have you understood the lecture contents?

well	ok	not-at-all	topic
☐	☐	☐	what is a graph?
☐	☐	☐	representing a graph
☐	☐	☐	graph notions (adjacent, etc.)
☐	☐	☐	connected component
☐	☐	☐	binary equivalence relation
☐	☐	☐	biconnected component