Taming Pointers

A Symbolic Approach

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Outline

- An Old Problem
- A New Strategy
- Taming Context Sensitivity
- Result and Conclusion

What is Pointer Analysis

```
char    *g, a, h1, h2;                    1
void main() {                             2
   char   *p, *q;                         3
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S1: p = getg( &q );                       5
S2: g = &a;                               6
   }                                      7
                                          8
char* getg( char** r ) {                  9
   char  **t;                            10
S0: t = &g;                              11
   if( g == NULL )                       12
S1:      alloc( t, &h2 );                13
S2: *r = *t;                             14
S3: return *r;                           15
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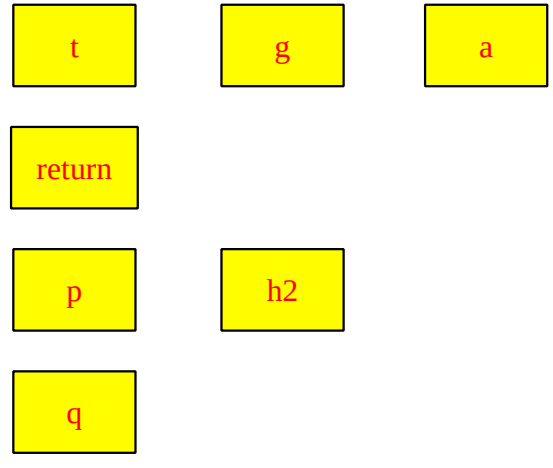
- Program variables
  - Named: globals/locals
  - Anonymous: return/malloc



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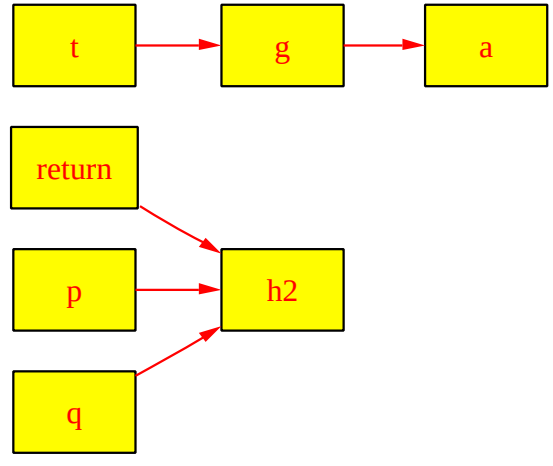
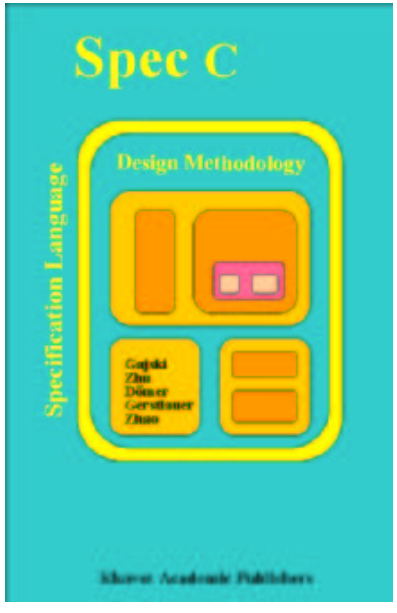
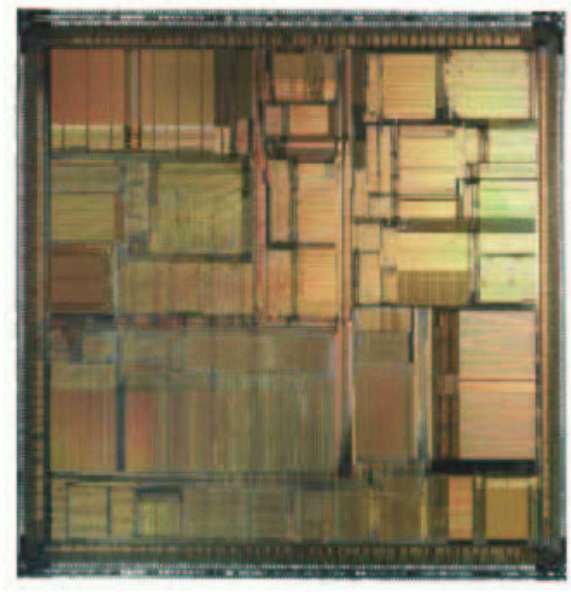
- Program state: Point-to graph



Why Do We Care: Silicon Compiler





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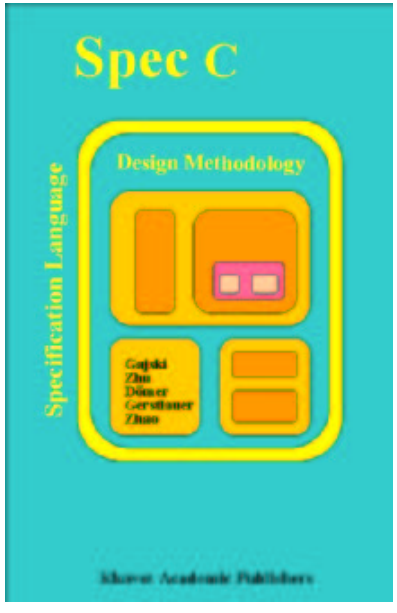

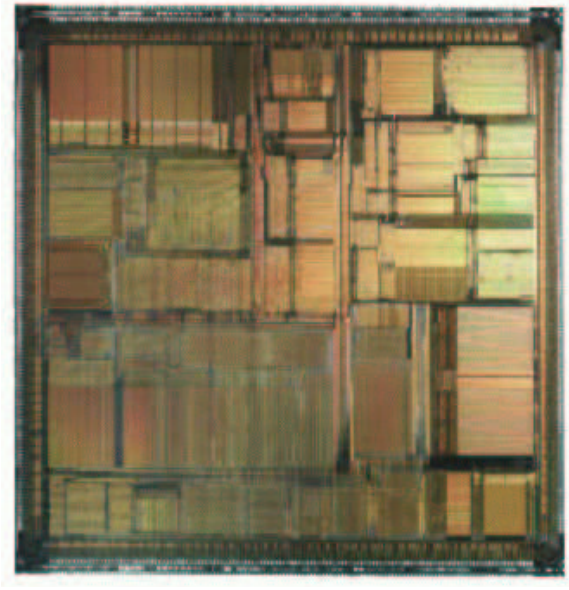

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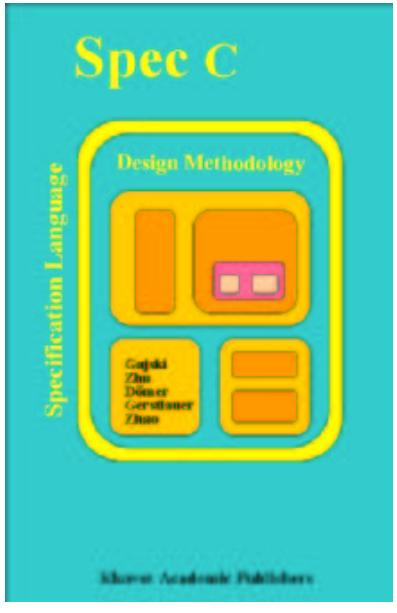

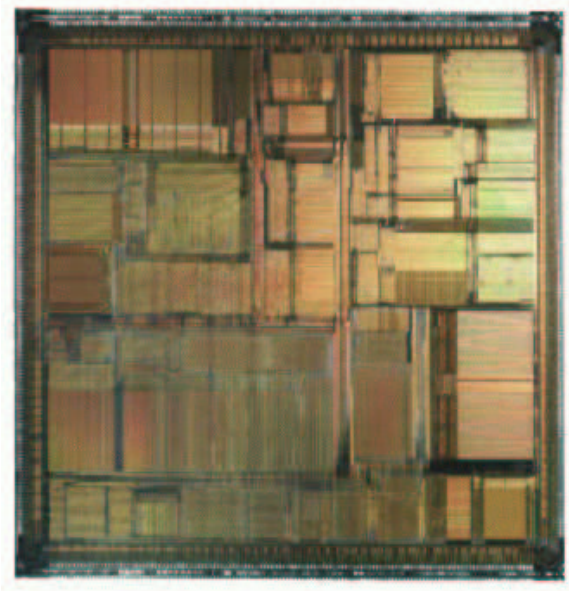

- Fine-grained parallelism
  - Feed values to functional units
  - Dependency test
- Coarse-grained parallelism
  - Feed data structures to processors/cores
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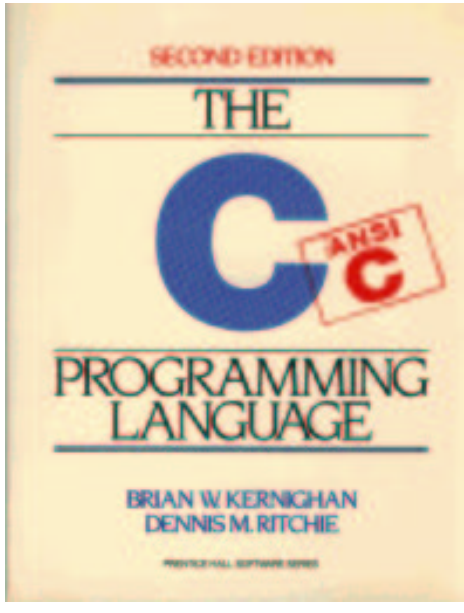

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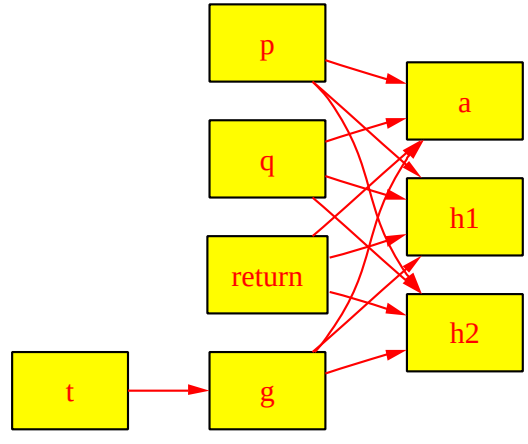

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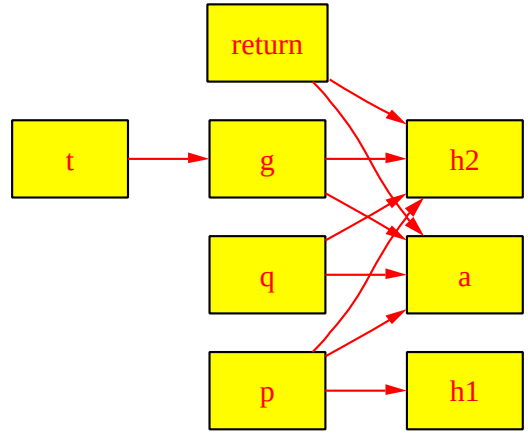

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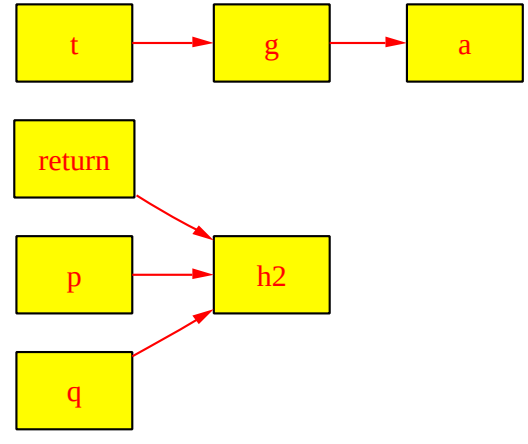

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  - Practice: program state at every program point!
- Context-sensitive analysis
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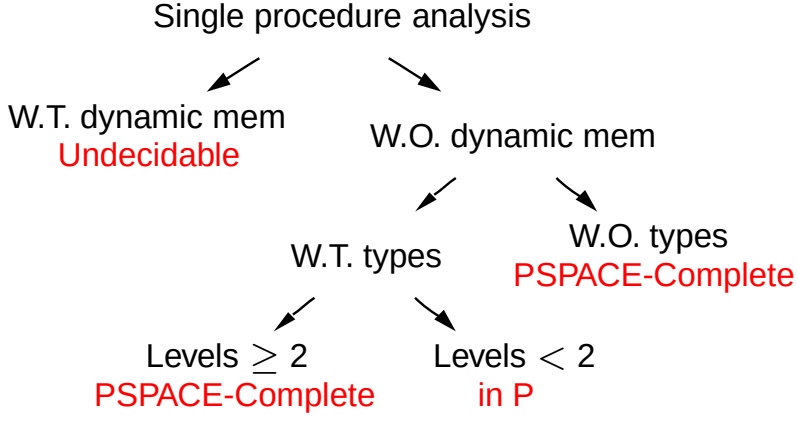

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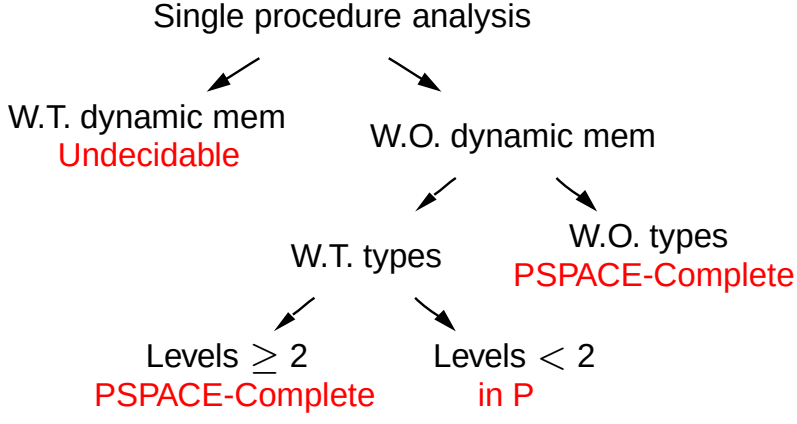

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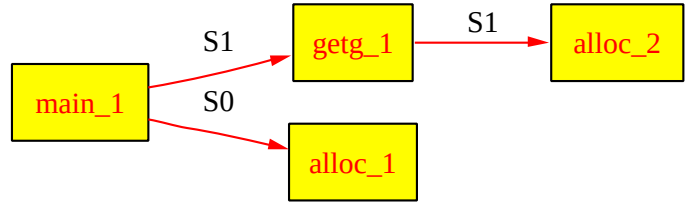

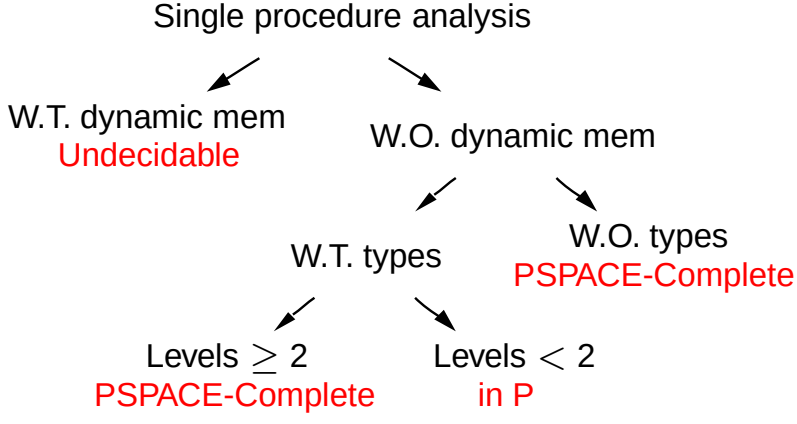

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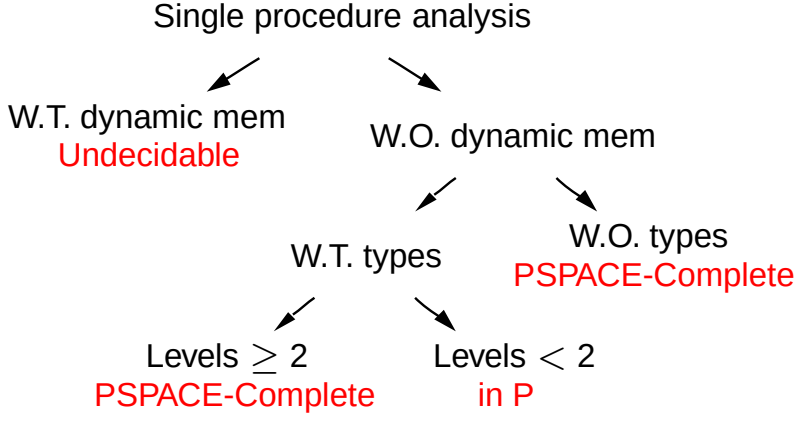

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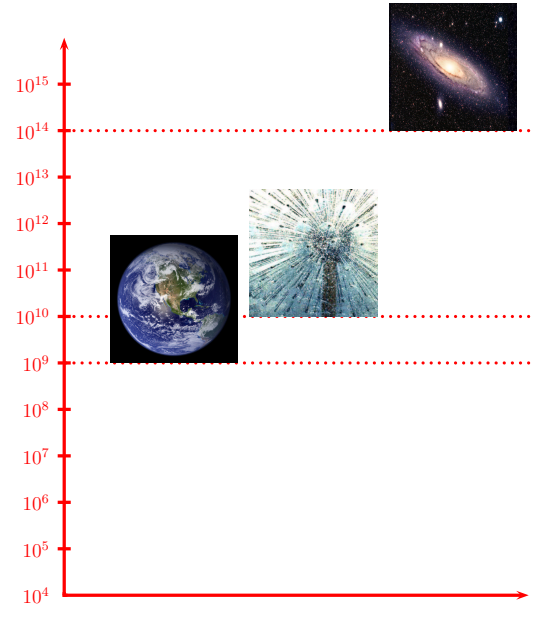

Outline

- An Old Problem
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- Result and Conclusion

Representing Set

- Discrete set isomorphic to integer set
- Given a finite discrete domain D
  - Sufficient to consider $D = [0, n-1]$
- Consider a set S in domain D: $S \subseteq D$
- Characteristic function $\lambda_S : D \mapsto \{0, 1\}$

$$\lambda_S(i) = \begin{cases} 0, & \text{if } i \notin S \\ 1, & \text{if } i \in S \end{cases}$$

Binary Decision Diagram (BDD)

x_2	x_1	x_0	λ_S
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
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- $D = [0, 7]$
- $S = \{3, 5, 7\}$
- $\lambda_S = \overline{x_2} x_1 x_0 + x_2 \overline{x_1} x_0 + x_2 x_1 x_0$

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x_1	0	0	1	1	0	0	1	1
x_0	0	1	0	1	0	1	0	1
λ_S	0	0	0	1	0	1	0	1

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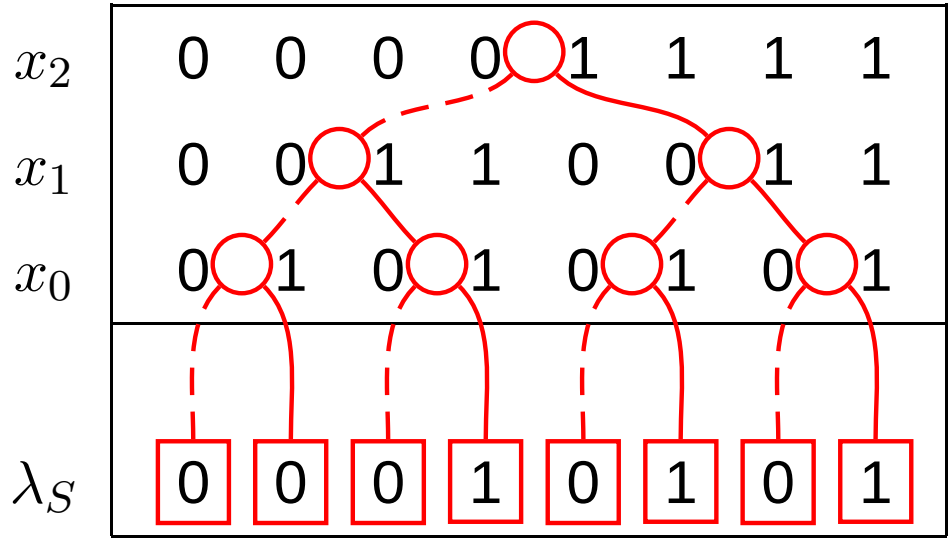
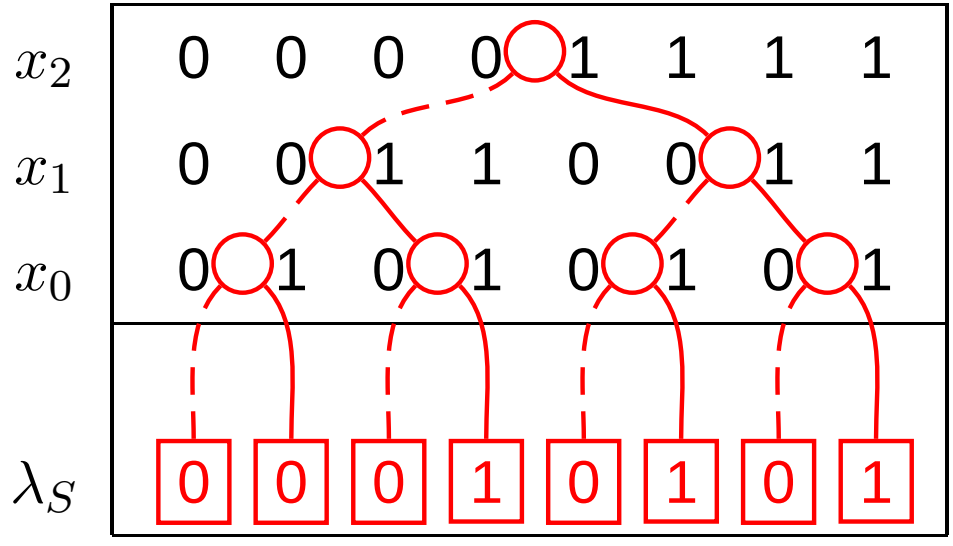


Reduced Ordered BDD (ROBDD)

- Reduction rules (Bryant'86)
  - var in fixed order



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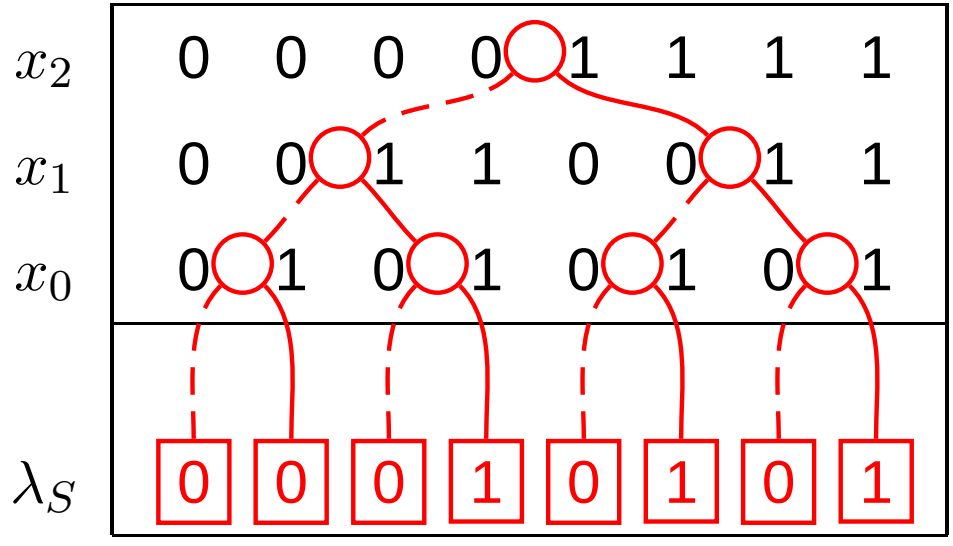

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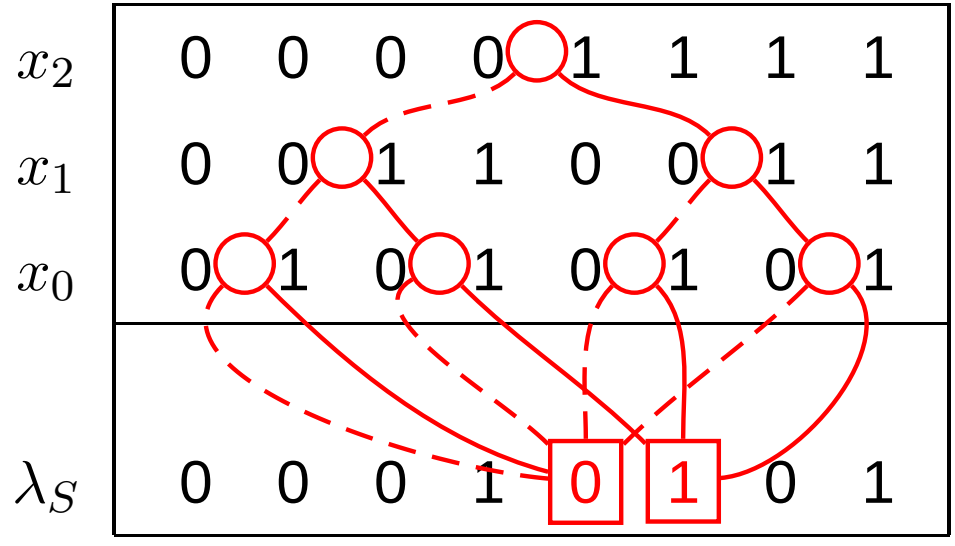

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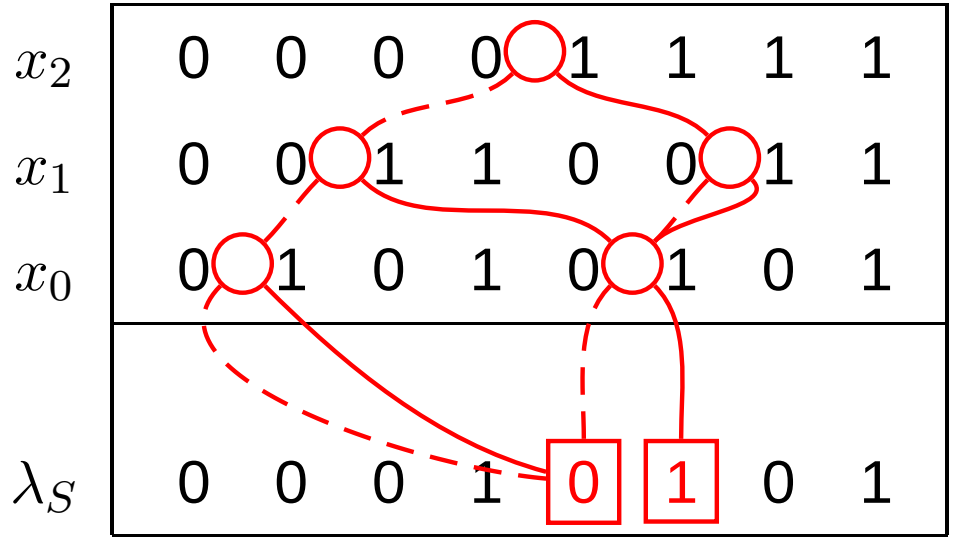

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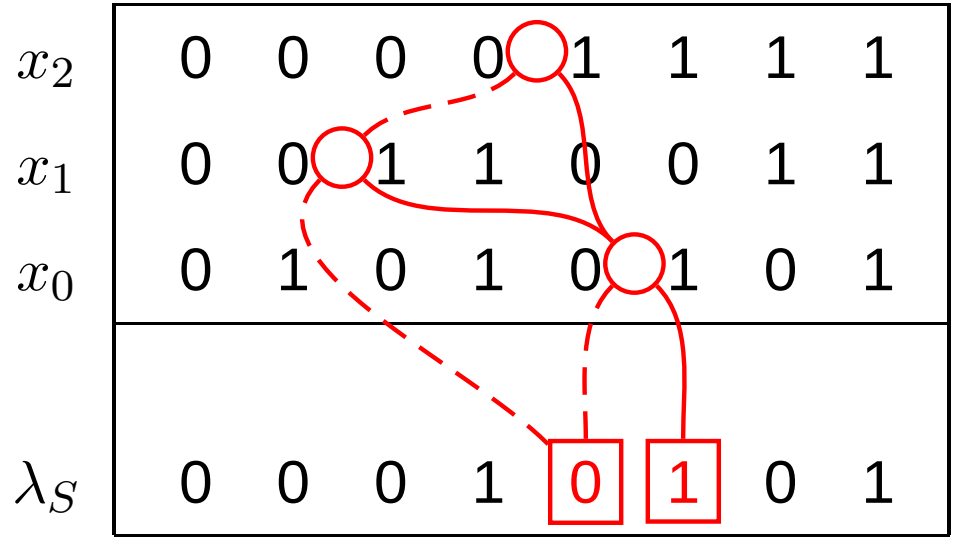

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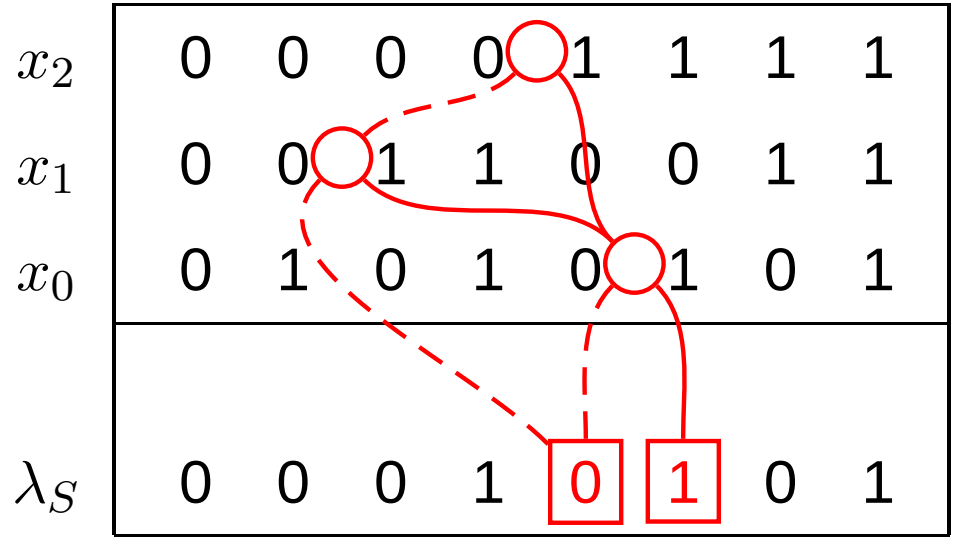

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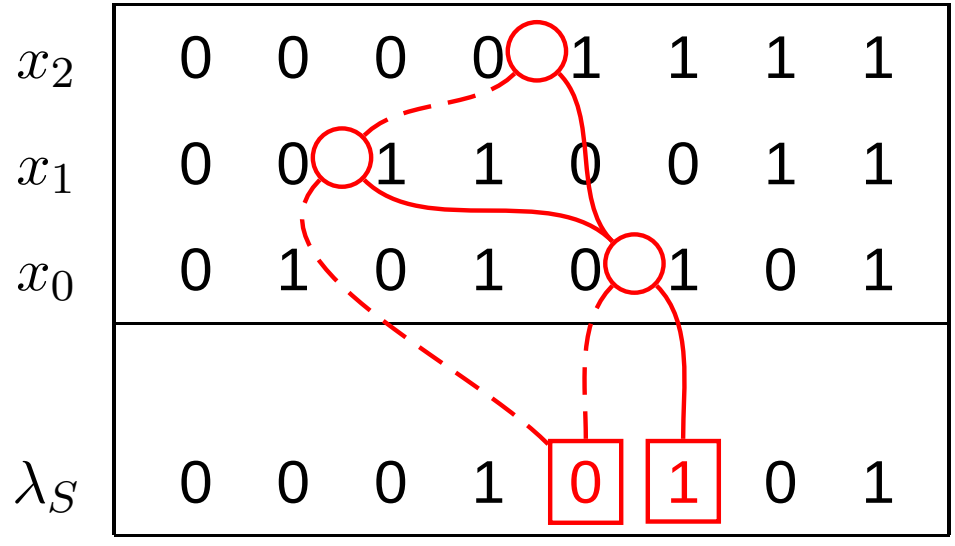

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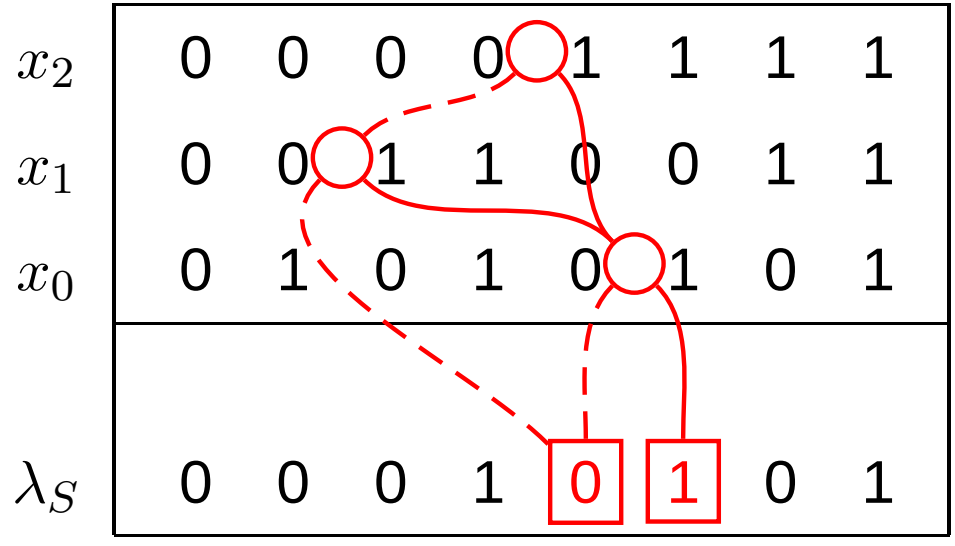

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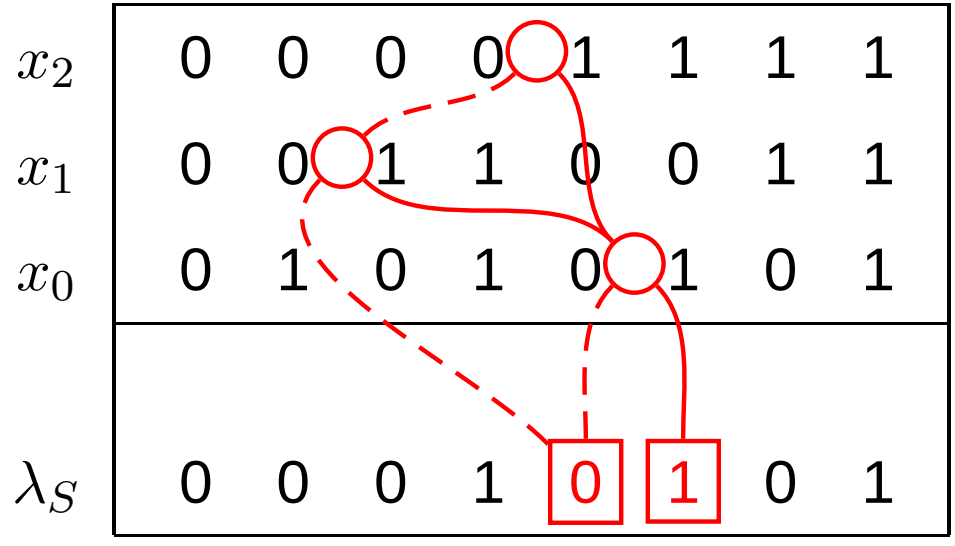

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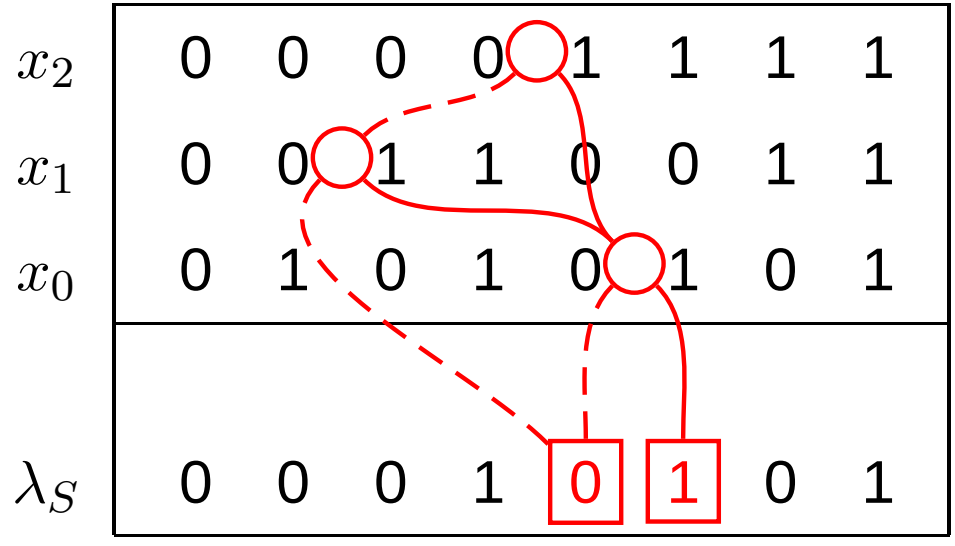

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Representing Relation

- Relation $R \subseteq A \times B$
  - Nothing but a set!
  - Characteristic function $\lambda_R : A \times B \mapsto \{0, 1\}$

$$\lambda_R(i,j) = \begin{cases} 0, & \text{if } \langle i,j \rangle \notin R \\ 1, & \text{if } \langle i,j \rangle \in R \end{cases}$$

- Consider graph $\langle V, E \rangle$
  - Let V^0, V^1 be two Boolean spaces for V
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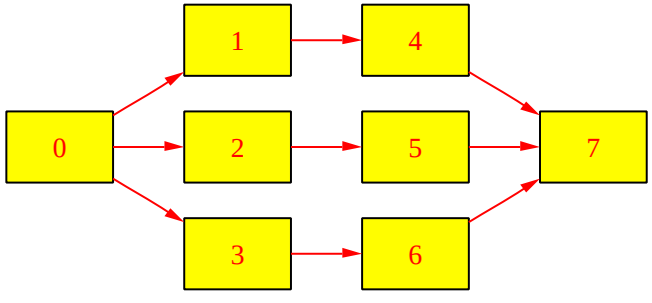

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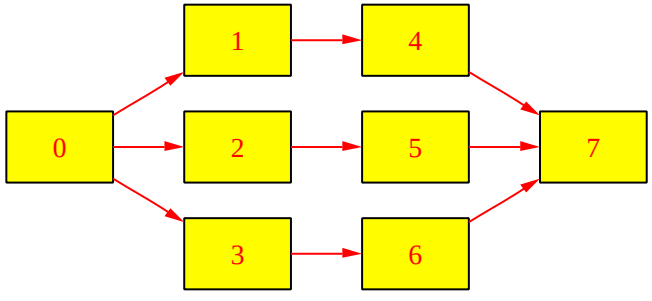

- Consider graph $\langle V, E \rangle$
  - Let V^0, V^1 be two Boolean spaces for V
  - Let V_i^0, V_j^1 be the ith, jth minterms of V^0, V^1
  - Product term $V_i^0 V_j^1$ represents an edge

$$\begin{aligned} \lambda_E &= V_0^0 V_1^1 + V_0^0 V_2^1 + V_0^0 V_3^1 \\ &+ V_1^0 V_4^1 + V_2^0 V_5^1 + V_3^0 V_6^1 \\ &+ V_4^0 V_7^1 + V_5^0 V_7^1 + V_6^0 V_7^1 \end{aligned}$$

Symbolic Computation

- First order logic in BDD
  - Negation: $\neg f$
  - Conjunction: $f \wedge g$
  - Disjunction: $f \vee g$
  - Existential abstraction: $\exists D^i . f$
  - Substitution: $f|_{D^i \mapsto D^j}$
  - Polynomial to BDD size
- Exercise: Finding successors

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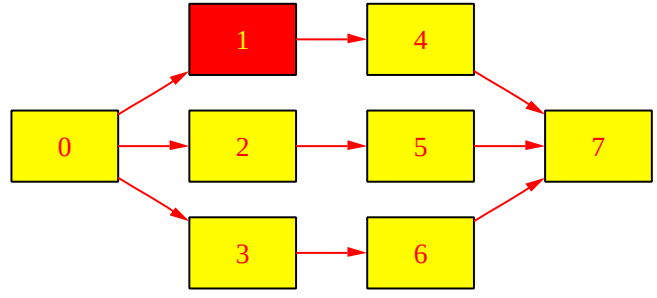

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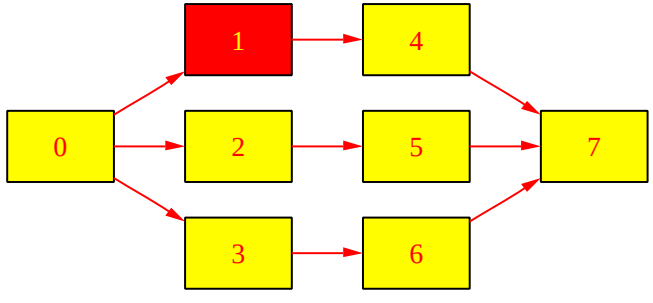

- Exercise: Finding successors
  - Given

$$
\begin{aligned}
r &= V_0^0 V_1^1 + V_0^0 V_2^1 + V_0^0 V_3^1 \\
&+ V_1^0 V_4^1 + V_2^0 V_5^1 + V_3^0 V_6^1 \\
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s &= V_1^0
\end{aligned}
$$

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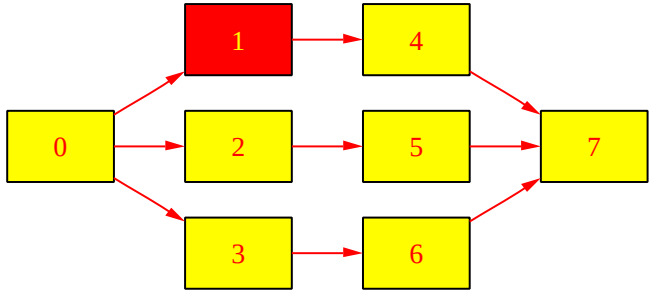

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  - Step 1: $s \wedge r = V_1^0 V_4^1$

Symbolic Computation

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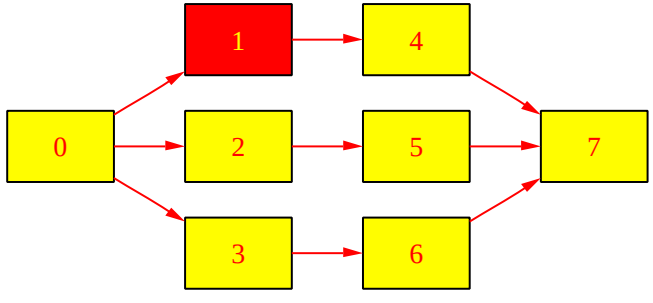

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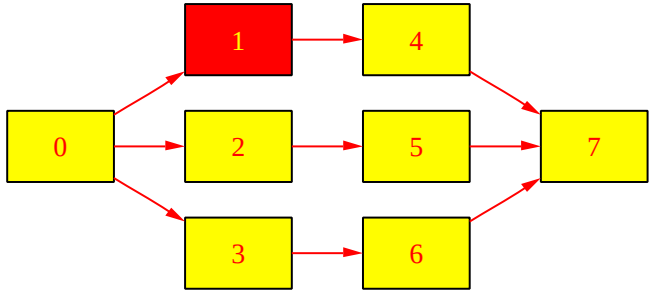

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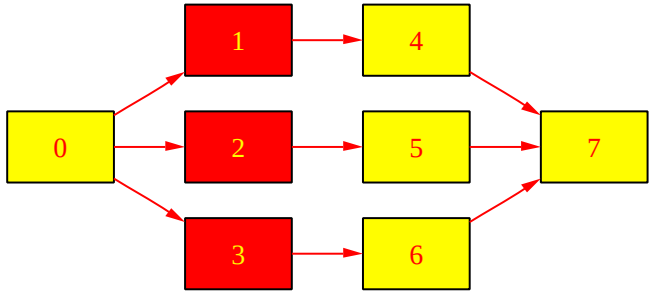
  - Step 1: $s \wedge r = V_1^0 V_4^1$
  - Step 2: $\exists V^0 . s \wedge r = V_4^1$
  - Step 3: $[\exists V^0 . s \wedge r]|_{V^1 \mapsto V^0} = V_4^0$

Image Computation

- s can be just a set
- Image (output) of a set (input) under a function



- Exercise again

Image Computation

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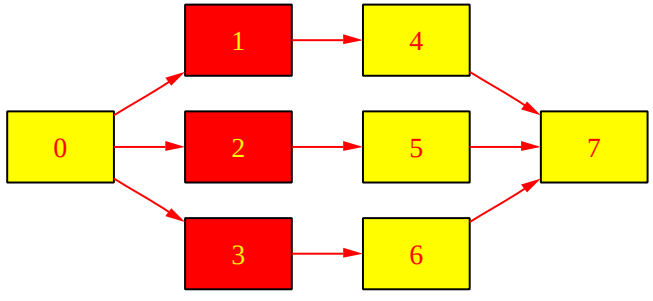

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s &= V_1^0 + V_2^0 + V_3^0
\end{aligned}
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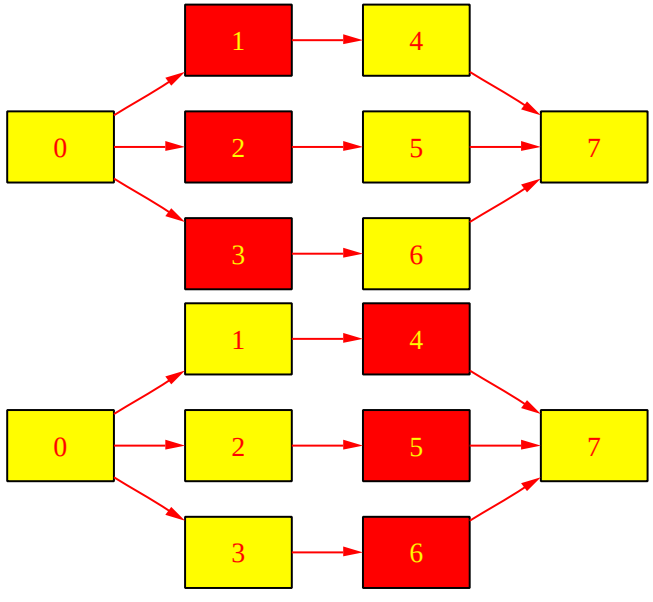

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  - Step 1: $s \wedge r = V_1^0 V_4^1 + V_2^0 V_5^1 + V_3^0 V_6^1$
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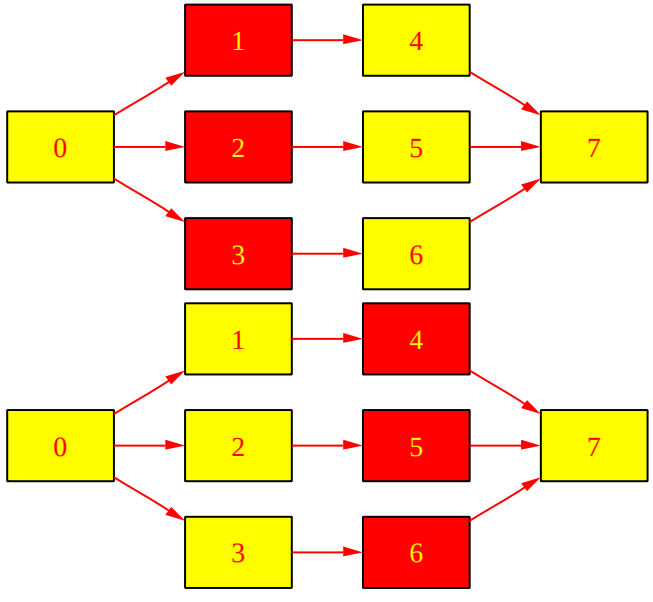

- Does not have to be slower!

- Exercise again
  - Given

$$\begin{aligned} r &= V_0^0V_1^1 + V_0^0V_2^1 + V_0^0V_3^1 \\ &+ V_1^0V_4^1 + V_2^0V_5^1 + V_3^0V_6^1 \\ &+ V_4^0V_7^1 + V_5^0V_7^1 + V_6^0V_7^1 \\ s &= V_1^0 + V_2^0 + V_3^0 \end{aligned}$$

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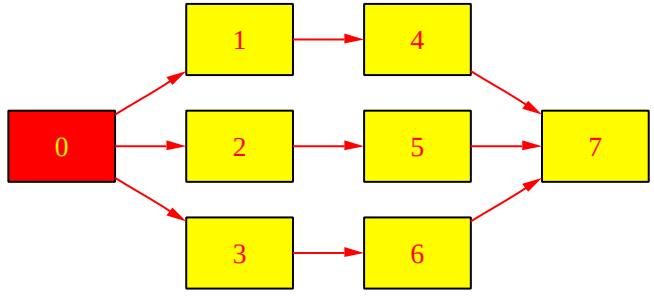
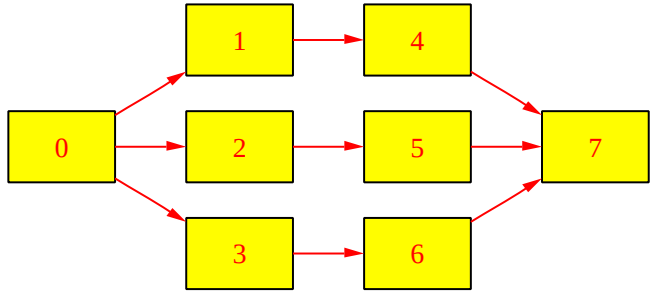
Graph Reachability

- image



- reached

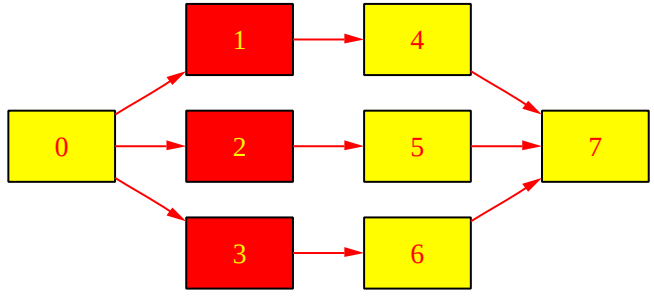


- Key for success of model checking
- Algorithm

```
reachable( s : V^0, g : V^0 × V^1 )                    1
  : V^0 {                                              2
  var image, reached : V^0;                            3
  image = s;                                           4
  reached = 0;                                         5
  while( image ≠ 0 ) {                                 6
    image = [∃V^0.image ∧ g]|_{V^1 ↦ V^0};             7
    reached = reached ∨ image;                         8
    image = image ∧ ¬reached;                          9
    }                                                  10
  return reached ;                                     11
 }                                                     12
```

Graph Reachability

- image



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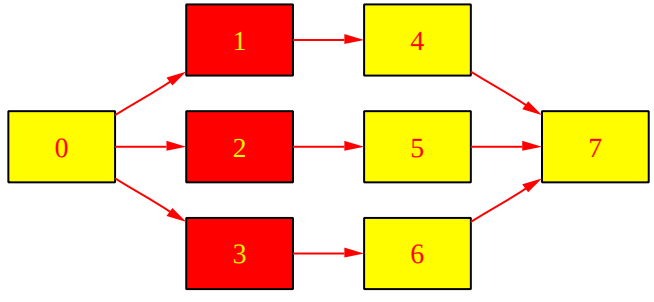

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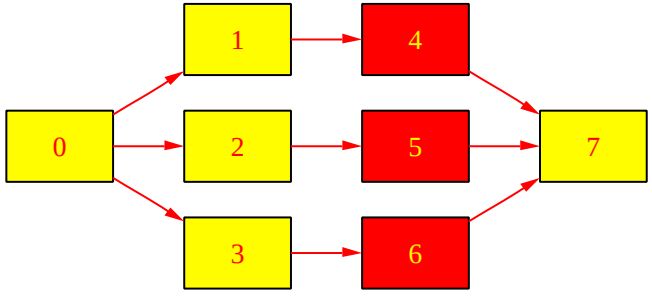

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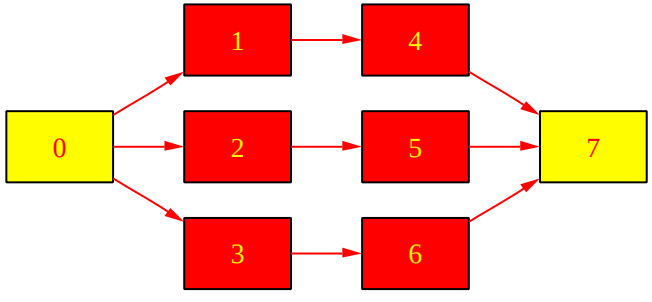

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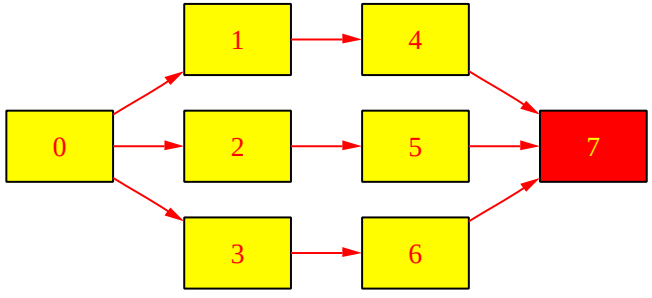

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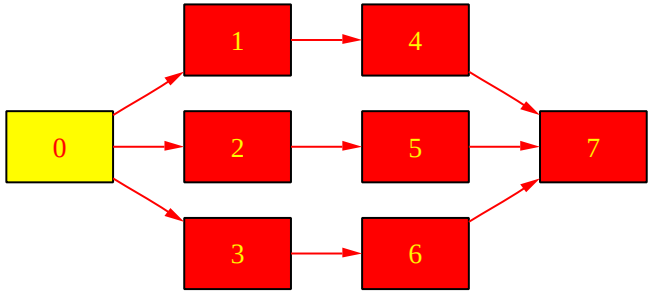

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    }                                                  10
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Superposed Computation

- Divide-and-Conquer paradigm
  - task $\mapsto$ many problem instances
- Parallel computing
  - One processor per instance
  - Exponential many instances?

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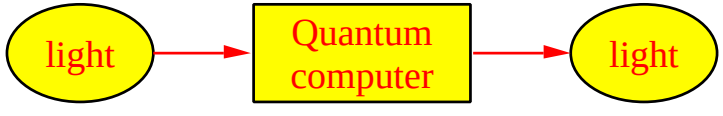

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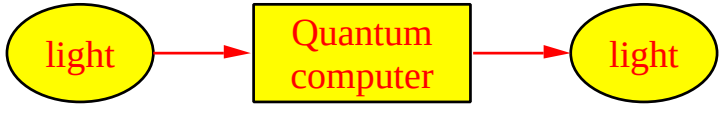

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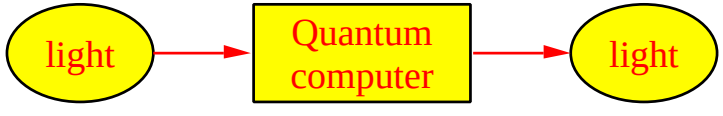

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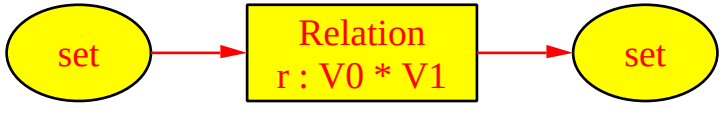

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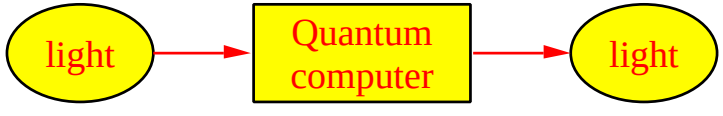

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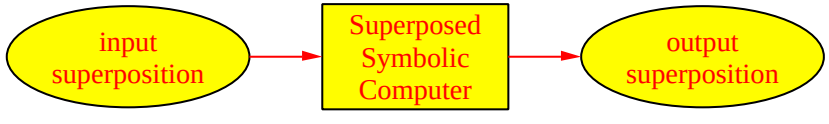

- Introduce domain of problem instances I
- Graph superposition: $I \times V^0 \times V^1$
- Input superposition: $I \times V^0$
- Superposed image computation
  - $s : I \times V^0, r : I \times V^0 \times V^1$
  - $s \wedge r : I \times V^0 \times V^1$
  - $\exists V^0 . s \wedge r : I \times V^1$
  - $[\exists V^0 . s \wedge r]|_{V^1 \mapsto V^0} : I \times V^0$

Transitive Closure

- Reachability problem for each node
- Algorithm

```
closure( g : V^0 × V^1 )                                   1
  : V^0 × V^1 {                                            2
  var image, reached : V^2 × V^0;                          3
  image = V^2 == V^0;                                      4
  reached = 0;                                             5
  while( image ≠ 0 ) {                                     6
    image = [∃V^0.image ∧ g]|_{V^1↦V^0};                   7
    reached = reached ∨ image;                             8
    image = image ∧ ¬reached;                              9
    }                                                      10
  return reached|_{V^2↦V^0, V^1↦V^1} ;                     11
}                                                          12
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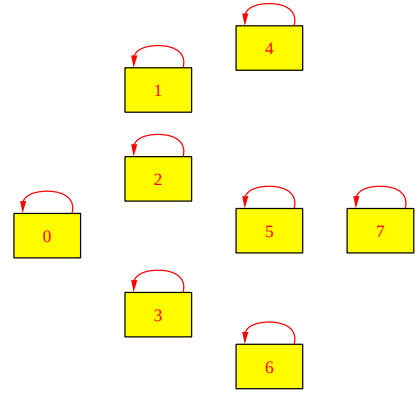
- image



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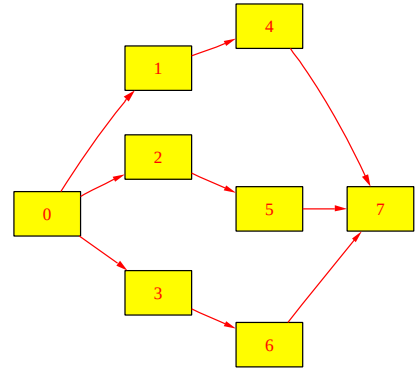

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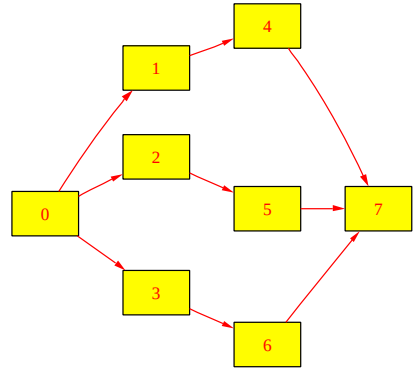

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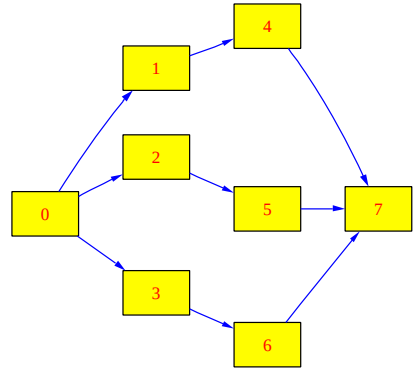

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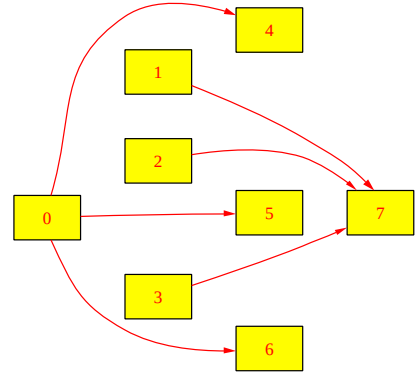
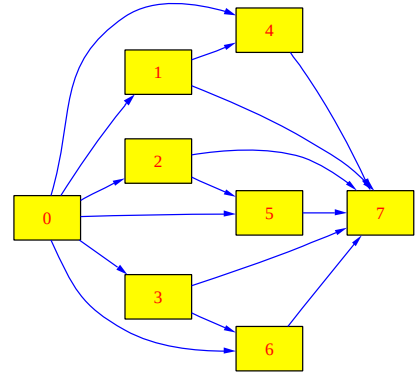
```

- image



- reached

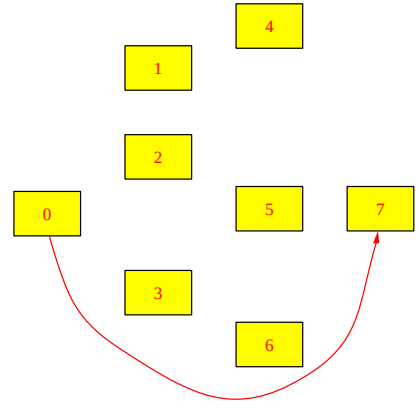


Transitive Closure

- Reachability problem for each node
- Algorithm

```
closure( g : V^0 × V^1 )                                   1
  : V^0 × V^1 {                                            2
  var image, reached : V^2 × V^0;                          3
  image = V^2 == V^0;                                      4
  reached = 0;                                             5
  while( image ≠ 0 ) {                                     6
    image = [∃V^0.image ∧ g]|_{V^1↦V^0};                   7
    reached = reached ∨ image;                             8
    image = image ∧ ¬reached;                              9
    }                                                      10
  return reached|_{V^2↦V^0, V^1↦V^1} ;                     11
}                                                          12
```

- image



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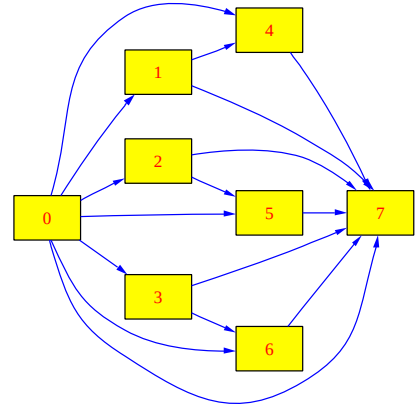

Outline

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- A New Strategy

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- Taming Context Sensitivity
- Result and Conclusion

Symbolic Framework (FICS)

- Given

$call$	:	$(M^0 \times S^0 \times C^0) \times (M^1 \times C^1)$
tr	:	$M^0 \times (B^2 \times P^0) \times (B^3 \times P^1)$
$path$	:	$P^0 \times F^0 \times P^1$
$inmap$	:	$M^0 \times S^0 \times (B^2 \times P^0) \times (B^3 \times P^1)$
$outmap$	:	$M^0 \times S^0 \times (B^2 \times P^0) \times (B^3 \times P^1)$

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\end{array}
$$

- Find

$$
\begin{array}{rcl}
s & : & B^0 \times F^0 \times B^1 \\
q & : & M^0 \times C^0 \times (B^2 \times P^0) \times B^0
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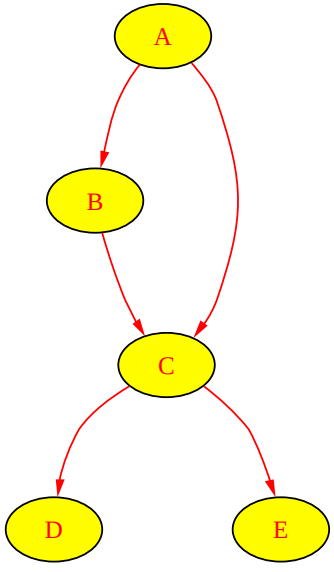
$$

- By Recurrence equations

$$
\begin{array}{rcl}
s & = & \text{apply}(tr, q) \\
q & = & \text{bind}(\text{query}(s, path), inmap, outmap, call)
\end{array}
$$

Symbolic Invocation Graph (SIG)



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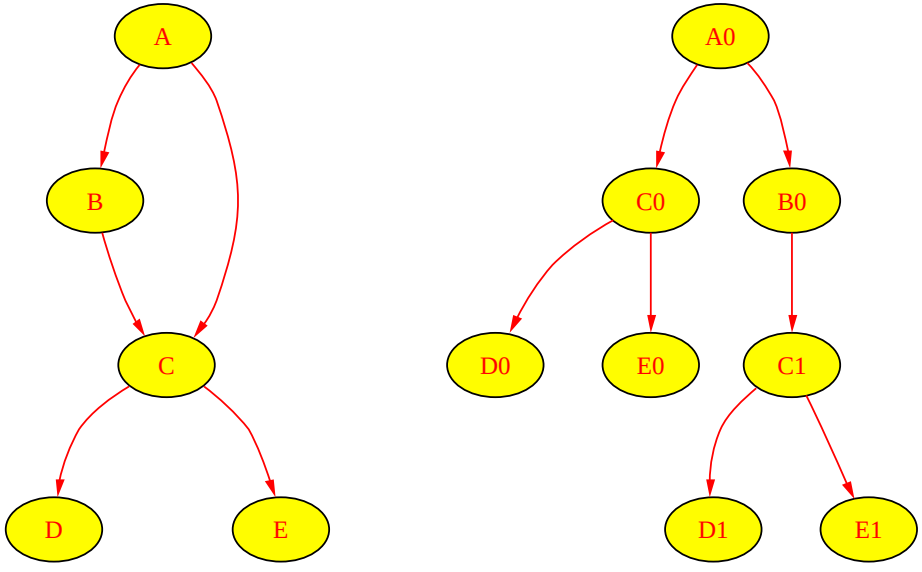

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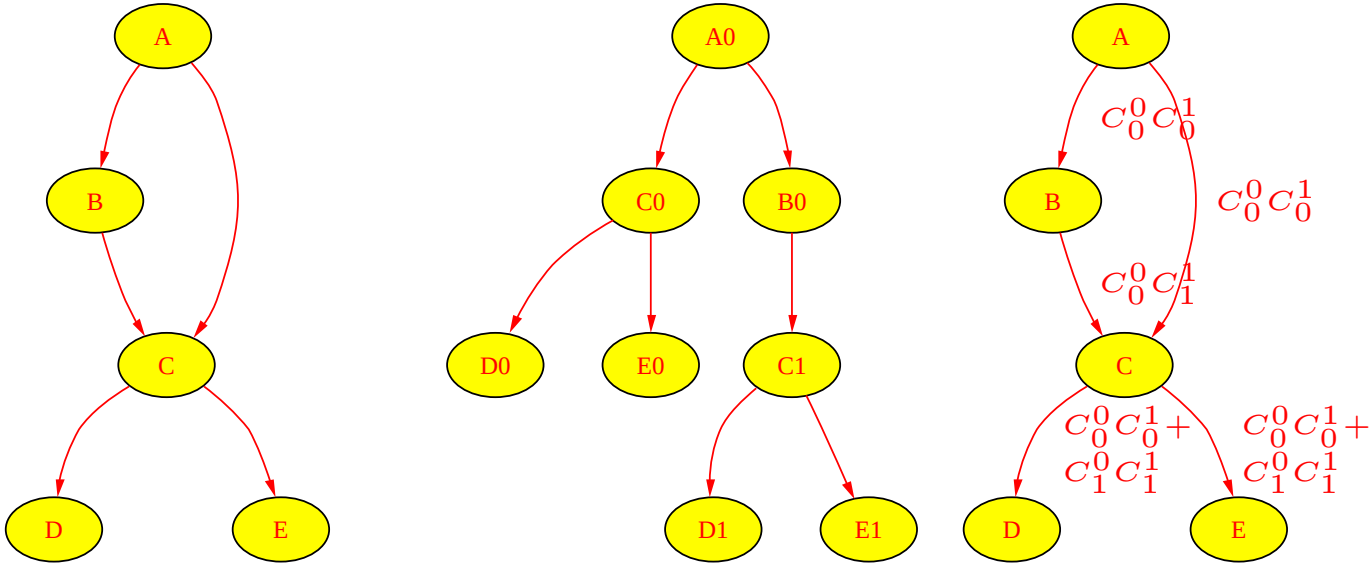

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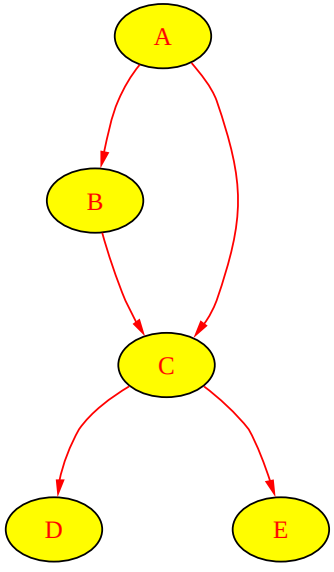
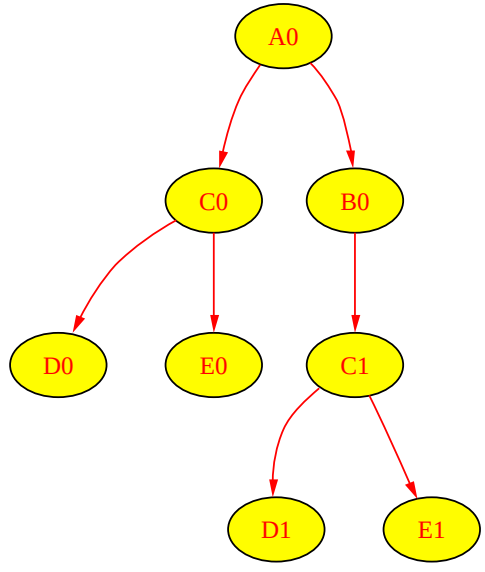
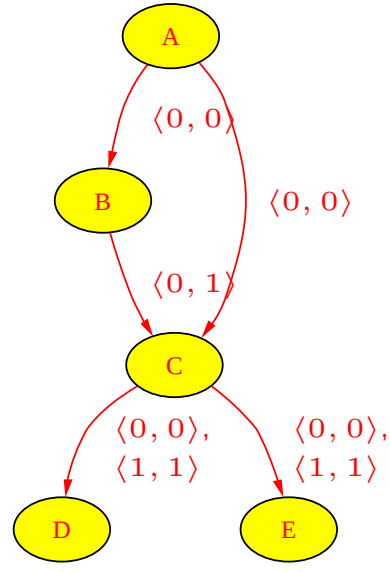
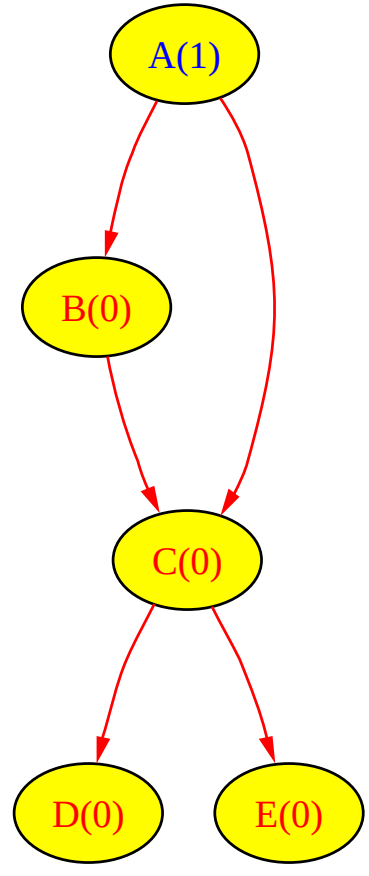






SIG Construction

- Step 1: Acyclic graph reduction
  - Presence of recursion
  - Identify Strongly connected components (SCCs)
  - Merge SCCs
- Step 2: Acyclic graph expansion
  - Top-down topological traversal
  - Annotate each call graph edge



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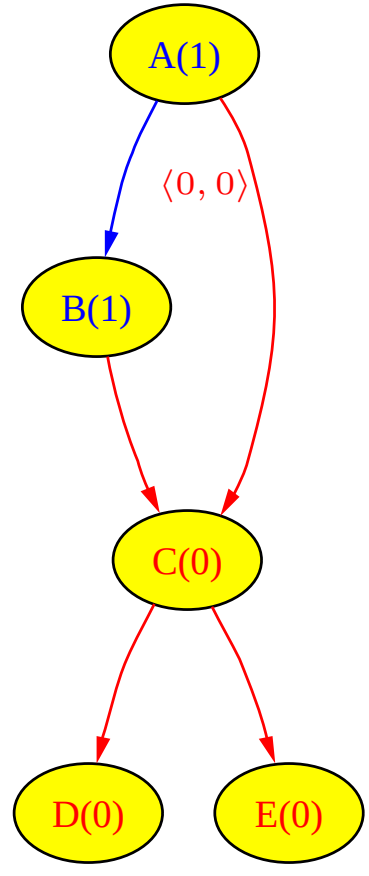

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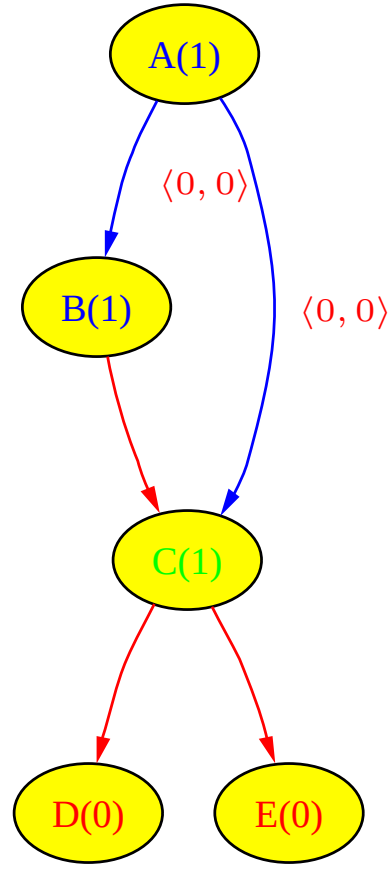

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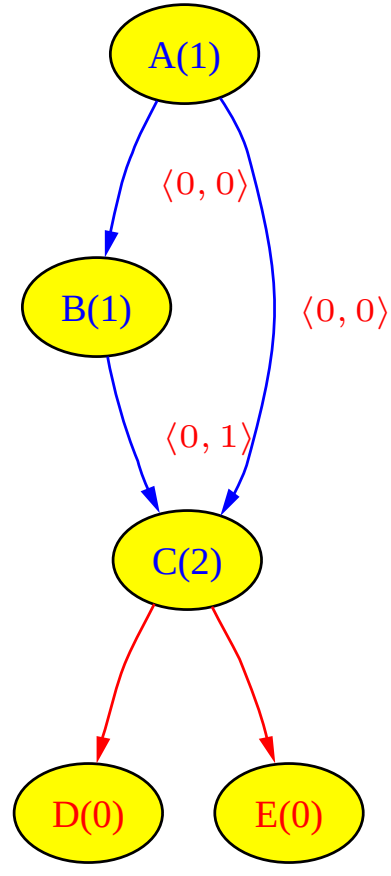

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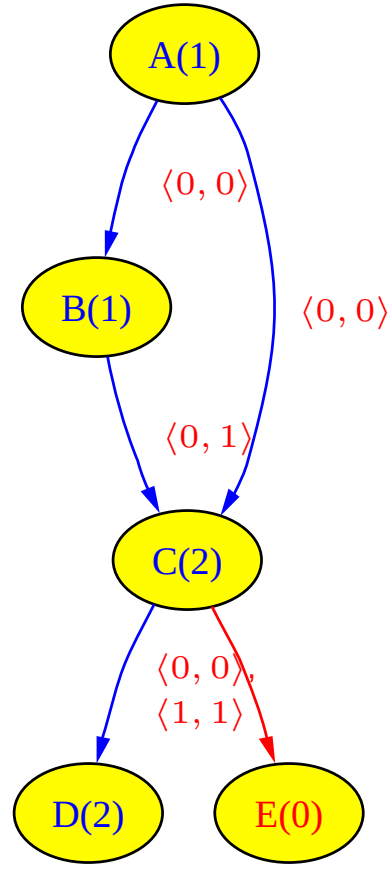

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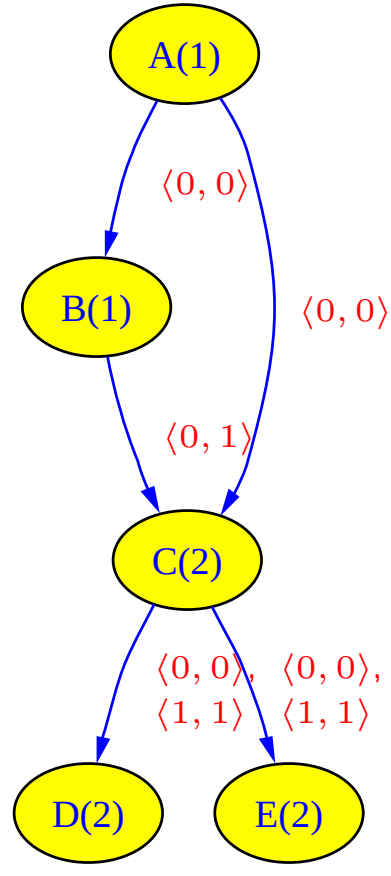
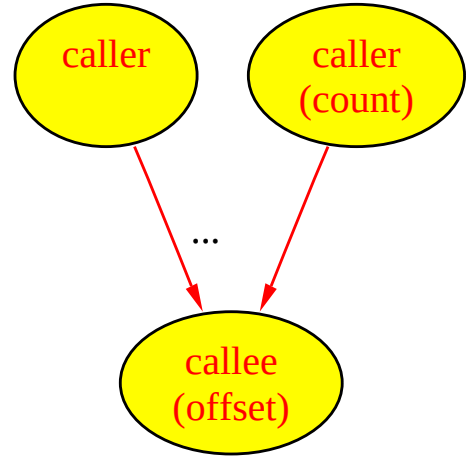


SIG Edge Construction

- Contexts for each procedure numbered continuously
- SIG edges for each CG edge
  - $\langle 0, \text{offset} \rangle$, $\langle 1, \text{offset} + 1 \rangle$, ...
    $\langle \text{count} - 1, \text{offset} + \text{count} - 1 \rangle$
  - Explicit construction is expensive



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- $\{\langle x, y\rangle\}$ satisfy two conditions
  - $y = x +$ offset
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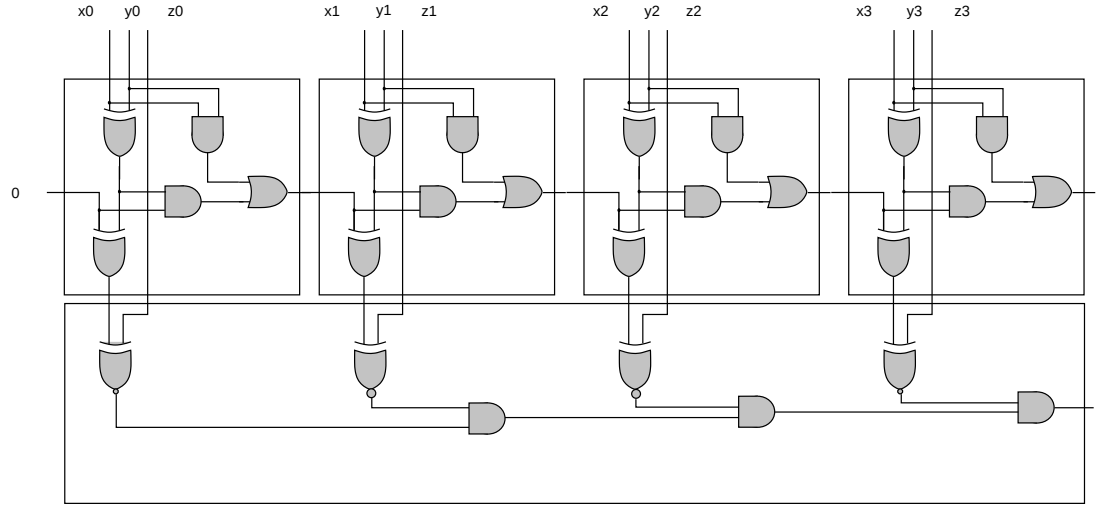
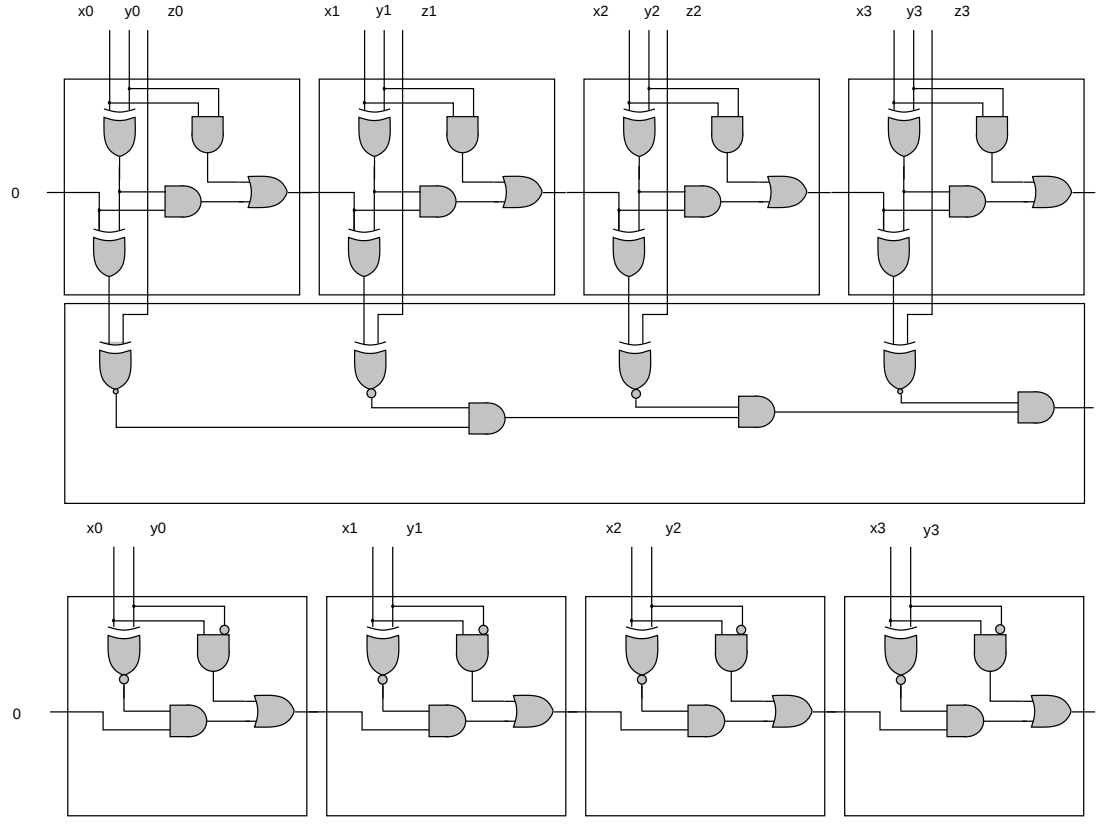


SIG Edge Construction

- $\{\langle x, y\rangle\}$ satisfy two conditions
  - $y = x +$ offset
  - $x <$ count
- Built by constructing circuits
  - Adder
  - Comparator



Insight in Complexity

- Algorithm polynomial to BDD size
  - BDD size in general is exponential
- Question
  Can we bound BDD size of SIG?
- Berman [TCAD'01]
  - Establish relation between circuit complexity and BDD complexity
  - BDD size $\propto 2^{\text{circuit cutwidth}} m$
  - m is number of BDD variables

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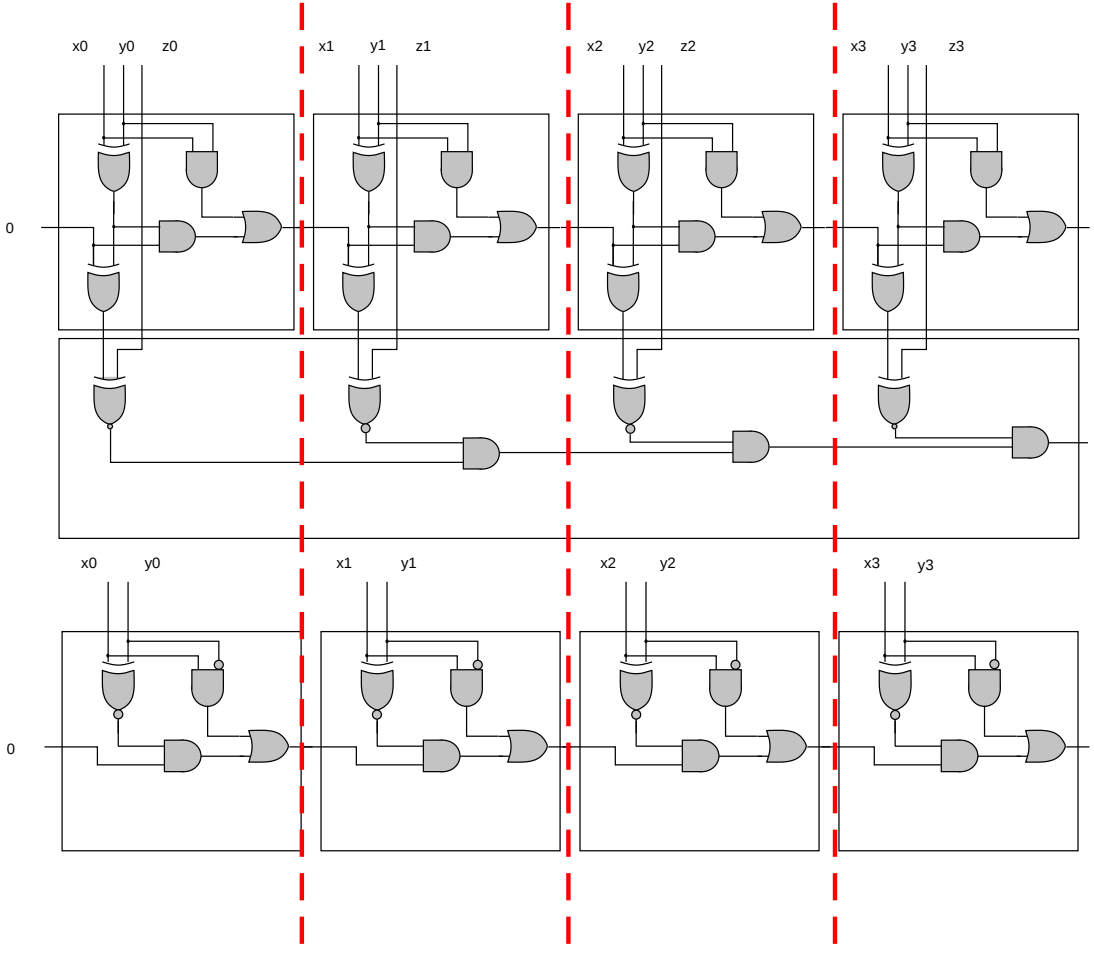

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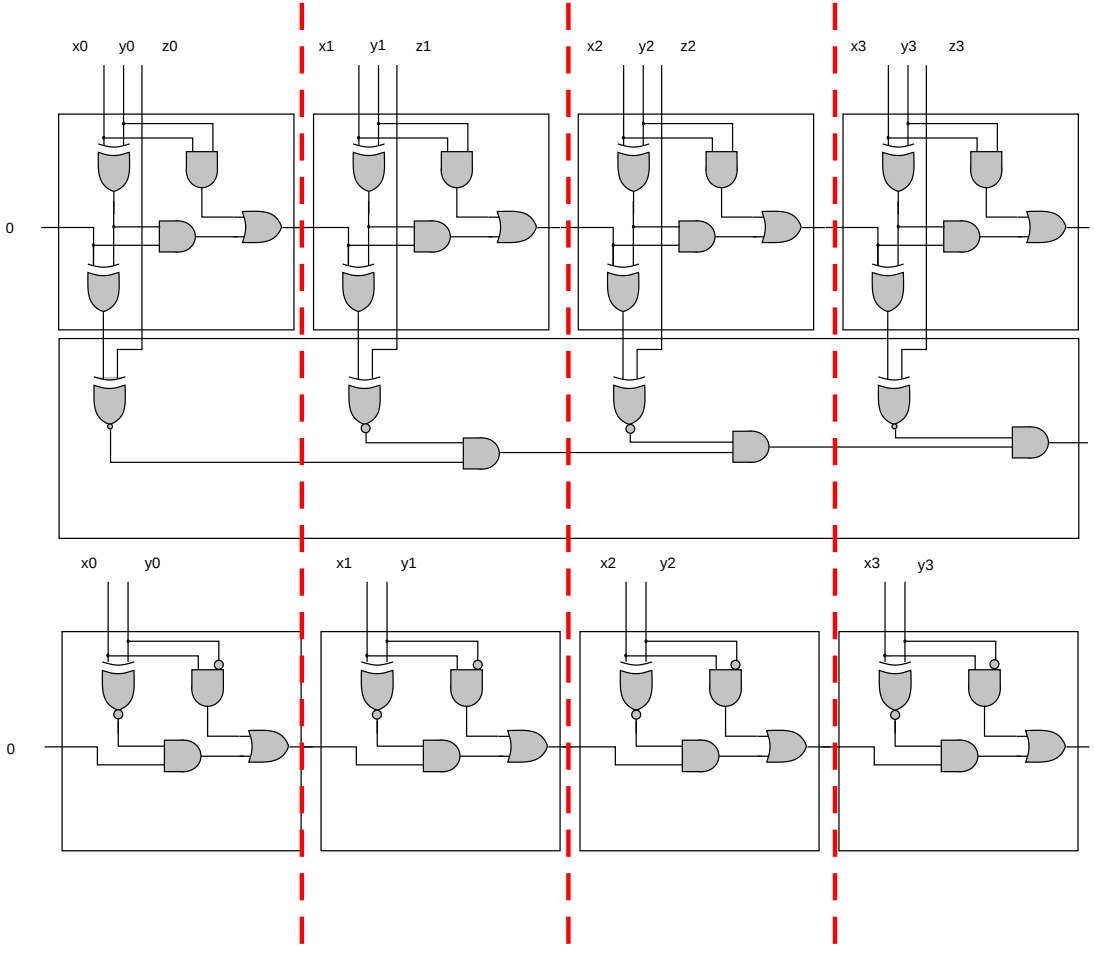

- Cutwidth is constant
- BDD size of adder, comparator linear!
- SIG construction polynomial

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Experiment Setup

- Sun Blade 150, 550MHz CPU, 128MB RAM
- Benchmarks
  - prolangs: pointer analysis community
  - MediaBench: embedded systems community
  - SPEC2K: computer architecture community

prolangs				MediaBench				SPEC2000			
name	#lines	#contexts	#blocks	name	#lines	#contexts	#blocks	name	#lines	#contexts	#blocks
315	1411	49	136	gsm	5473	267	1124	vortex	67211	$9.217*10^{10}$	18433
TWMC	24032	6522	4613	pegwit	5503	1968	1121	bzip2	4665	495	995
simulator	3558	8953	1316	pgp	28065	199551	5265	vpr	16984	179905	4318
larn	9933	1750823	6180	mpeg2dec	9823	44979	2748	crafty	19478	317378	5282
moria	25002	318675286	9446	mpeg2enc	7605	1955	2997	twolf	19756	5538	4231

Experiment Goal

- Goal: evaluate scalability
- Comparative study
  - FICS: flow-insensitive, context-sensitive analysis
  - FSCI: flow-sensitive, context-insensitive analysis
  - FSCS: flow-sensitive, context-sensitive analysis
  - FSCS*: flow-sensitive, context-sensitive, field-sensitive analysis

Experimental Result: prolangs

Benchmarks			Flow Time (s)	Solver Time (s)	Total Time (s)
prolangs	315	CS	0	0.01	0.01
		FS	0.01	0	0.01
		FSCS	0.04	0.01	0.05
		FSCS*	0.04	0.01	0.05
	T-W-MC	CS	0	3.72	3.72
		FS	4.43	0.59	5.02
		FSCS	8.71	1.76	10.47
		FSCS*	8.86	1.68	10.54
	larn	CS	0	0.83	0.83
		FS	0.67	0.19	0.86
		FSCS	21.83	1.64	23.47
		FSCS*	21.94	1.68	23.62
	moria	CS	0	1.47	1.47
		FS	2.24	0.47	2.71
		FSCS	40.60	4.4	45.00
		FSCS*	40.87	4.47	45.34
	simulator	CS	0	0.10	0.10
		FS	0.06	0.04	0.1
		FSCS	0.76	0.10	0.86
		FSCS*	0.79	0.14	0.93

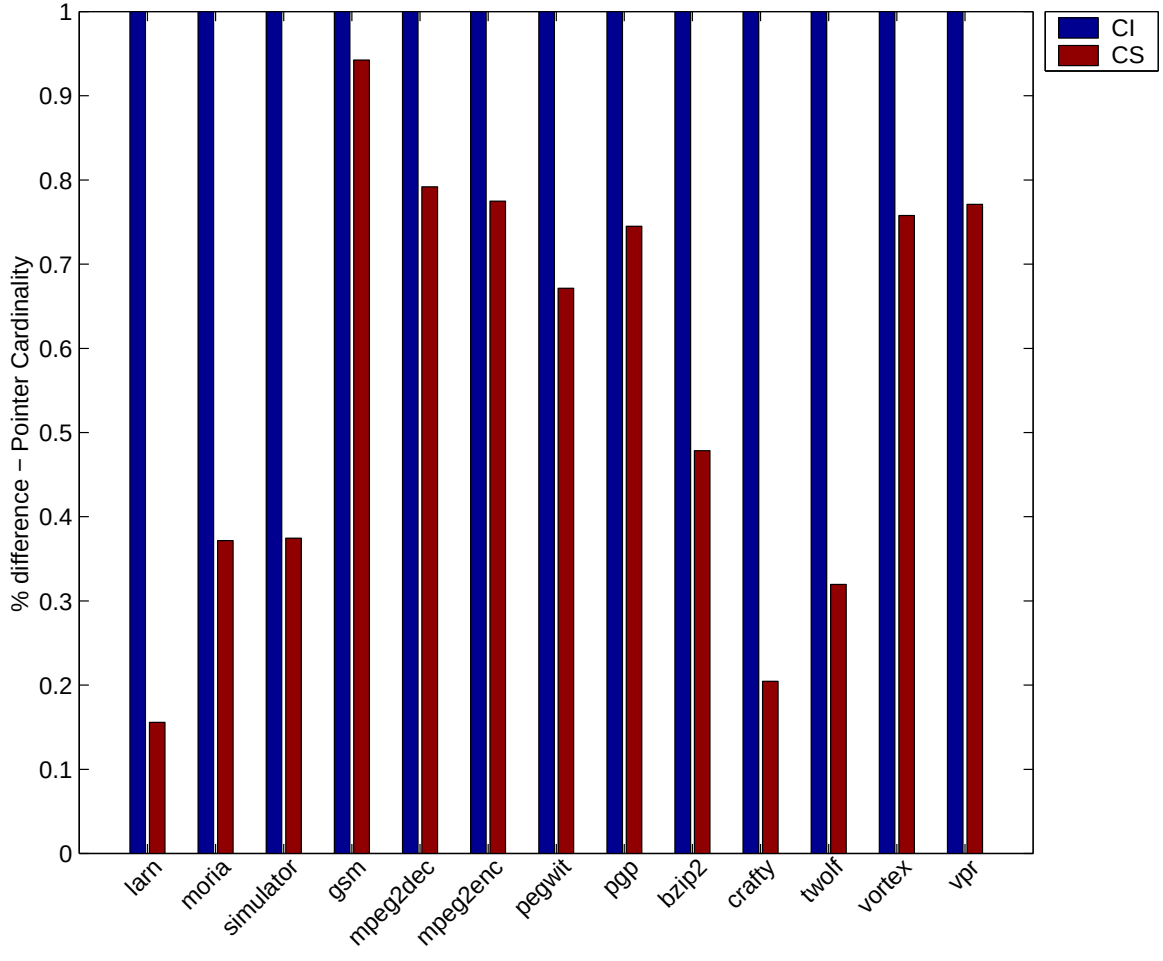
Experimental Result: MediaBench

Benchmarks			Flow Time (s)	Solver Time (s)	Total Time (s)
MediaBench	gsm	CS	0	0.05	0.05
		FS	0.19	0.05	0.24
		FSCS	0.55	0.1	0.65
		FSCS*	0.59	0.08	0.67
	mpeg2dec	CS	0	0.20	0.20
		FS	0.29	0.23	0.52
		FSCS	3.15	0.54	3.69
		FSCS*	3.23	0.48	2.19
	mpeg2enc	CS	0	0.19	0.19
		FS	0.95	0.14	1.09
		FSCS	1.44	0.27	1.71
		FSCS*	1.52	0.23	1.75
	pegwit	CS	0	0.12	0.12
		FS	0.09	0.09	0.18
		FSCS	0.97	0.26	1.23
		FSCS*	0.96	0.25	1.21
	pgp	CS	0	1.05	1.05
		FS	2.32	0.73	3.05
		FSCS	26.28	6.03	32.31
		FSCS*	26.19	5.90	32.09

Experimental Result: SPEC2K

Benchmarks			Flow Time (s)	Solver Time (s)	Total Time (s)
SPEC2000	255.vortex	CS	0	4.32	4.32
		FS	10.53	1.31	11.84
		FSCS	135.59	7.49	143.08
		FSCS*	136.68	7.71	144.39
	186.crafty	CS	0	0.53	0.53
		FS	3.1	0.44	3.54
		FSCS	12.12	2.06	14.18
		FSCS*	12.17	2.09	14.26
	256.bzip2	CS	0	0.02	0.02
		FS	0.06	0.02	0.08
		FSCS	0.3	0.06	0.36
		FSCS*	0.32	0.06	0.38
	300.twolf	CS	0	0.09	0.09
		FS	5.25	0.09	5.34
		FSCS	9.19	0.26	9.45
		FSCS*	9.21	0.20	9.41
	175.vpr	CS	0	0.23	0.23
		FS	1.37	0.1	1.47
		FSCS	5.96	0.4	6.36
		FSCS*	5.84	0.34	6.18

Experimental Result: Precision



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- Leads to new milestones in solving the pointer analysis problem
- Lessons learned
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Further Readings

References

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