combinatorics 001 handout

k-sequence of n-set

a.k.a. *ordered k-list with repetition*

- def'n: ordered arrangement $(a_1, a_2, \ldots, a_k)$ of elements of n-set
- e.g. all 3-sequences of $\{a, b, c, d, e\}$? (how many? 125)

$$(a, a, a)\ (a, a, b)\ (a, a, c)\ (a, a, d)\ (a, a, e)\ (a, b, a) \ldots (e, e, e)$$

- number of k-sequences of n-set is … n^k
  - n choices for each a_j
  - product rule
  - $n \times n \times \ldots \times n = n^k$

k-permutation of n-set

a.k.a. *ordered k-list*

- def'n: k-sequence with no repetition
- e.g. all 3-permutations of $\{1, 2, 3, 4, 5\}$? (how many? 60)

$$(1{,}2{,}3)\ (1{,}2{,}4)\ (1{,}2{,}5)\ (1{,}3{,}2)\ (1{,}3{,}4)\ (1{,}3{,}5)\ (1{,}4{,}2) \ldots (5{,}4{,}3)$$

- number of k-permutations of n-set is … $P(n, k) = n!/(n-k)!$
  - $n - j$ choices for each a_j
  - product rule
  - $n \times (n-1) \times \ldots \times (n-(k-1)) = (n!)/(n-k)!$

k-multiset of n-set a.k.a. *k-combination with repetition*

- def'n: unordered collection $\{a_1, a_2, \ldots, a_k\}$ of elements of n-set
- e.g. all 3-multisets of $\{a, b, c, d, e\}$? (how many? 35)

$$\{a,a,a\}\ \{a,a,b\}\ \{a,a,c\}\ \{a,a,d\}\ \{a,a,e\}\ \{a,b,b\}\ \ldots \{e,e,e\}$$

- number of k-multisets of n-set is … $C(k+n-1, k)$
  - represent k-multiset with k marks and $n-1$ dividers
  - from $k+n-1$ positions, choose positions of k marks

k-subset of n-set a.k.a. *k-combination*

- def'n: k-multiset with no repetition
- e.g. all 3-subsets of $\{1, 2, 3, 4, 5\}$? (how many? 10)

$$\{1,2,3\}\ \{1,2,4\}\ \{1,2,5\}\ \{1,3,4\}\ \{1,3,5\}\ \{1,4,5\}\ \ldots \{3,4,5\}$$

- number of k-subsets of n-set is … $C(n,k) = \binom{n}{k} = \dfrac{n!}{k!(n-k)!}$
  - taking all k-permutations of all k-subsets yields each k-permutation exactly once, so
  - $C(n,k) = P(n,k)/(k!) = \dfrac{n!}{k!(n-k)!}$