CMPUT 671 Assignment One
Fall 2003 Due Oct. 2, 2003

1. **Graph List Coloring** (an easy one to start with...)
INSTANCE Input: A graph $G = (V, E)$ and for each vertex $v \in V$ a list $avail[v]$ of colors available to that vertex.
QUESTION: Can we assign each vertex v a color $c[v] \in avail[v]$ so that for all edges $\{u, v\} \in E$, $c[u] \neq c[v]$?

   (a) Give an example graph with two colors per list which has no odd cycle, but which cannot be colored. Note: the union of the lists may have more than 2 colors.

   (b) Show that if each list has at most two colors the problem is in P, even if the union of the lists has a total of more than two colors.

   (c) Show that for lists of length three or more the problem is NP-complete.

2. Let us say that a clause is *positive* if every literal is a positive occurrence of a variable, and *negative* if all literals are negated variables. We say an instance (U, C) of CNF SAT is *pos-neg* if all clauses are either positive or negative. (See also the next question).

   A hypergraph is a pair $H = (V, E)$ where V is a finite set and E is a set of subsets of V. In general, the subsets may vary in size. If $\forall e \in E, |e| = \alpha$ we say that the graph is an α-ary hypergraph. What we have been calling graphs are just 2-ary hypergraphs.

   Starting with the fact that 3-SAT is NP-complete, show that each of the following is NP-Complete. You may solve these in any order, and use intermediate results to show subsequent problems are NP-complete. You are not allowed to use other NP-completeness results.

   pos-neg 3-SAT

   Input: An instance of pos-neg 3-SAT; i.e. a pos-neg instance in which every clause has exactly 3 literals.
   Query: Does there exist a satisfying truth assignment?

   Symmetric pos-neg 3-SAT

   We define *symmetric* pos-neg 3-SAT as a subset of pos-neg 3-SAT in which for any instance $\langle x \vee y \vee z \rangle \in C \iff \langle \overline{x} \vee \overline{y} \vee \overline{z} \rangle \in C$.
   Query: Is there a satisfying assignment?

   Hypergraph 2-coloring

   Input: A (general) hypergraph Graph H.
   Query: Is there a two coloring of the vertices V such that every edge in E has at least one vertex of each color?

   3-ary Hypergaph 2-coloring

   Input: A 3-ary hypergraph H
   Query: Is there a two coloring of the vertices V such that every edge in E has at least one vertex of each color?

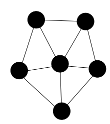
3. Consider the pos-neg 2-SAT instance

$$\langle x \vee y \rangle \, , \langle x \vee z \rangle \, , \langle y \vee z \rangle \, , \langle \overline{x} \vee \overline{y} \rangle \, , \langle \overline{x} \vee \overline{z} \rangle \, , \langle \overline{y} \vee \overline{z} \rangle$$

   (a) Use a simple counting argument to show the instance is unsatisfiable.

   (b) Use resolution to show it is unsatisfiable.

4. Let W_{2n} be the wheel on $2n$ vertices, $n \geq 2$ and consider the reduction from k-coloring to CNF SAT given in class for $k = 3$.



W_6

   Let $R(n)$ be the number of clauses used in a minimum resolution proof that the resulting SAT instance is unsatisfiable. Estimate as closely as you can $R(n)$.

5. In class (it is in Garey and Johnson and most texts with an NP-completeness section also) we showed that CNF SAT can be reduced to 3 CNF SAT. Prove that any resolution proof for a transformed instance is at least as large as a minimum resolution proof of the original instance.

6. ($*$) Define a suitable class of pos-neg 3-SAT instances such that there is a simple argument to show each member of the class is unsatisfiable, but such that the shortest resolution proof is exponential in the number of variables. Prove any claims you make. Hint: you may use any results from class or related texts or papers.

Due to past experience, the following seems to be a necessary statement. I am aware that there are answers to many of these questions in various texts and papers. I do not want to see those results, I want to see the results of your efforts. If you get stuck and must consult me, other students or texts or papers, then you MUST append a bibliography citing all such inputs. I do not want to see several assignments with essentially the same answer sets. If there is too much co-operation and/or too much similarity between assignments, the mark will be severely reduced, and if it is apparent there is copying, or significant consultation without appropriate reference, the students in question will be referred to the dean with all attendant consequences. If you are in doubt as to what is appropriate, then please consult with me before offering or accepting any consultation.