

## 7. Windows

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## Playing with Search Windows

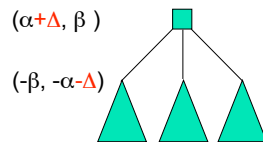
- There are many ways that the search window can be altered
- Use this to:
  - Improve search efficiency
  - Answer questions
  - Low overhead exploratory searches

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## Improving $\alpha$

Improve  $\alpha$  means that the children have a smaller threshold for achieving a cutoff

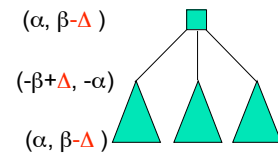


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## Improving $\beta$

Improve  $\beta$  means that the grand children have a smaller threshold for achieving a cutoff



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## Aspiration Search

- Normally search the root using  $(-\infty, +\infty)$
- What if you have a good idea of where the root value really is?
- Assume you believe the root value is close to  $V$ , plus or minus  $\Delta$ ?
- Search the root using  $(V-\Delta, V+\Delta)$
- But...

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## Aspiration Search

- If result of the search is in  $(V-\Delta, V+\Delta)$ , then everything is OK and you have obtained the root value with less search
- What if the value,  $v$ , is  $\leq V-\Delta$ ?
  - Fail low
  - Re-search using window  $(-\infty, v)$
- What if the value,  $v$ , is  $\geq V+\Delta$ ?
  - Fail high
  - Re-search using window  $(v, +\infty)$

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## Aspiration Search

- What is a good guess for the value of a search?
  - Use the value returned for the depth  $d$  search as the guess for the  $d+1$  search
- How big should  $\Delta$  be?
  - Smaller  $\Delta$  means less search
  - Use experimentally
- Note: aspiration search can build trees smaller than the minimal tree -- why?

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## Aspiration Search

```
guess = 0;
for( depth = 1; TimeAvailable(); depth++ ) {
  alpha = guess - Δ; beta = guess + Δ;
  /* Search moves: maximize score */
  if( score >= beta ) {
    alpha = score; beta = ∞;
    /* Search with new window */
  } else if( score <= alpha ) {
    alpha = -∞; beta = score;
    /* Search with new window */
  }
  guess = score;
}
```

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## Narrow Windows

- The idea of narrowing a window can be taken to the extreme!
- What are the semantics of  $(v, v+1)$ ?
- Minimal or null window
- Answers a Boolean question:
  - Is the value  $\leq v$  or is it  $>v$

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## CUT Nodes Revisited

- We know that at CUT nodes, with high probability the best move is being searched early
- That implies that the rest of the moves are inferior to the best
- Can we reduce search effort by exploiting this observation?

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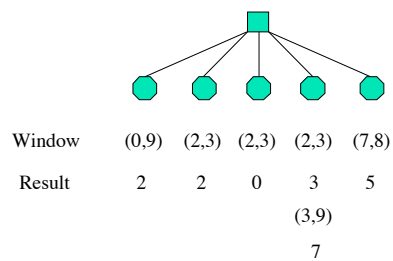
## NegaScout <sup>[1]</sup> and PVS <sup>[2]</sup>

- Search best move with the full window
- Search the remaining moves with a minimal window attempting to prove them inferior to the best move
  - Could be wrong and may need to re-search
  - Re-search may not be expensive because of saved TT results
  - With good move ordering, number of re-searches is usually very small

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## Example



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## NegaScout

```
score = -AlphaBeta( Successor( 1 ), -beta, -alpha, d );
if( score < beta ) {
  for( child = 2; child <= NumSuccessors( s ); child++ ) {
    lbound = MAX( score, alpha ); ubound = lbound + 1;
    result = -AlphaBeta( Successor( child ), -ubound, -lbound );
    if( result >= ubound && result < beta ) {
      result = -AlphaBeta( Successor( child ), -beta, -result );
    }
    if( result > score ) score = result;
    if( result >= beta ) break;
  }
}
return( score );
```

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## Taking MWs to the Extreme

- Pearl first proposed the idea of doing minimal window searches (Scout) [3]
- Used them to answer a Boolean question about the search tree
- Can we couch the entire alpha-beta search as a series of Boolean questions?

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## Scout Searches

- To determine the value of root do a series of minimal window searches to narrow in on the value.
- Could do a divide and conquer approach, using each search to cut the possible alpha-beta range in half

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## Scouting

- Assume value lies in range [-100,100]
  - Possible values are -100..100
  - Search with window [0,1] gives result of 1
    - Possible values are 1..100
  - Search with window [50,51] gives result 48
    - Possible values are [1..48]
  - Search with window [24,25] gives result 24
    - Possible values are [1..24]
  - And so on...

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## MTD(f)

- Divide-and-conquer is too slow.
- We have a good idea where the true value is... start from there.
- Use the value of the previous iteration to see the starting minimal window.

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## MTD(f) [4,5]

```
lbound = -∞; ubound = +∞;
score = ResultOfPreviousIteration();
repeat {
  if( score == lbound )   window = score;
  else                   window = score - 1;
  score = AlphaBeta( s, window, window + 1, d );
  if( score < window )   ubound = score;
  else                   lbound = score;
} until ( lbound >= ubound );
```

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## MTD(f)

- Note that all searches go down (except the last one), or all go up (except the last one)
- This only makes sense with a TT so that results of previous searches can be reused
- For chess an average of 3-5 iterations were needed to converge
- Node reductions of 5-15%

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## MTD(f)

- By changing the initial guess and the choice of bounds, SSS\* and DUAL\* can be derived.
- There are subtle points that may result in MTD(f) not converging
  - Bugs in your algorithm
  - Search depth changes
  - TT side effects

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## References

- [1] Alexander Reinefeld. "An Improvement to the Scout Tree Search Algorithm", *ICGA Journal*, vol. 6, no. 4, pp. 4-14, 1983. See also [4].
- [2] Murray Campbell and Tony Marland. "A Comparison of Minimax Tree Search Algorithms", *Artificial Intelligence*, vol. 20, pp. 347-367, 1983.
- [3] Judea Pearl. "Scout: A Simple Game-Searching Algorithm with Proven Optimal Properties", AAAI National Conference, 1980.
- [4] [www.cs.vu.nl/~aske/mtdf.html](http://www.cs.vu.nl/~aske/mtdf.html)
- [5] Aske Plaat, Jonathan Schaeffer, Wim Pijls, and Arie de Bruin. "Best-first Fixed-depth Minimax Algorithms", *Artificial Intelligence*, vol. 87, no. 1-2, pp/ 1-38, 1996.