Generalized Thermography: A New Approach To Evaluation in Computer Go

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Abstract

Go is a game of creating territories. Evaluation, or counting territory, is to a large extent a local process. Combinatorial game theory provides tools which are more efficient for localized analysis of games than full-board minimax search. One of these tools, thermography, has recently been generalized to local games that contain *ko* loops in their game graph.

Generalized thermography computes a *mast value* and a *temperature* for such games, which can be used to estimate the full board game score and the value of moves precisely.

We report the first implementation of an algorithm for generalized thermography, which can be used to evaluate almost arbitrary local Go positions. To illustrate the power and scope of our program, we give examples of ko positions and their thermographs.

1 Local Analysis of Go Positions

Solving full-board Go positions by traditional minimax search methods is feasible only for very small board sizes [Thorp and Walden, 1972] or extremely late endgames. A program based on combinatorial game theory can do much better: it can solve ko-free Go positions containing many independent local games of moderate size [Müller, 1995].

A drawback of previous combinatorial game methods was that they could not handle ko. Ko fights often affect the evaluation of local situations [Müller and Gasser, 1996].

Generalized thermography [Berlekamp, 1996] is a powerful new method for analyzing local Go positions, including those that contain ko. For each local game a data structure called *thermograph* is computed. This information is not enough to guarantee perfect full-board play, but it yields simple and good local algorithms. There is evidence that the move chosen by such algorithms is sound in the large majority of real game situations, and it is provably optimal in several simplified models of play.

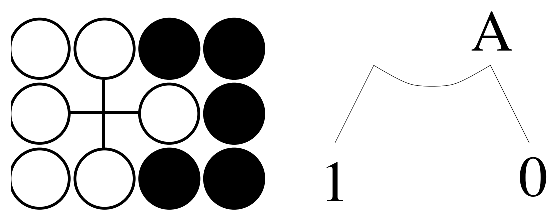

Figure 1: A simple ko and its game graph

We report the first implementation of generalized thermography. Such an algorithm can be used as a building block for evaluation in Computer Go. It can be used to count the score and find the value of moves.

The structure of this paper is as follows: Section 1 explains the combinatorial game approach to Go and reviews the theoretical background of combinatorial game theory and generalized thermography. Section 2 describes the algorithm and discusses some limitations in case of multiple ko. Section 3 contains a catalog of representative examples taken from textbooks and master games, proceeding from simple kos to iterated ko, *mannen ko* and the famous *"3 Points Without Capturing"*.

1.1 The Combinatorial Game Approach to Go

Necessary steps for playing Go by the methods of combinatorial game theory are board partition, local search, local evaluation, and move selection. We give a brief overview of these phases. Detailed descriptions can be found in [Müller, 1995].

Board Partition and Subgame Identification

The precondition for applying combinatorial game theory to a game is that it decomposes into subgames. In Go, this happens when parts of the board are separated by walls of safe stones. Moves in one part have no effect on other parts across such a wall.

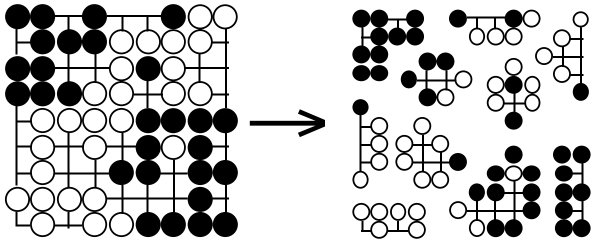

Figure 2: Board partition

Local Search

Local search differs from minimax search because successive moves by the same player have to be considered. Search builds a *game graph*, which is a directed cyclic graph in Go. We avoid using the traditional term *game tree* because it is misleading here.

Local Evaluation

Each local game graph can be evaluated by computing its *thermograph*, which yields the *mast value* and the *temperature*. This step is explained in detail below.

Full Board Move Selection

Algorithms such as Berlekamp's *Sentestrat* estimate the score and decide a move by using the results of local evaluation.

Heuristic Game Play

In analogy to minimax search, the ideas of combinatorial game theory can be used in a heuristic setting [Müller, 1995]. Before walls of safe stones are complete, heuristic partitioning methods can be used to define local games.

Selective local search expands only some lines of play, and statically evaluates stable-looking positions. This yields approximate mast values and temperatures. Integrating such techniques into a full scale heuristic Go program remains a challenging research topic. The technique presented here is the first evaluation procedure that makes such an approach to Computer Go seem feasible.

1.2 Combinatorial Game Theory and Thermography

The basic combinatorial game concepts of temperature and thermographs have been known for a long time [Berlekamp *et al.*, 1982]. Generalized thermography [Berlekamp, 1996] is a new extension to this theory.

Leftscore and *Rightscore* form a connection between combinatorial games and minimax theory: they are the minimax values of a game that result if players move alternately and Left (Black) or Right (White) play first, respectively.

Cooling imposes a tax t on every move. This technique simplifies a game while retaining much of its structure.

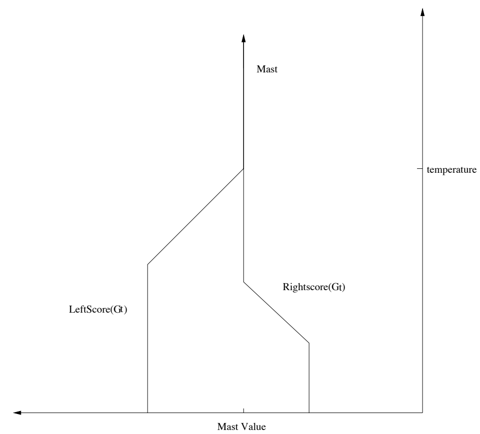

Figure 3: Sample thermograph

The *temperature* of a game is the smallest nonnegative amount t that makes Leftscore and Rightscore of the cooled game equal. This score is called the *mast value* of the game. The *thermograph* shows the process of cooling:

Let G_t be the game G cooled by t. The thermograph of G shows both *Leftscore*(G_t) and *Rightscore*(G_t) plotted along the reversed x-axis, as a function of the temperature t on the y-axis.

Thermographs provide a powerful tool for determining mast values and temperatures. The thermograph of a game G can be computed from the thermographs for its options G^L and G^R by applying the tax and selecting the best option at each t:

$$LeftScaffold(G) = max_{G^L}(Rightscore(G_t^L) - t)$$
$$RightScaffold(G) = min_{G^R}(Leftscore(G_t^R) + t)$$

The temperature of G is the value of t at which the scaffolds meet. At values of t less than the temperature of G, the scaffolds become the *walls* of the thermograph of G. At higher values, the left and right walls coincide to form a *vertical mast*, which starts at the point where the scaffolds meet.

1.3 Komaster and Pass-ban

Komaster is a player that is able to win all kos because of a surplus of ko threats. The opponent is the *koloser*. Komaster cannot win ko fights for free, however: after starting to play a ko, komaster has to continue to play locally. This rule ensures that once a ko is started, it will be won at the same temperature. Komaster has enough ko threats to win the ko but not enough threats to lower the temperature during the ko fight.

In classical thermography, players have the option of passing locally at each move. If Komaster had this option as well, she could win all ko fights too easily: she would move into a position where the opponent could

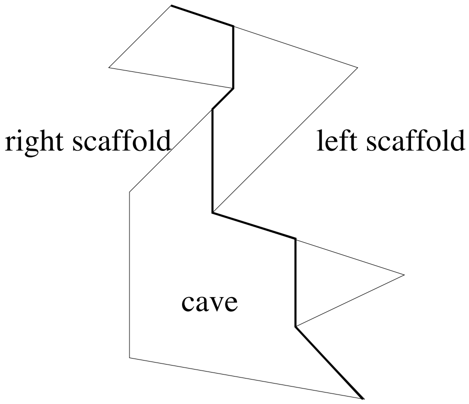

Figure 4: The balloon's path through a cave

not win in one move, then wait until the overall temperature is very low to play the remaining moves and resolve the ko. Whenever the opponent tried to play the ko, she would just use her surplus of threats to revert the ko to the previous state.

To calculate the mast value for a ko, Berlekamp's *pass ban* rule states that once komaster starts playing in a ko she has to go ahead and win it.

1.4 Generalized Thermography for Games with Loops

Generalized thermographs differ from classical thermographs. This is a consequence of the fact that the mast of pass-banned thermographs is non-vertical.

In thermographs of loopfree games, the slopes of the walls indicate whether the difference in the number of moves played by both players is zero or one. In *sente* regions, the difference is zero, and the wall is vertical. In *gote* regions, the difference is one, and the wall is diagonal. A player's profit increases as the tax decreases.

Generalized thermography allows other differences in the number of moves played by both players at a given temperature. The ko ban rule may force komaster to spend two or more moves in a row locally to win a ko. Slopes such as $2, 3, 4, \ldots$ result.

Generalized thermographs can also bend backwards, in situations where the koloser starts the ko, then forces komaster to spend two or more moves to win and eliminate it. In this case, koloser gains profit as the tax increases.

With ko, the scaffolds used to construct a thermograph can intersect more than once, leading to several *cave* and *hill* regions. A hill region corresponds to the classical case, with the scaffolds defining the walls of the thermograph. In a cave, the left scaffold is to the right of the right scaffold. The left and right walls of the thermograph coincide and follow a "balloon's path" through the cave, as shown in Figure 4.

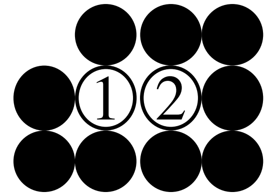

Figure 5: 3 at 1. Suicidal two-move White-only loop

Placid and Hyperactive Ko

Given a komaster, we can compute the thermograph of a loopy game. The start of the vertical mast defines the temperature of the game. In contrast to loopfree games, the mast value cannot always be interpreted as the mean value of a game. Kos that have the same mast value independent of who is komaster are called *placid*. The mast value of *hyperactive* ko depends on who is komaster. Examples for both types of ko are given in section 3.

2 An Algorithm for Generalized Thermography

Classical thermographs for ko-free games can be computed recursively directly from the definition. This direct approach does not work for ko because of the loops in the game graph. In generalized thermography, loops are broken by the pass ban rule.

2.1 Pass-banned Thermographs

Because of the komaster and pass ban rules some nodes in the game graph obtain two different states: if a node in a loop is entered for the first time, all moves from it are allowed. The thermograph of the node is computed recursively from its options in the standard way. However, when a node is reentered, both the komaster rule and the pass ban rule change the computation:

- The komaster rule removes loop-closing moves by the koloser.
- The pass ban rule removes the option of passing for komaster.

2.2 Preliminaries: Elimination of Single-Player Loops and Thermographs of Ko-free Subgames

In Go, single-player loops are possible when the rules allow suicide (see Figure 5). Such loops contain at least one bad move and can be eliminated by pruning the loop-closing move [Müller, 1995]. After this preprocessing, each remaining loop in the game graph contains moves by both players.

If the game graph reachable from a node contains no ko, i.e. if it is a tree, we can compute thermographs in this subtree using the classical recursive algorithm. Such loopfree subtrees always exist, for example terminal nodes of a game.

2.3 Computing Generalized Thermographs

Before a thermograph can be computed, a loop-breaking move must be selected for each loop in the game. In the most common case of 2-move loops the choice is unique: there is only one move by each player in the loop. After marking loop-breaking nodes, thermograph computation starts. *ComputeNode()* determines whether all necessary thermographs of options needed to compute pass-banned or non-banned thermographs are known.

If a game graph contains several stages of loops, the order of computation of thermographs can be intricate. Our program repeatedly traverses the game graph, calling *ComputeNode* to identify nodes where new thermographs can be computed.

```
boolean ComputeNode (node, komaster)
{
  canCompute = FALSE;
  if (retrieve all komaster options:
    pass-banned thermographs of nodes in loop,
    non-banned thermographs of all others)
  {
    if (retrieve pass-banned thermograph)
      compute non-banned thermograph next:
      koOptions = {};
    else
      find koOptions of koloser;

    if (retrieve all non-banned thermographs
        of {koloser options - koOptions} )
    {
       canCompute = TRUE;
       compute Black (White) scaffold
       from maximum (minimum) of walls
       of all options;
       apply taxes to both scaffolds;

       determine type of new thermograph:
       if (koOptions = {})
         type = non-banned;
       else
         type = pass-banned;

       generate thermograph;
       store(node, thermograph, type);
    }
  }
  return canCompute;
}
```

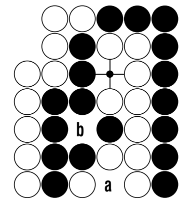

Figure 6: Double ko

Limitations for Multiple Ko

Generalized thermography works well in local games where at most one ko is relevant at any given time. The ko-ban methodology computes a value of playing in this ko by comparing it with the value of moves elsewhere. If there is more than one concurrent ko locally, it must be clear which one is currently being fought: a move in this ko must dominate all moves in other local kos.

The algorithm fails in positions with two or more simultaneous ko, as in Figure 6. In normal ko, komaster can eventually win any ko or series of ko by ignoring sufficiently many threats. In double ko or triple ko, the only good moves for both players are to stay in the loop. Such a ko can never be resolved into a terminal position where there are no more good moves. Since all loops are broken by the algorithm, the capture of one ko cannot be balanced by taking another ko.

3 Examples

The format of the examples is as follows: Each example shows the picture of a Go position, followed by its game graph. For complex games, only selected lines of play are shown. Omitted parts of the game graph are indicated by dots. The third picture in each example shows the thermographs for Black as komaster using solid lines, and for White as komaster using dashed lines. For display purposes, White's thermographs have been shifted slightly to the left, in order to see both thermographs in regions where they coincide.

All examples presented here were computed using Japanese scoring. To give an indication of the values and temperatures involved, a scale with integer unit is drawn at temperature 1. A much larger collection of examples is given in [Müller *et al.*, 1996].

Diagrams showing the game graph are similar to those used for game trees in combinatorial games [Berlekamp *et al.*, 1982]. Non-loop moves are represented by a straight line. A two move loop is drawn as an arc. It can be traversed in both directions. Lines for black moves initially lead down and to the left, white moves lead down and to the right. Terminal positions such as 0 and 1 in Figure 7 are marked by their value. The scores in the game graph don't show *dame* points.

The simple ko of Figures 7 and 8 contain just one loop. They differ only in the size of the ko. Iterated ko have two or more stages. The 2-iterated ko in Figures 9 and 10 appears frequently as a subgame of other, more complicated situations.

Figures 11 to 14 show an approach move ko (yose ko). Scoring of multi-move approach ko depends on whether the rules demand capturing the stones. The examples were computed under the assumption that stones must be captured eventually.

Ten-thousand year ko (mannen ko) in Figures 15 to 17 contain a latent big ko fight, yet their temperature is low because it is difficult to start the ko. The position in Figure 16 is unusual: if Black is komaster then White is as good as dead. Both players pass even at temperature zero. Scoring depends on the rules of the game, whether Black has to capture White or not and whether this costs any points.

Figures 18 and 19 show small hyperactive ko. They contain a position where one player can choose to increase the stakes in the ko fight. The "rogue" position of Figure 18 was the first hyperactive ko to be studied in detail. Figure 19, Kao's ko, is a hyperactive ko in a very small area. After taking the ko, White can increase the stakes by pushing in from the right side.

"3 Points Without Capturing" in Figures 20 to 21 is a classical position which appears in every rulebook.

4 Summary and Outlook

Combinatorial game theory provides tools for efficient local analysis of Go. The most recent development, generalized thermography, allows the analysis of positions containing ko, which is crucial for developing a general purpose evaluation procedure for Computer Go. We have presented an implementation of this method and shown that it can compute masts and temperatures for a wide range of local Go positions. This makes it possible to apply efficient combinatorial game algorithms to Computer Go.

In this paper we present precise computations for tightly surrounded areas. We think that heuristic methods based on the same approach can produce good evaluations for more open local situations. This is a topic for future research.

5 Glossary

Cooling A technique for simplifying a combinatorial game by applying a tax

Game graph A graph showing the moves and final positions of a game

Gote Opposite of sente, losing the initiative

Hyperactive ko A ko where mast value depends on who is komaster

Komaster A player who can win a ko

Koloser Opposite of komaster

Leftscore Minimax score if Left (Black) plays first

Mast value A measure of the score of a local game

Placid ko Opposite of hyperactive

Rightscore Minimax score if Right (White) plays first

Scaffold The taxed options of a combinatorial game, used to compute the walls of the thermograph

Sente The initiative, the right to move

Tax A real number, the amount that a player has to pay for making a move

Temperature A measure for how urgent it is to move in a combinatorial game

Thermograph A data structure consisting of walls that can be used to determine temperature and mast value of a combinatorial game

Wall The left (right) wall of a thermograph shows the Leftscore (Rightscore) of a cooled game.

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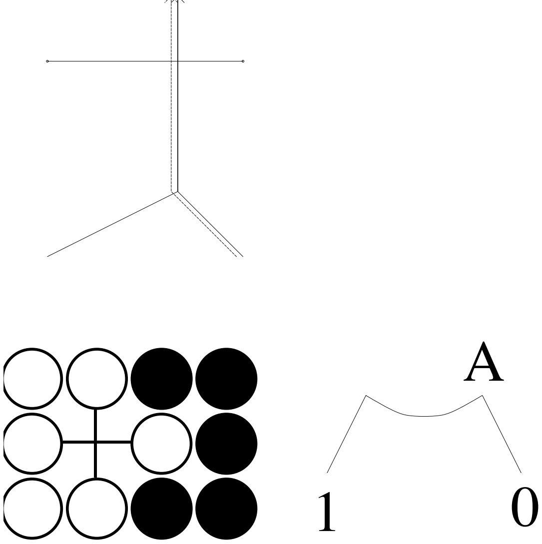

Figure 7: One point ko, node A

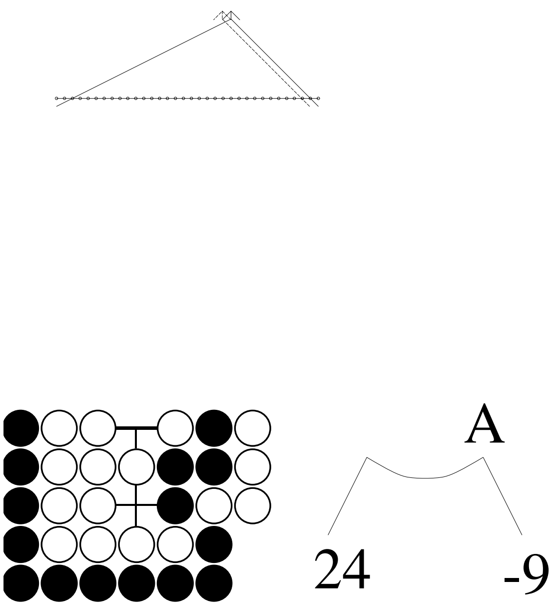

Figure 8: 33 point ko, node A

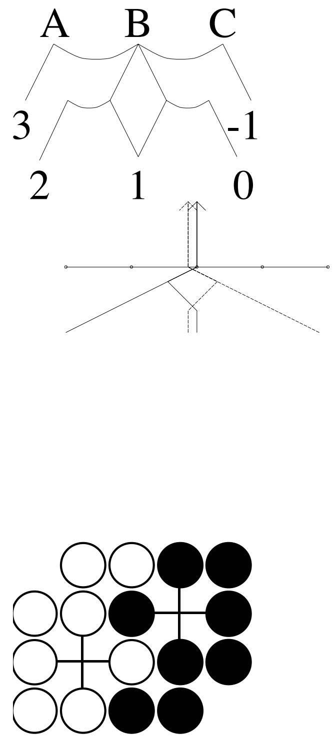

Figure 9: 2-iterated ko, node B

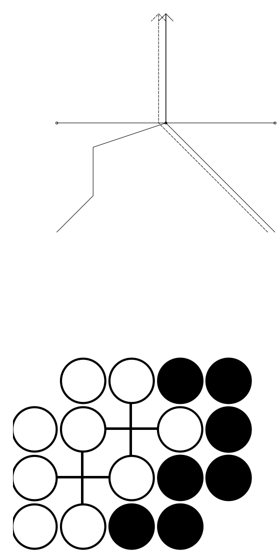

Figure 10: 2-iterated ko, node C

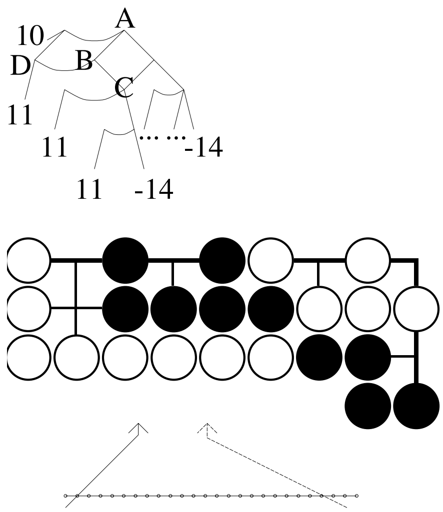

Figure 11: Approach ko, node A

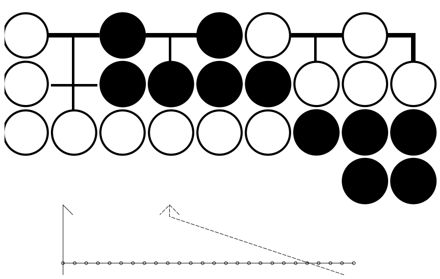

Figure 12: Approach ko, node B

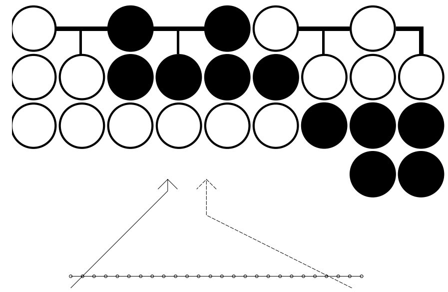

Figure 13: Approach ko, node C

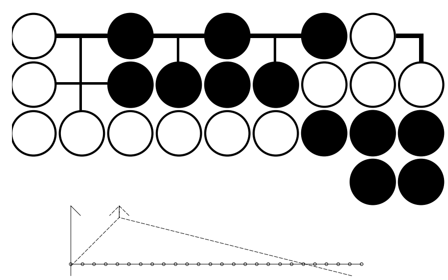

Figure 14: Approach ko, node D

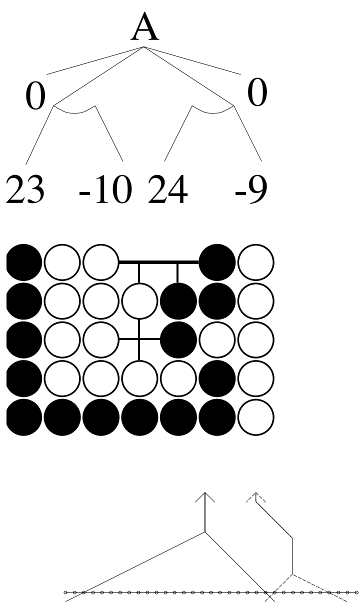

Figure 15: Mannen ko (1), node A

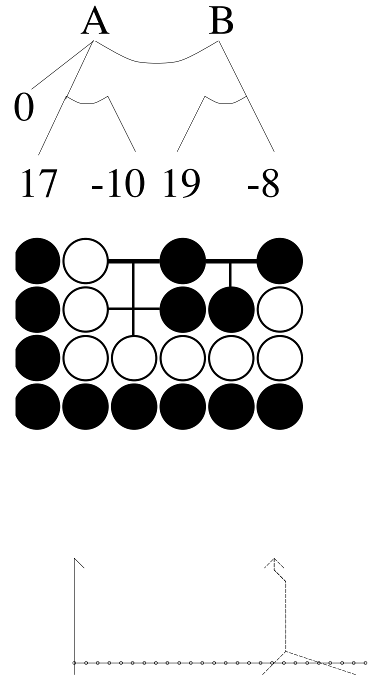

Figure 16: Mannen ko (2), node A

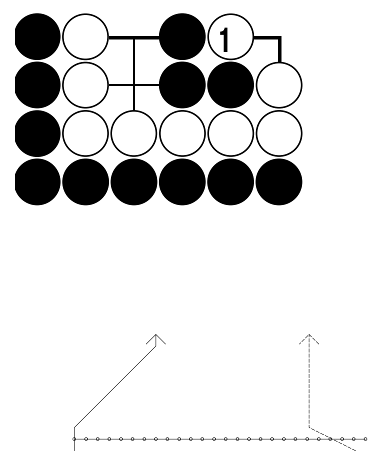

Figure 17: Mannen ko (2), node B

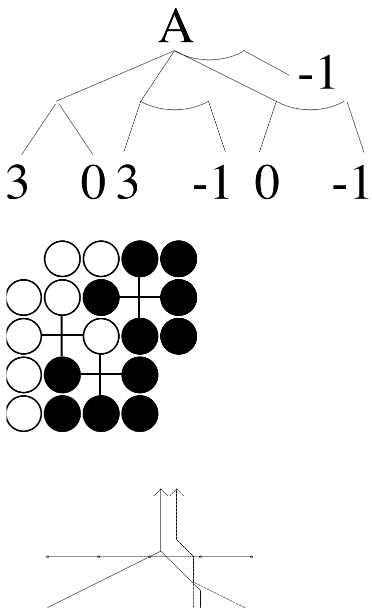

Figure 18: Rogue, node A

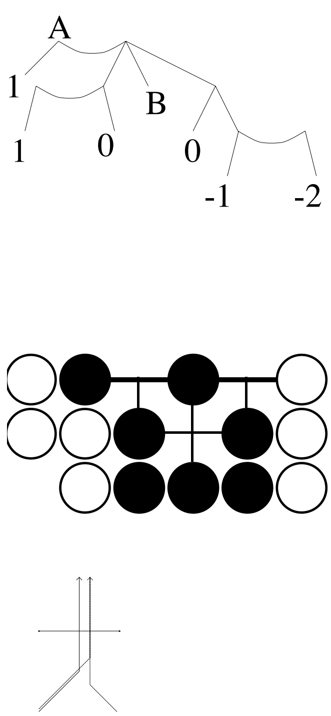

Figure 19: Kao's ko, node A. Node B in the game graph is equivalent to B in Fig. 9

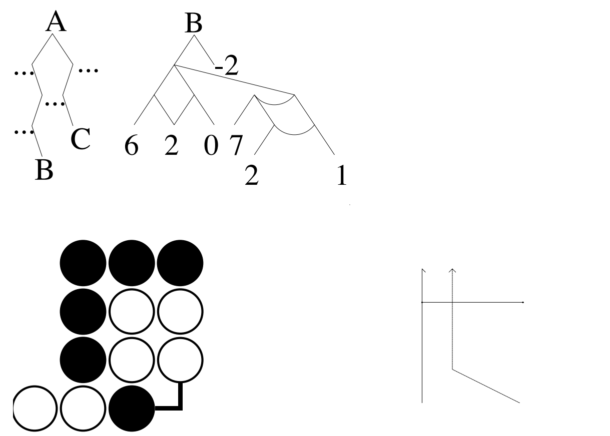

Figure 20: 3 Points Without Capturing, node A

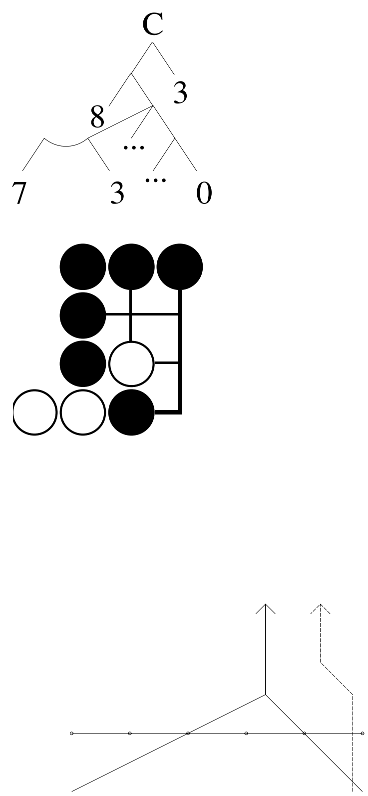

Figure 21: 3 Points Without Capturing, node C