Near-Optimal Bidirectional Search

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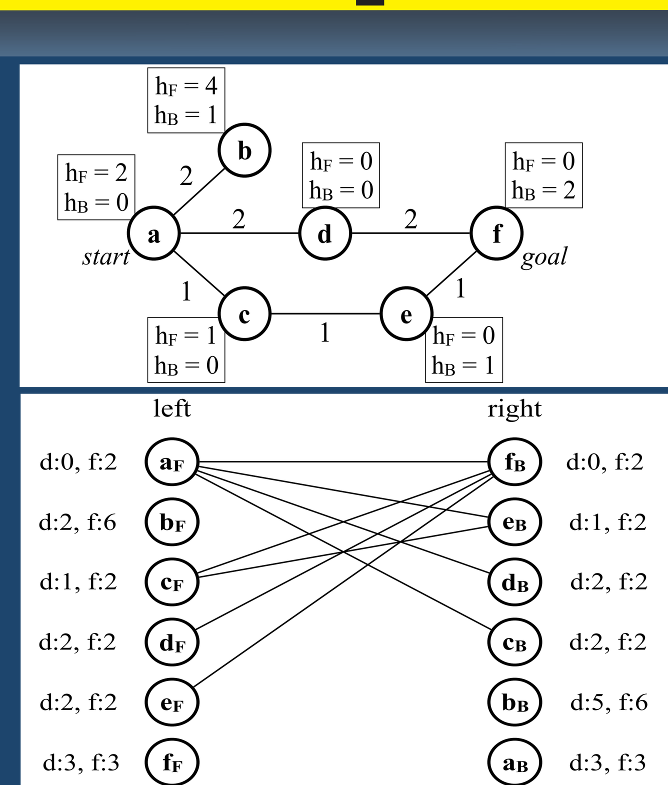
Necessary Node Expansions

- Unidirectional algorithms must expand all states with $f < C^*$. Such states are called "surely expanded" (s.e.).
- Bidirectional algorithms do not have s.e. states, but s.e. pairs (Eckerle *et al.*, 2017): at least one of u and v must be expanded for all pairs with $lb(u,v) < C^*$, where

  $lb(u,v) = max\{f_F(u), f_B(v), g_F(u)+g_B(v)\}$.
- Surely Expanded pairs can be represented by a bipartite graph (Must-Expand Graph).



A sample problem instance and its Must-Expand Graph.

What Must-Expand Graph gives us:

- The set of states expanded by any admissible algorithm must be a vertex cover in Must-Expand Graph.
- A new framework to analysis the necessary node expansions for all bidirectional algorithms:
- The minimum number of node expansions among all possible algorithms is equal to the size of minimum vertex cover in Must-Expand Graph.

New Algorithm: NBS

We present a new bidirectional front-to-end algorithm, Near-Optimal Bidirectional Search (NBS), with following properties:

- It will always return optimal solution.
- It will never do more than twice the minimum necessary node expansions.
- Its bound, two, is tight.
- It can be implemented with efficient data structure to be practical.

Pseudocode of NBS

While lb_{min} < currentSolution

  Choose the pair (u,v) with min lb

  $lb_{min} = lb(u,v)$

  Forward-expand u, backward-expand v

End While

Highlights of NBS implementation:

- NBS is adapted from a greedy vertex cover algorithm (Papadimitriou and Steiglitz, 1982).
- The tie-breaking matters. We choose to break ties towards pairs with low f-cost.
- Naive implementation of pair selection needs all-pair computation, which requires $O(n^2)$ time per selection operation.
- Our efficient data structure for pair selection only needs $O(\log n)$ amortized time per selection.

Performance

The general trend:

- When the heuristic is very strong, A* performs best.
- As the heuristic get weaker, or the problems get harder, the bidirectional approaches become competitive.

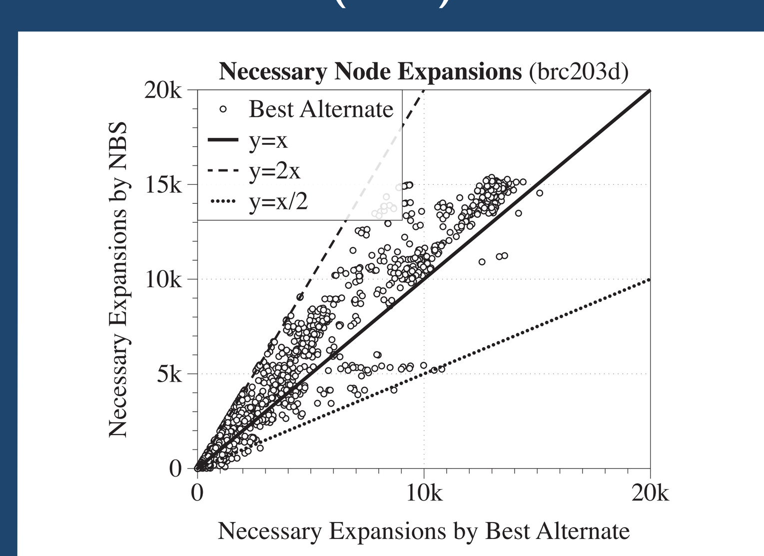
NBS is practical:

- NBS can do less node expansion and can run faster on some domains.

NBS is an insurance:

- When NBS is not the best approach, it is never far from the best; when NBS is the best approach, it could be much better than existing alternatives. NBS is the only algorithm with bounded suboptimality.

A comparison between necessary expansions by NBS(y-axis) and best alternative(x-axis) on each instance



Average running time to solve an instance (in seconds)

Domain	h	A*	BS*	MMe	NBS
DAO	Octile	0.005	0.006	0.015	0.007
Mazes	Octile	0.035	0.022	0.060	0.019
TOH4	12+2	3.23	2.44	4.17	3.54
TOH4	10+4	52.08	23.06	30.64	16.60
Pancake	GAP	0.00	0.00	0.00	0.00
Pancake	GAP-2	14.16	4.91	5.25	5.23
Pancake	GAP-3	*N/A*	212.33	72.13	77.17
15 puzzle	MD	47.68	29.59	41.38	37.67

Average states expansions to solve an instance

Domain	h	Strength	A*	BS*	MMe	NBS	MM0
DAO	Octile	+	9,646	11,501	13,013	12,085	17,634
Mazes	Octile	-	64,002	42,164	51,074	34,474	51,075
4 peg TOH	12+2	++	1,437,644	1,106,189	1,741,480	1,420,554	12,644,722
4 peg TOH	10+4	-	19,340,099	8,679,443	11,499,867	6,283,143	12,644,722
16 Pancake	GAP	+++	125	339	283	335	*unsolvable*
16 Pancake	GAP-2	-	1,254,082	947,545	578,282	625,900	*unsolvable*
16 Pancake	GAP-3	--	*unsolvable*	29,040,138	7,100,998	6,682,497	*unsolvable*
15 puzzle	MD	+	15,549,689	12,001,024	13,162,312	12,851,889	*unsolvable*