A Guide to Budgeted Tree Search

Nathan R. Sturtevant
University of Alberta
Amii Fellow, CIFAR Chair

Malte Helmert
Universität Basel

Talk Overview

- Budgeted Tree Search (BTS) is a new algorithm with better worst-case guarantees than IDA*

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- Why do we need BTS?
- How does BTS work?

Start

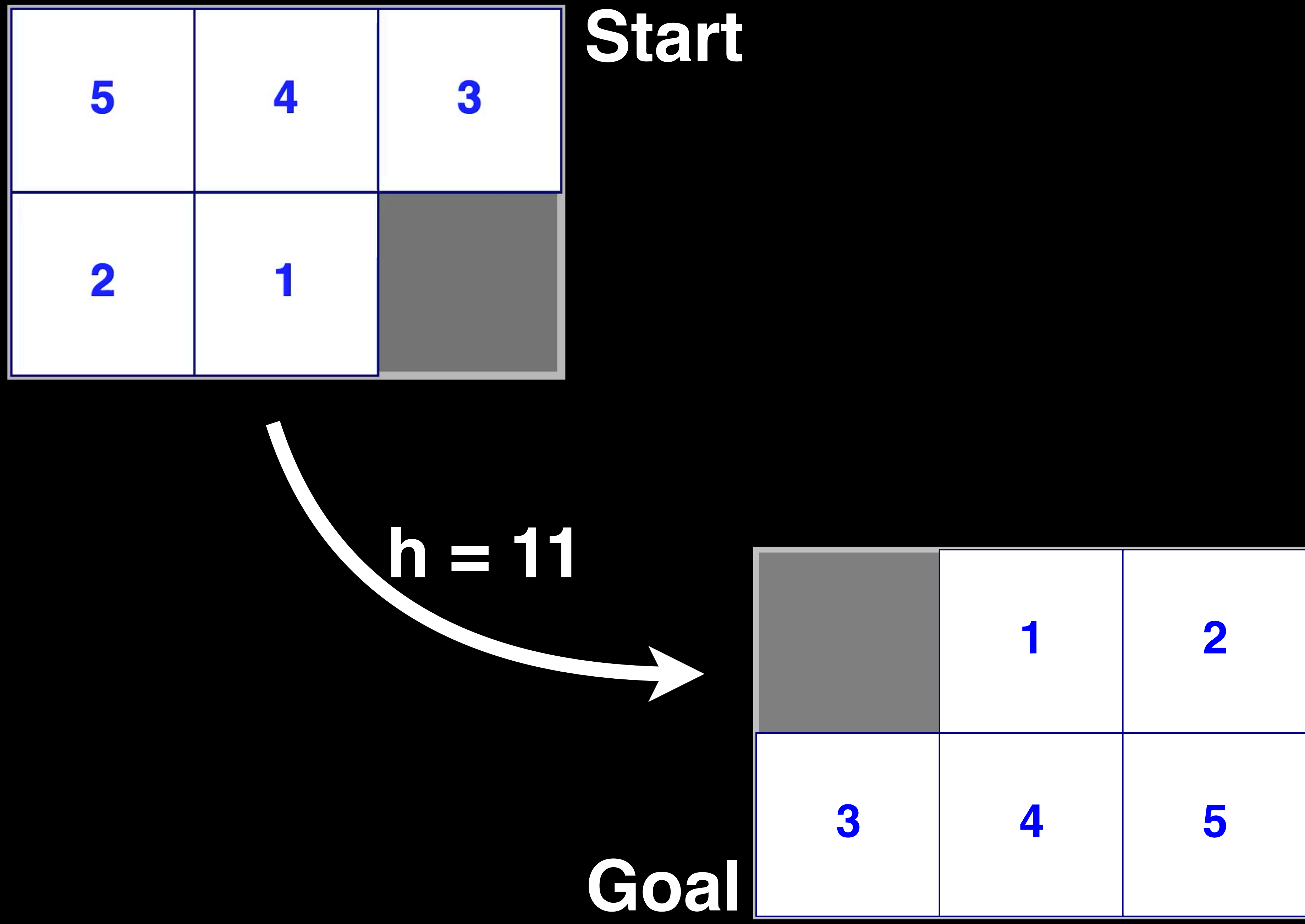
5	4	3
2	1	

Start

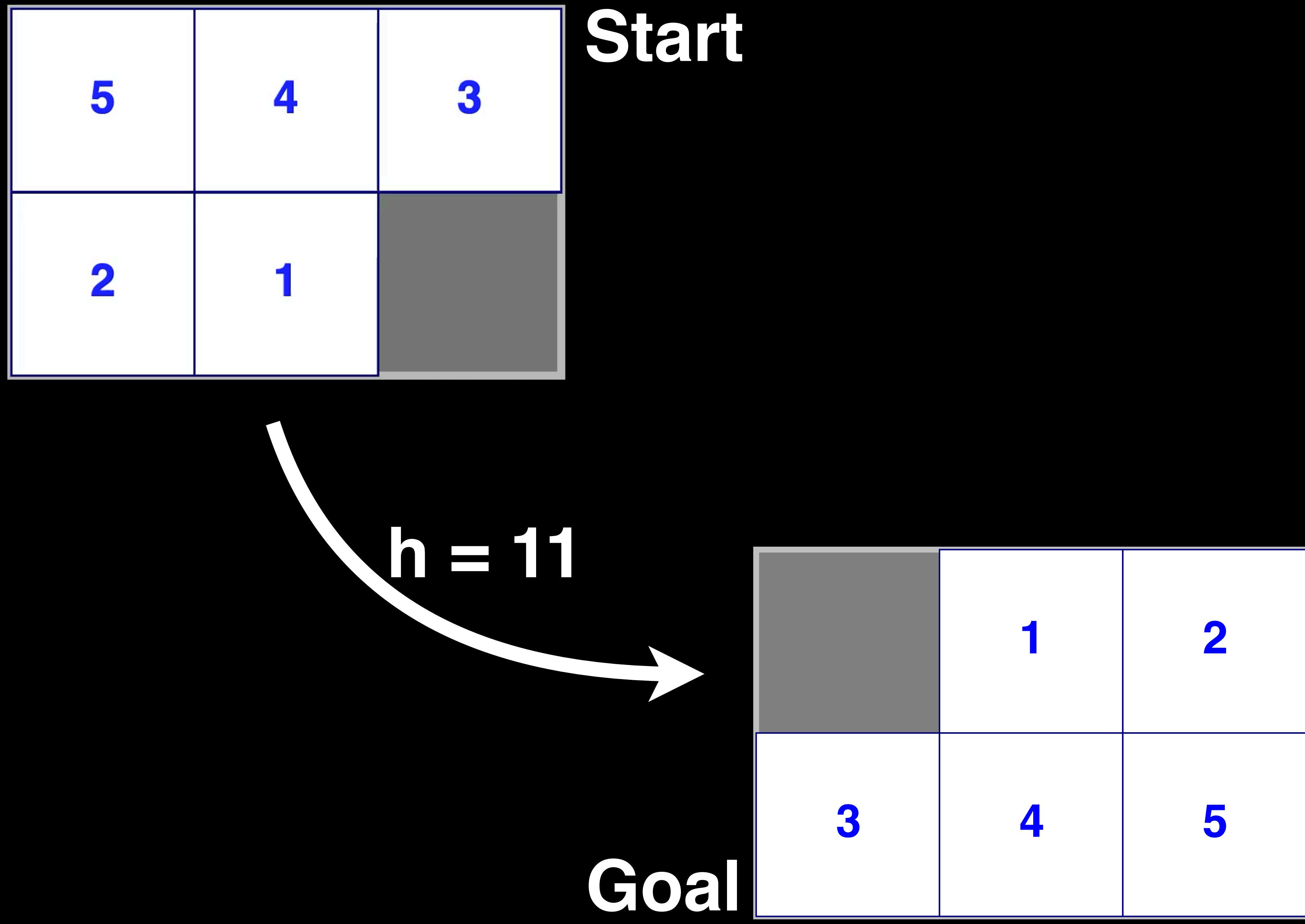
5	4	3
2	1	

Goal

	1	2
3	4	5



Start
5
4
3
2
1
h = 11
1
2
3
4
5
Goal


Start
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Goal

Search Problem

- Given:

Search Problem

- Given:
  - Start state

Search Problem

- Given:
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  - Goal state

Search Problem

- Given:
  - Start state
  - Goal state
  - Successor function

Search Problem

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← Defines implicit graph

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- Find:

Search Problem

- Given:
  - Start state
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  ← Defines implicit graph

  - Heuristic function
- Find:
  - Optimal path between start/goal

Why do we need BTS?

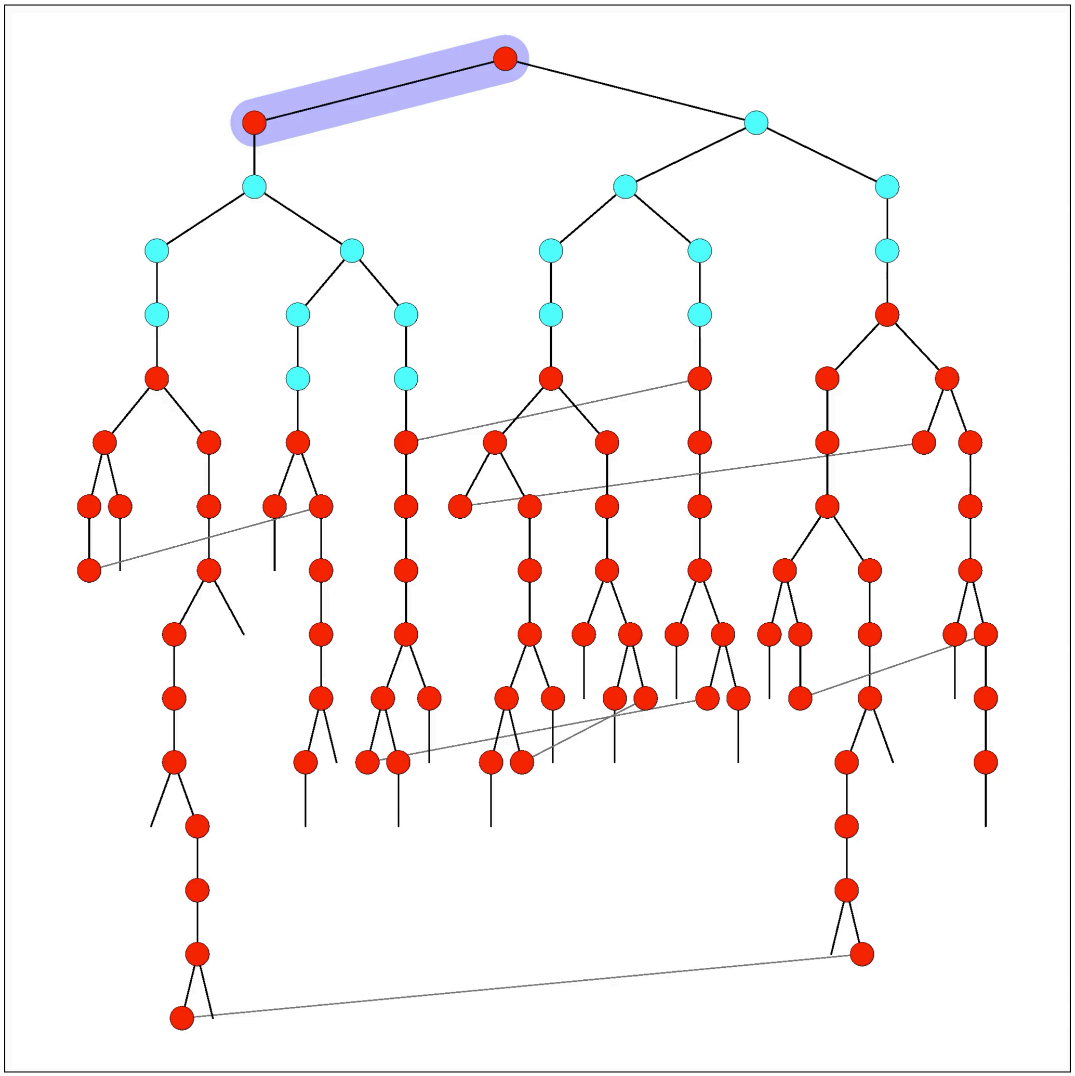
IDA* Refresher

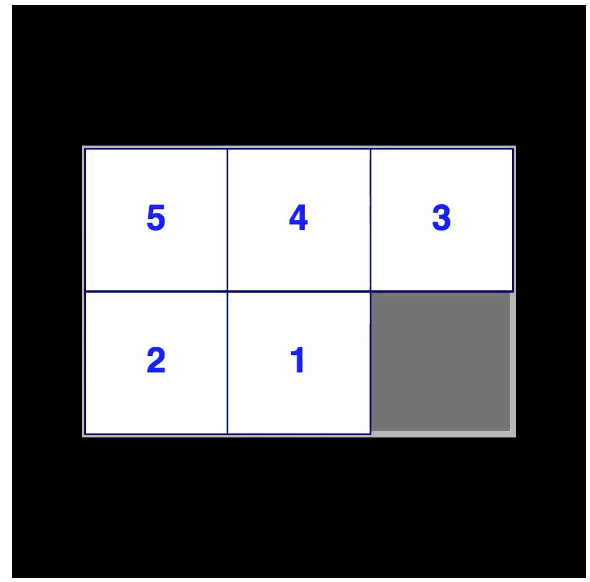
- IDA* does *iterative deepening* search on *f*-costs
  - $f(n) = g(n) + h(n)$

IDA* Refresher

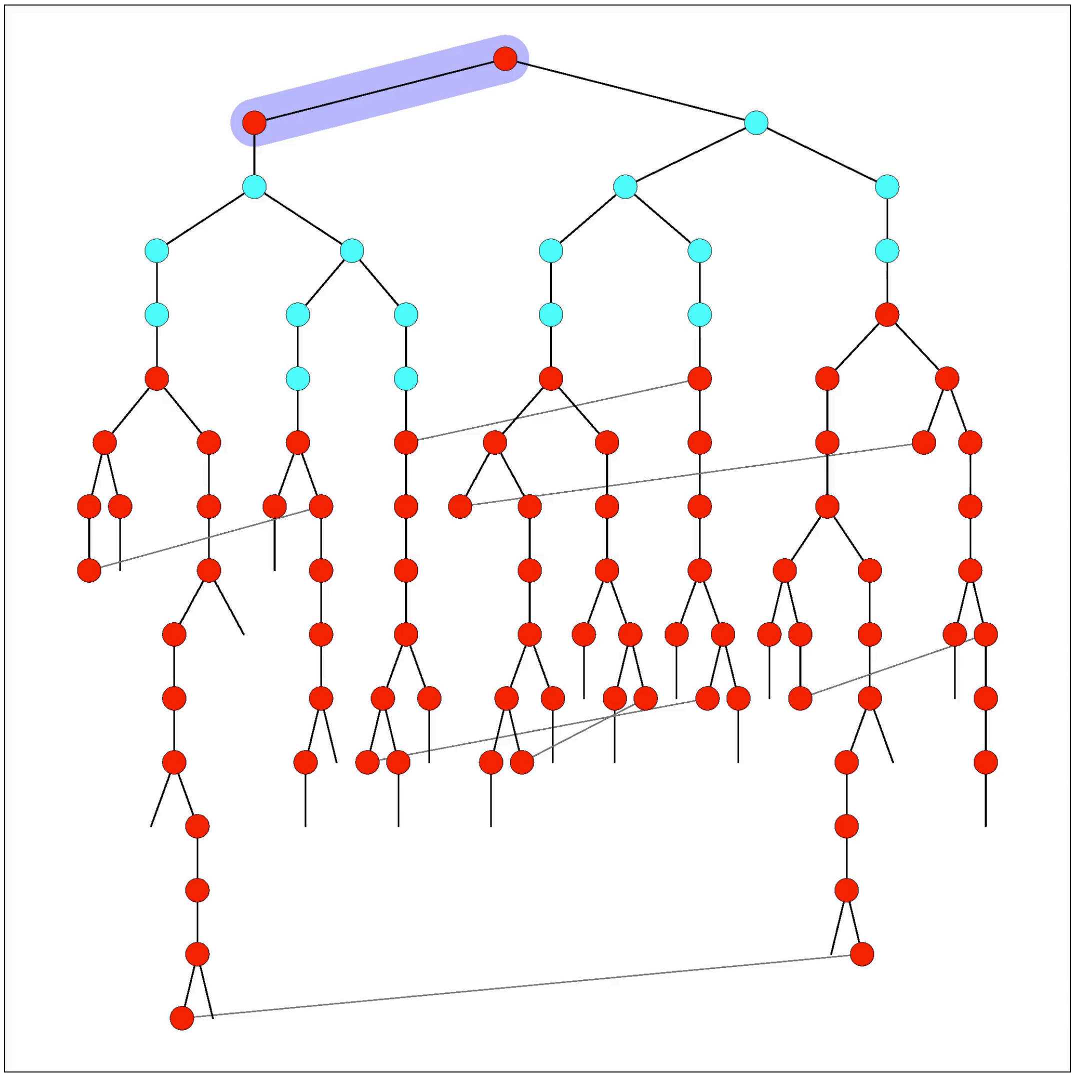
- IDA* does *iterative deepening* search on *f*-costs
  - $f(n) = g(n) + h(n)$
- Next iteration *f*-cost:
  - Smallest unexplored from previous iteration

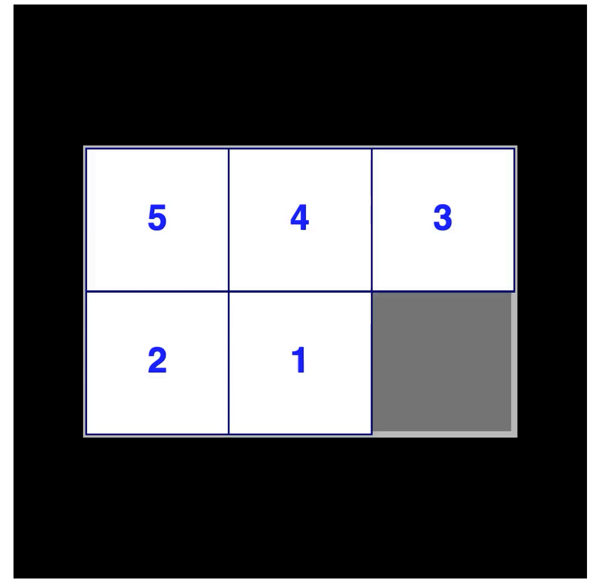
IDA* - Unit Costs



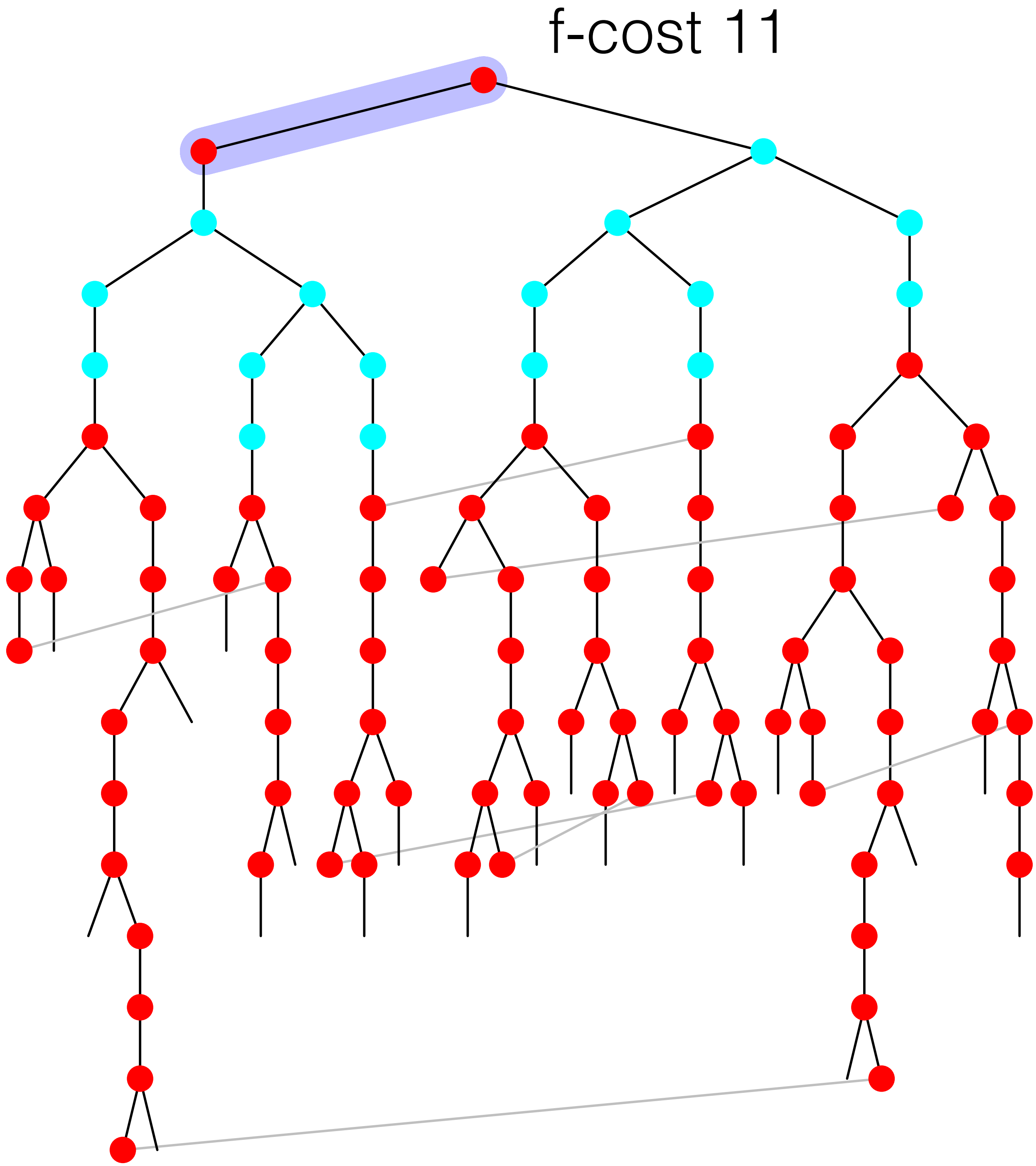


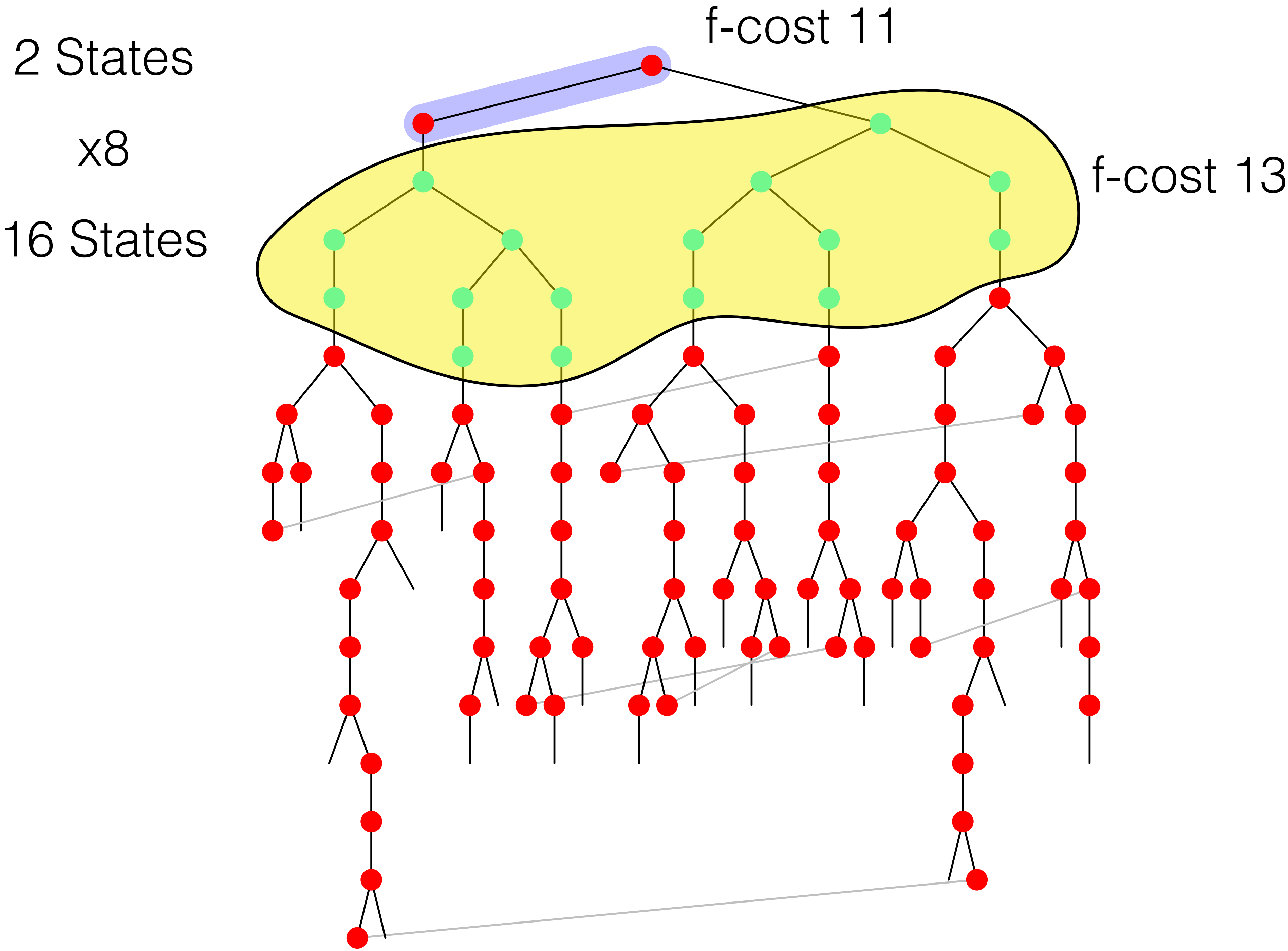
IDA* - Unit Costs

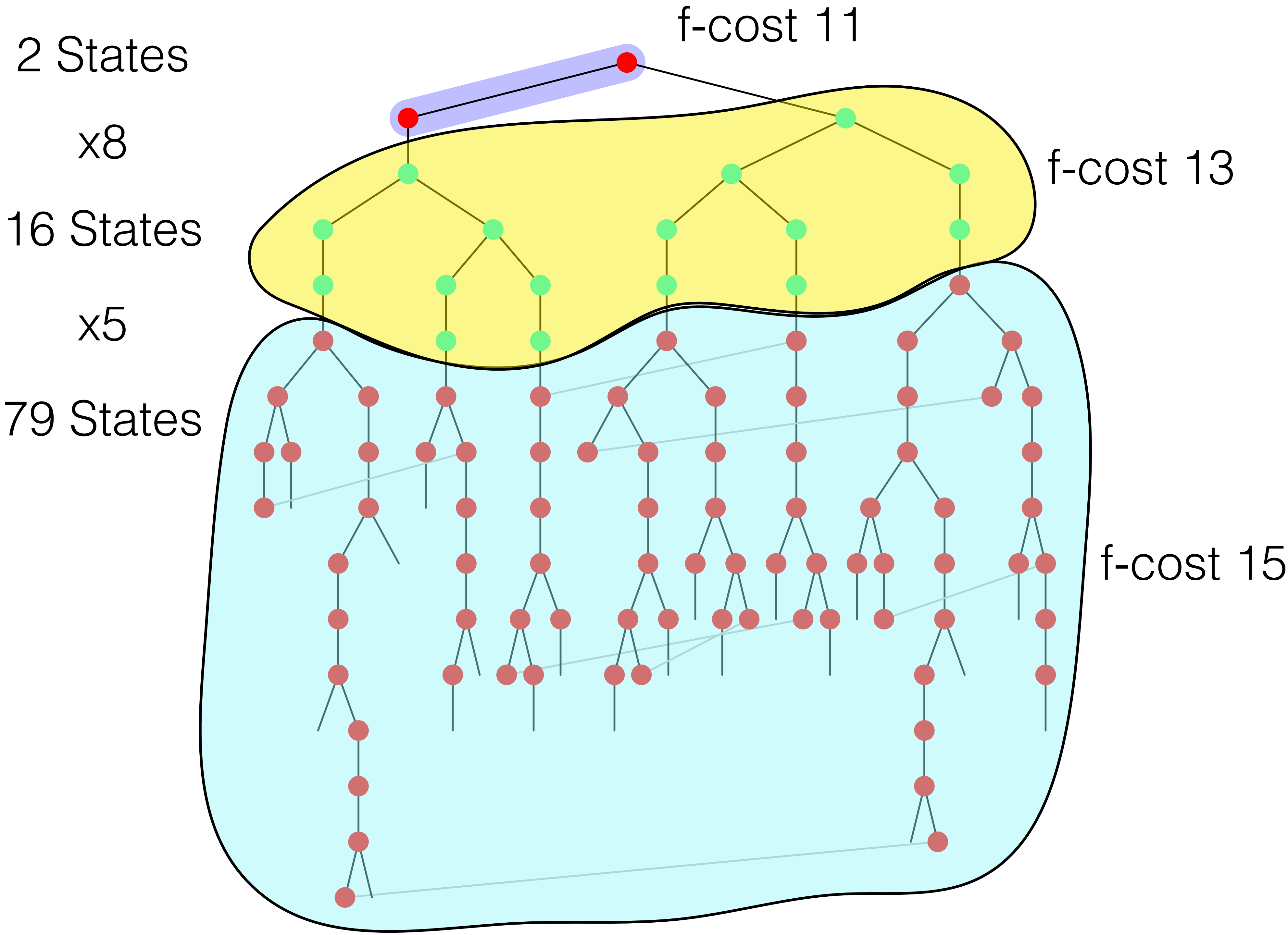




2 States




2 States
x8
16 States
f-cost 11
f-cost 13


f-cost 11
2 States
x8
f-cost 13
16 States
x5
79 States
f-cost 15

IDA* Worst Case

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  - 1

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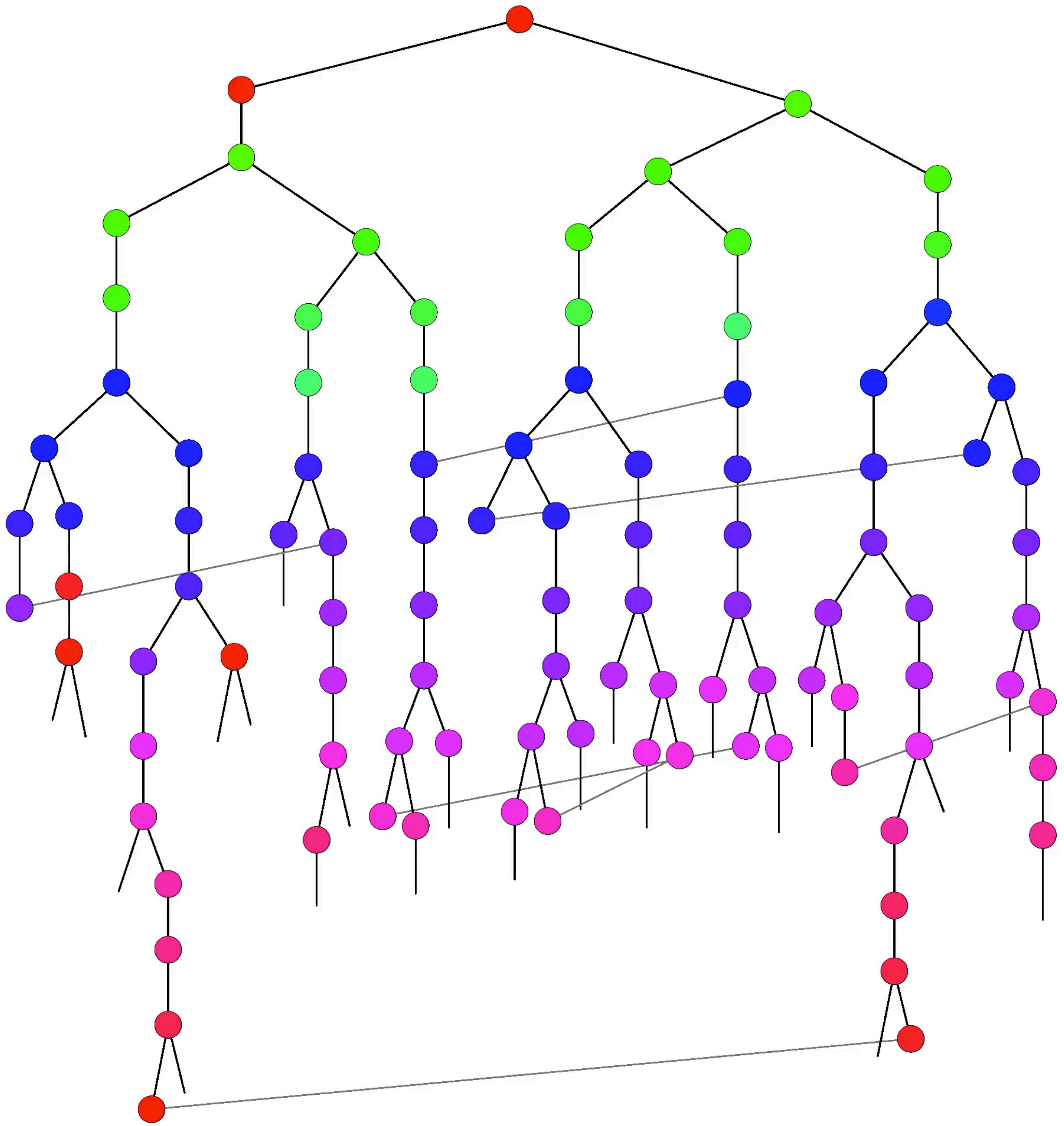
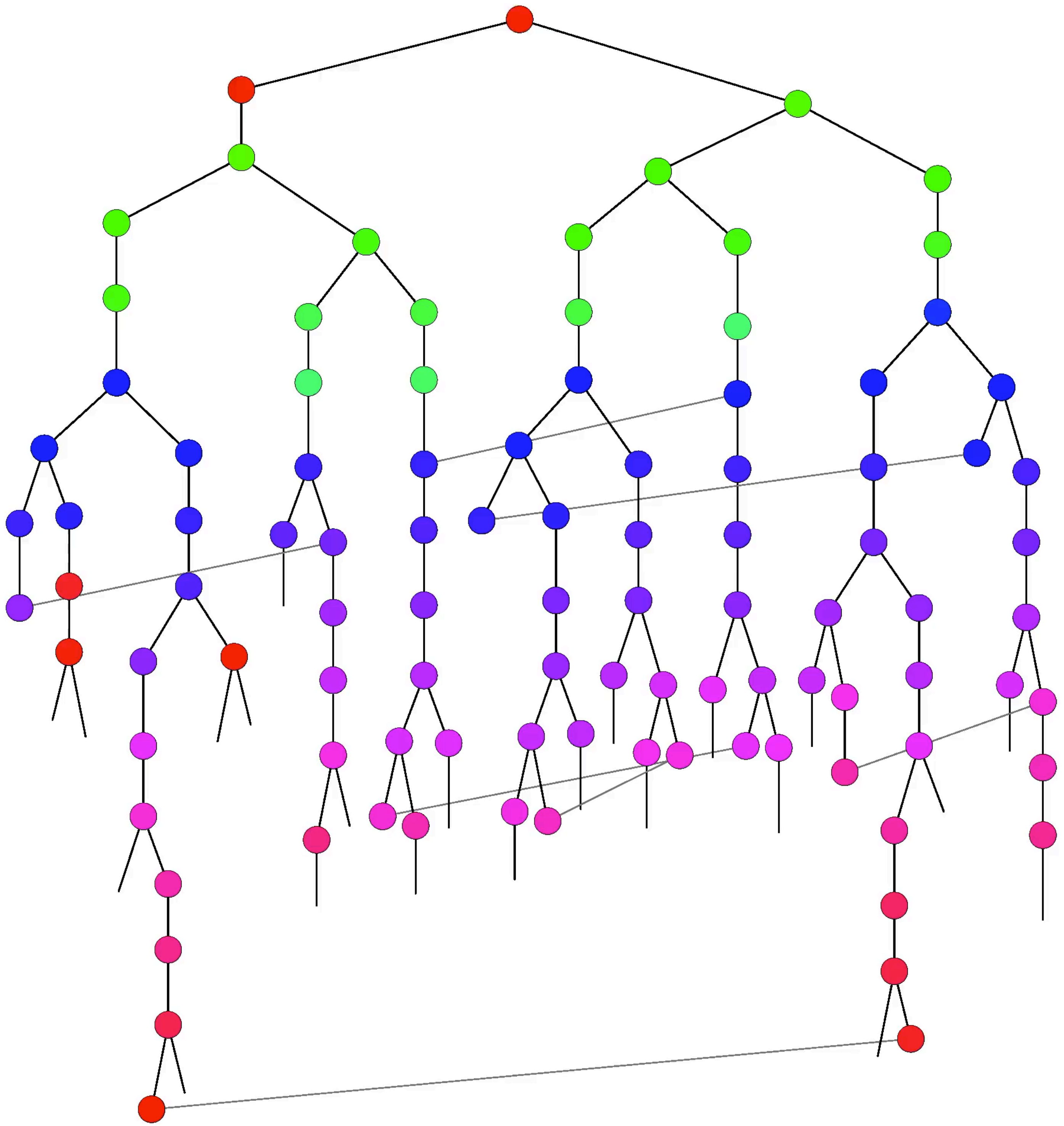
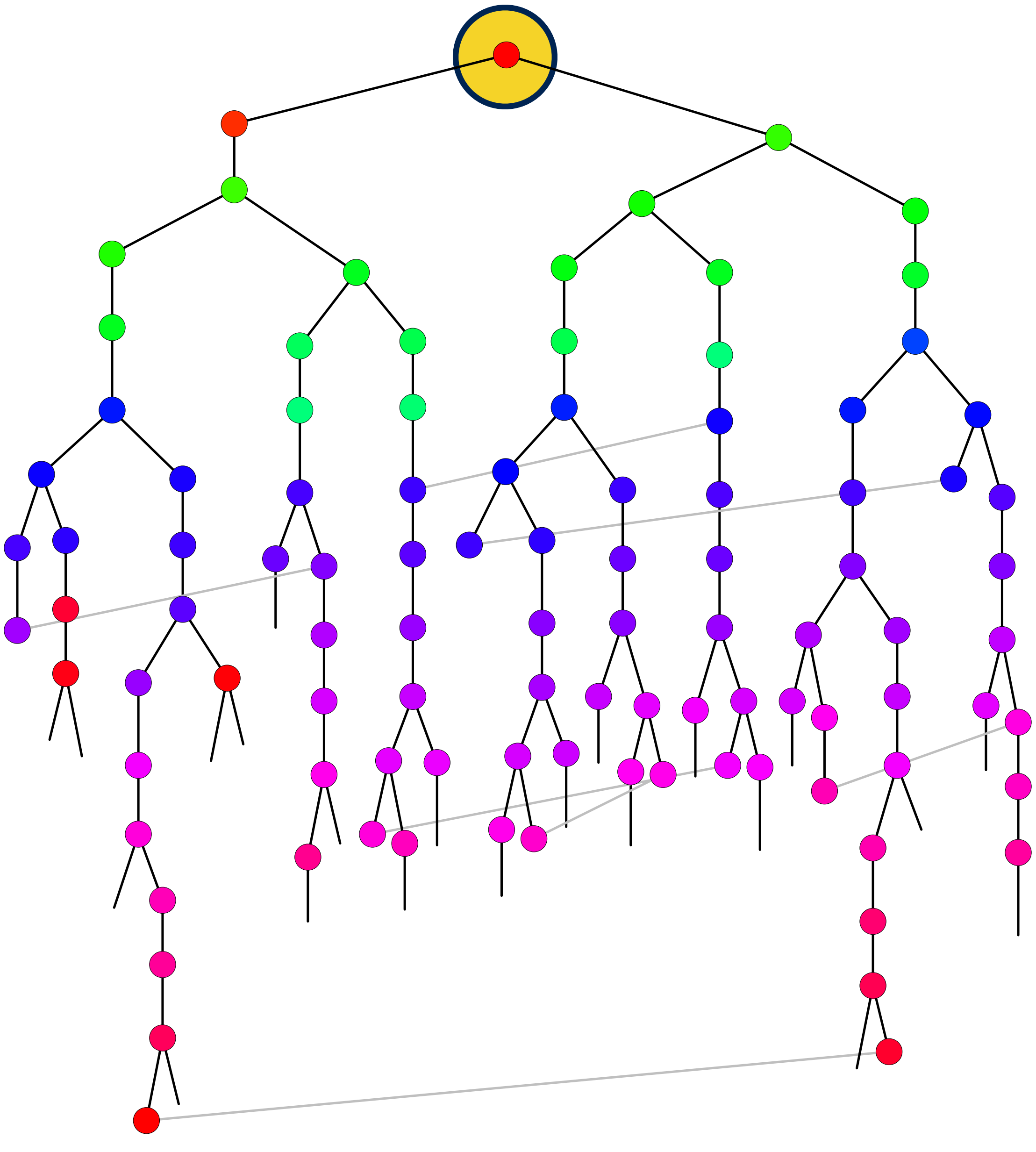
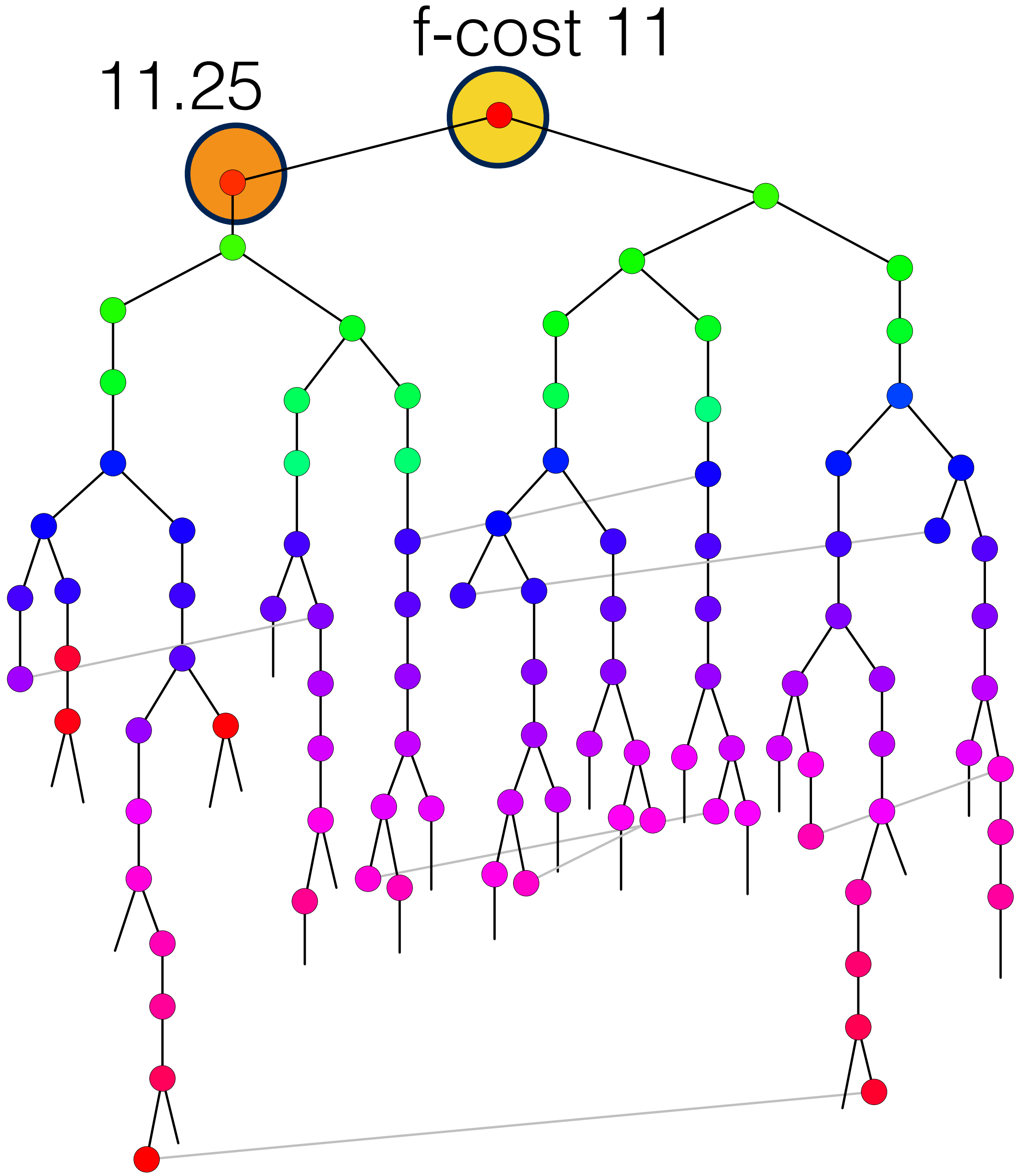
Tile	Cost
1	$\frac{1+2}{1+1} = 1.5$
3	$\frac{3+2}{3+1} = 1.25$
7	$\frac{7+2}{7+1} = 1.125$
9	$\frac{9+2}{9+1} = 1.1$





f-cost 11





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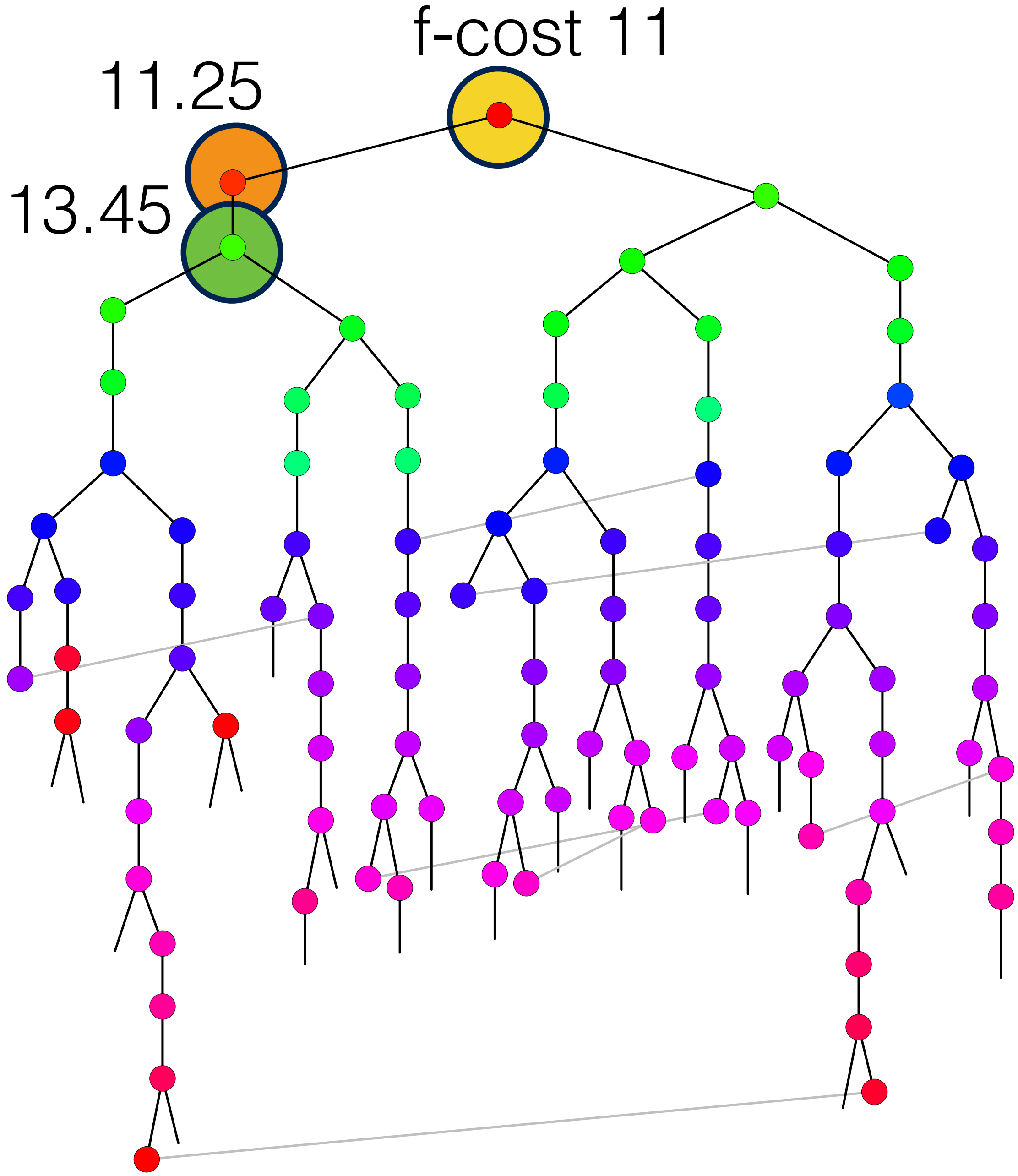
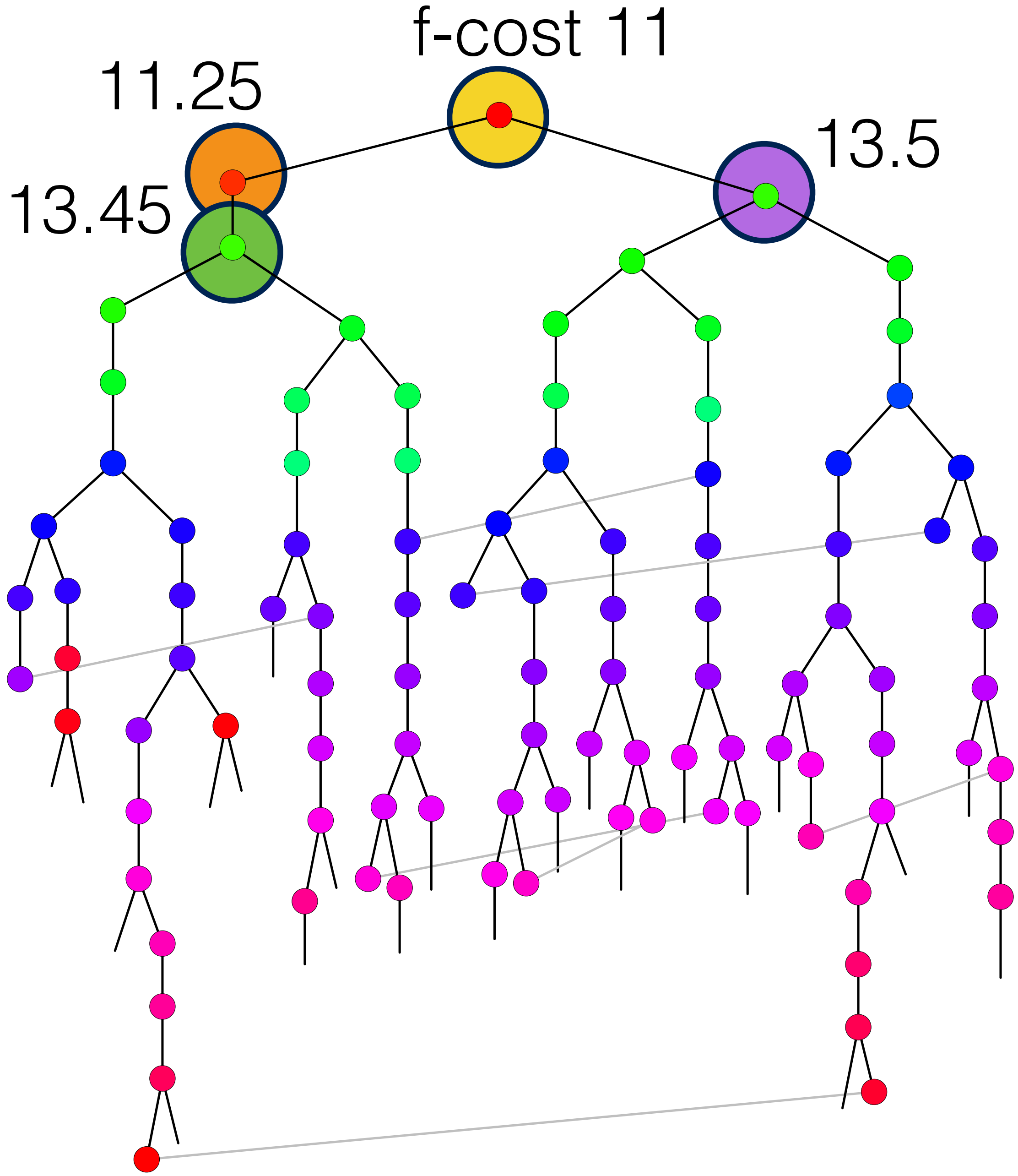
11.25


f-cost 11
11.25
13.45



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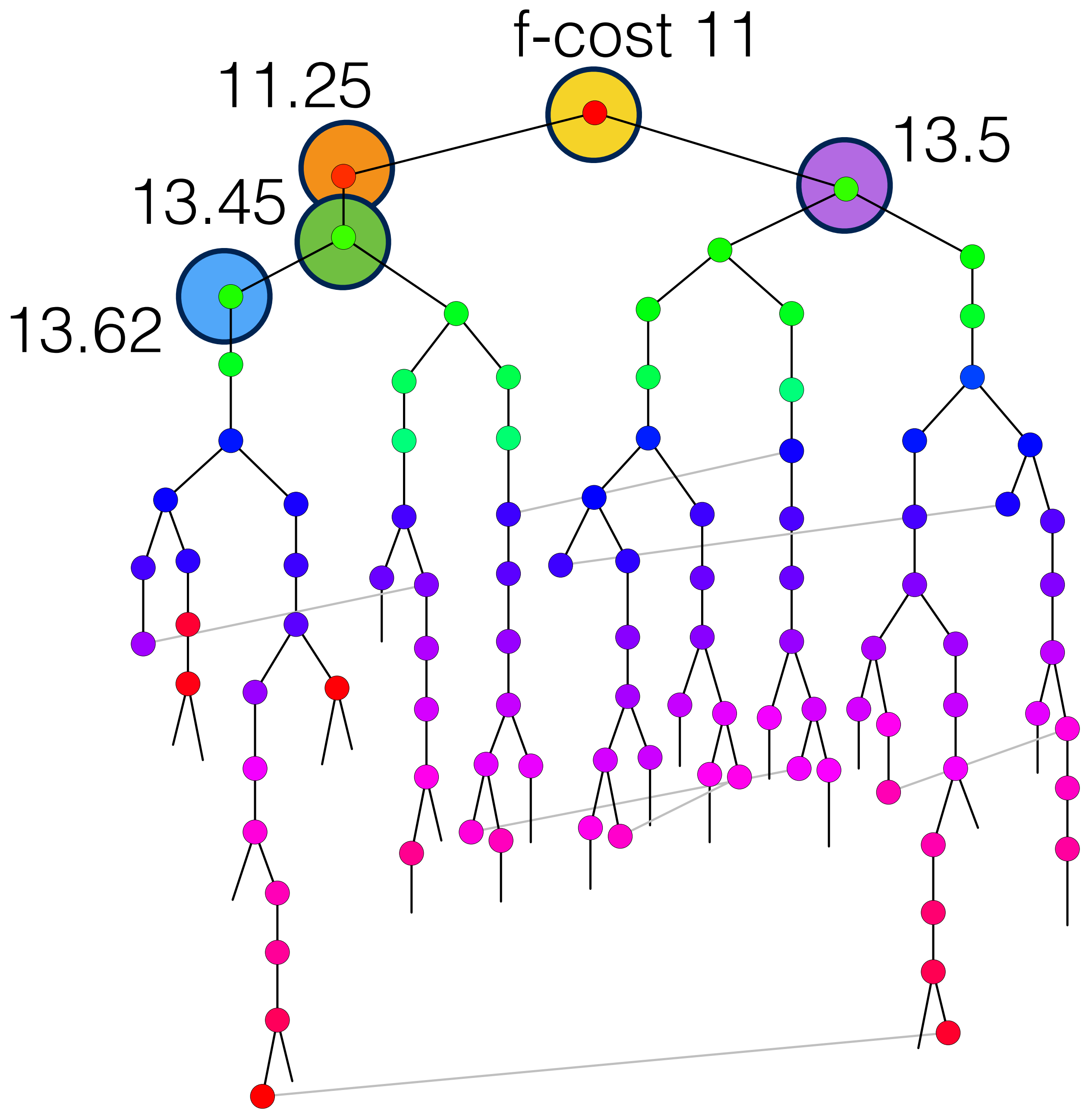
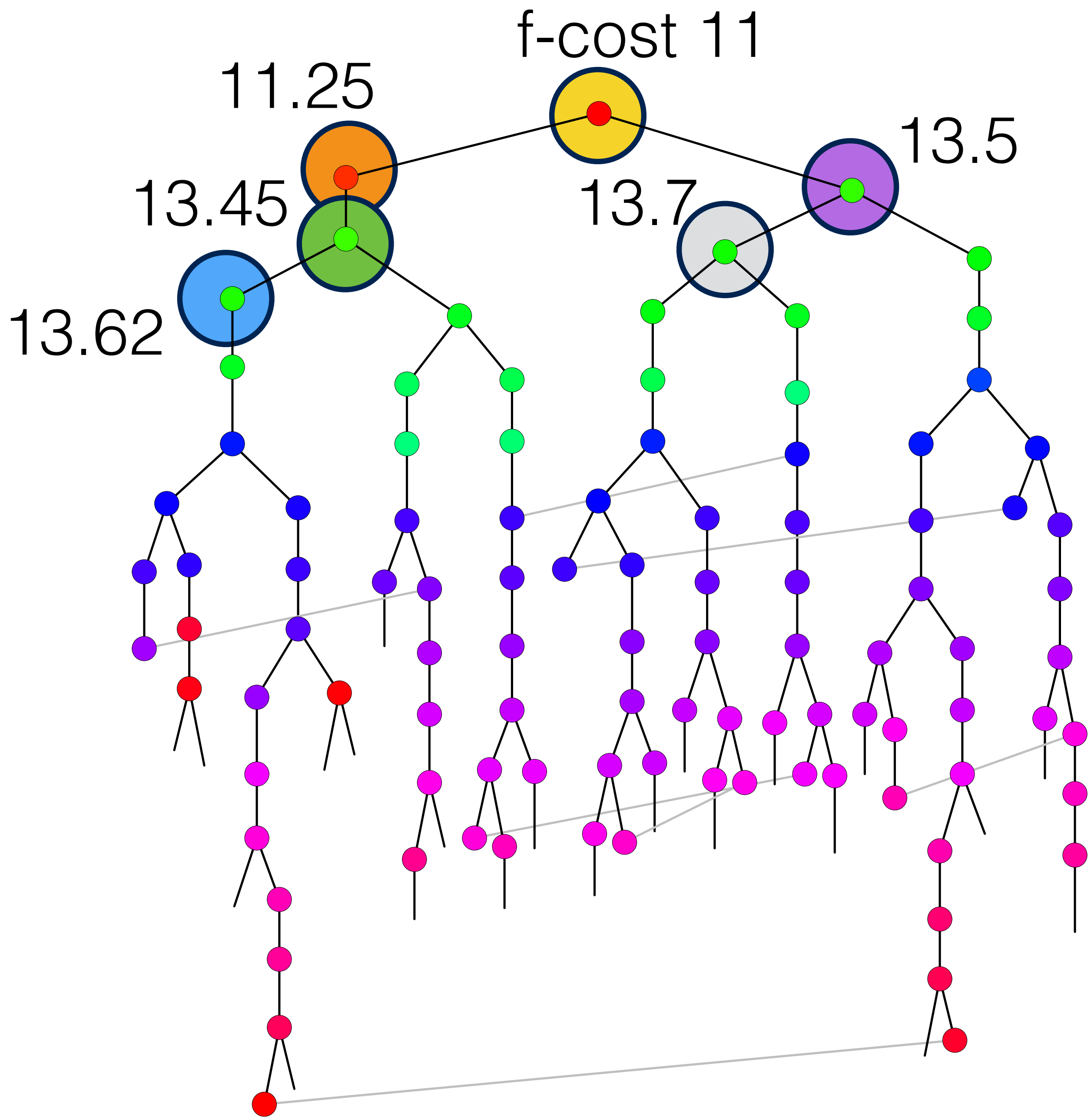
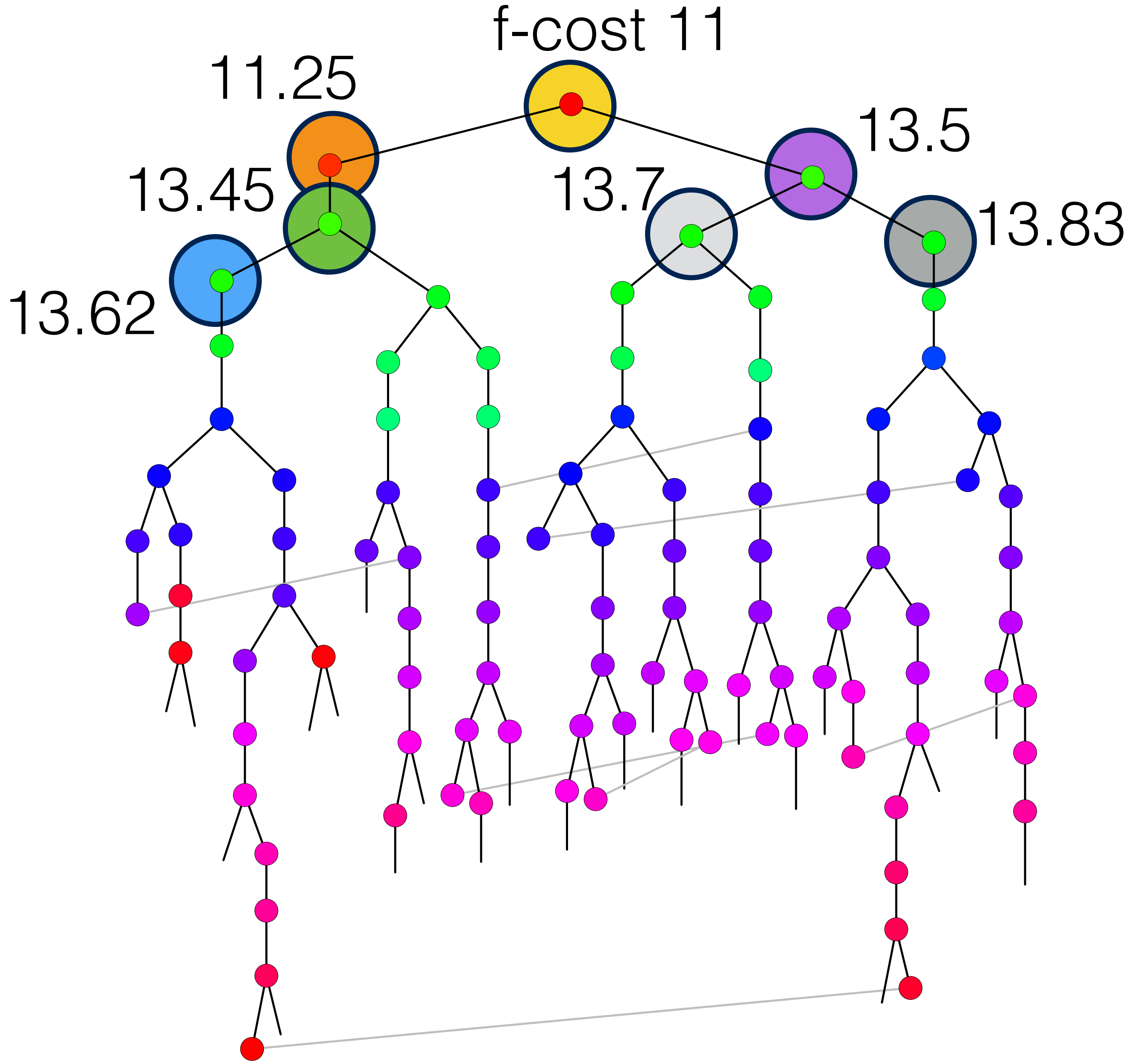
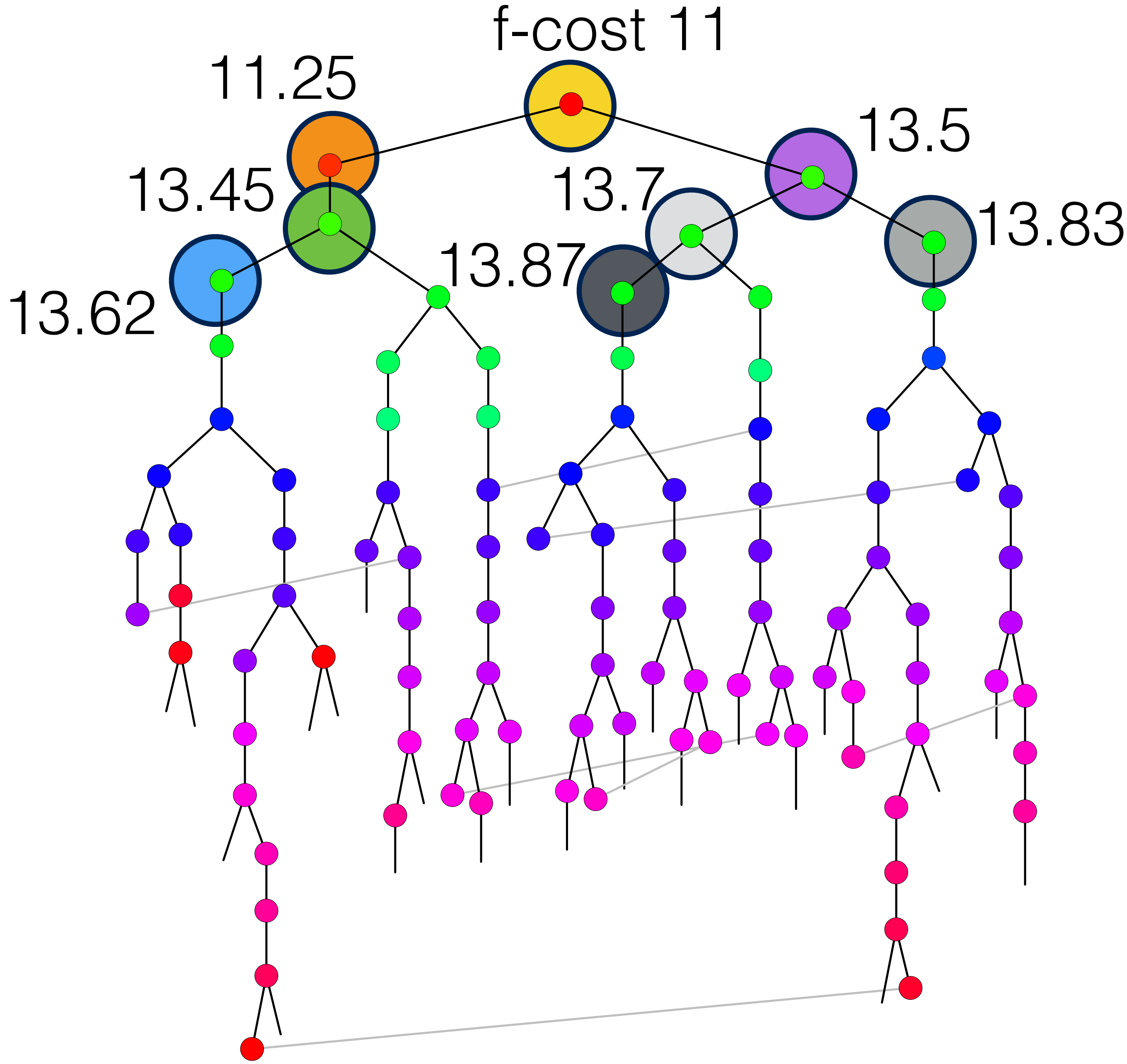

f-cost 11
11.25
13.5
13.45
13.62


f-cost 11
11.25
13.5
13.45
13.7
13.62


f-cost 11
11.25
13.5
13.45
13.7
13.83
13.62


f-cost 11
11.25
13.5
13.45
13.7
13.83
13.87
13.62

Why do we need BTS?

If the nodes in each iteration
do not grow exponentially

Getting the next bound

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- Can be conservative:
  - IDA* (Korf, 1985)

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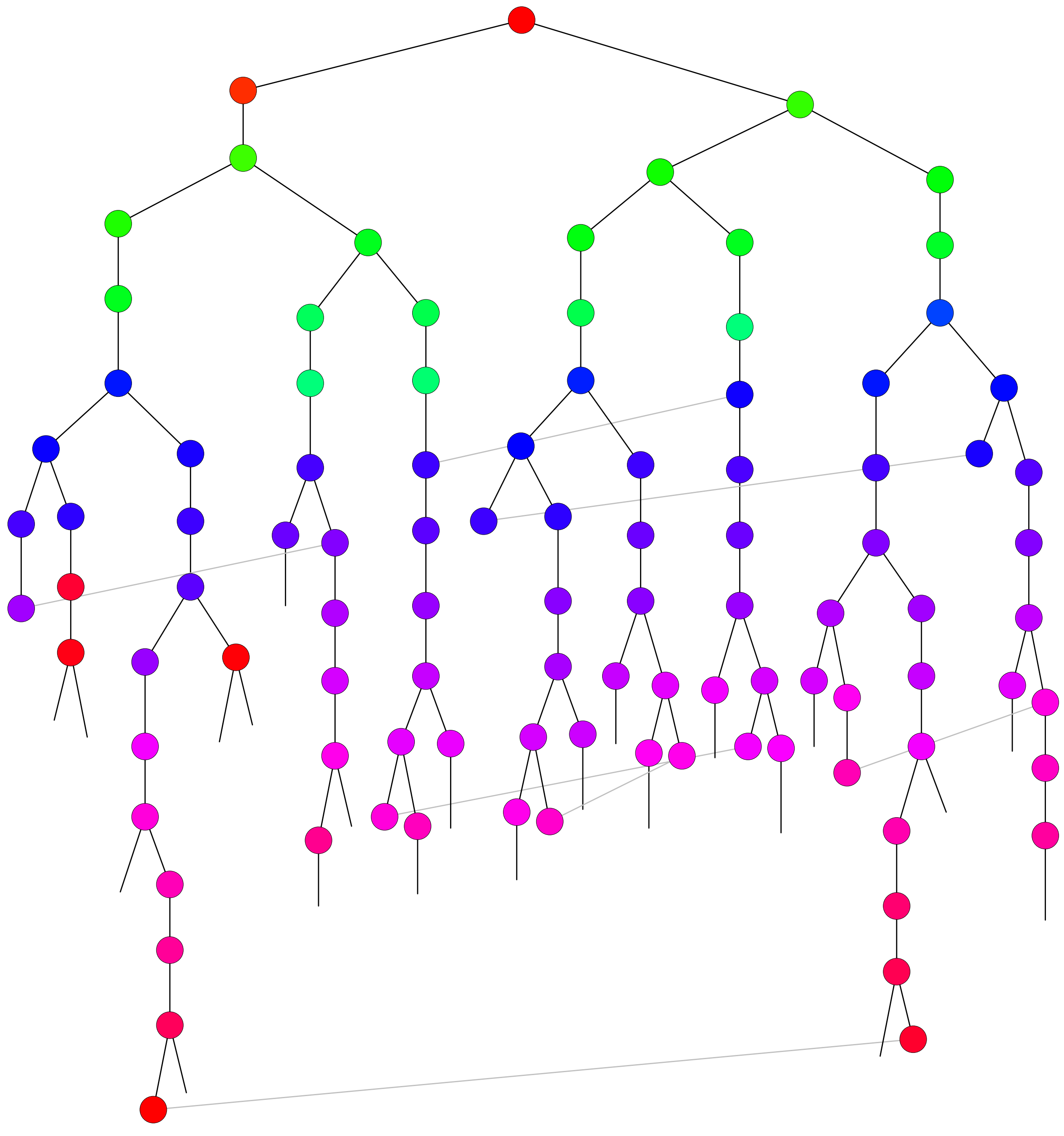
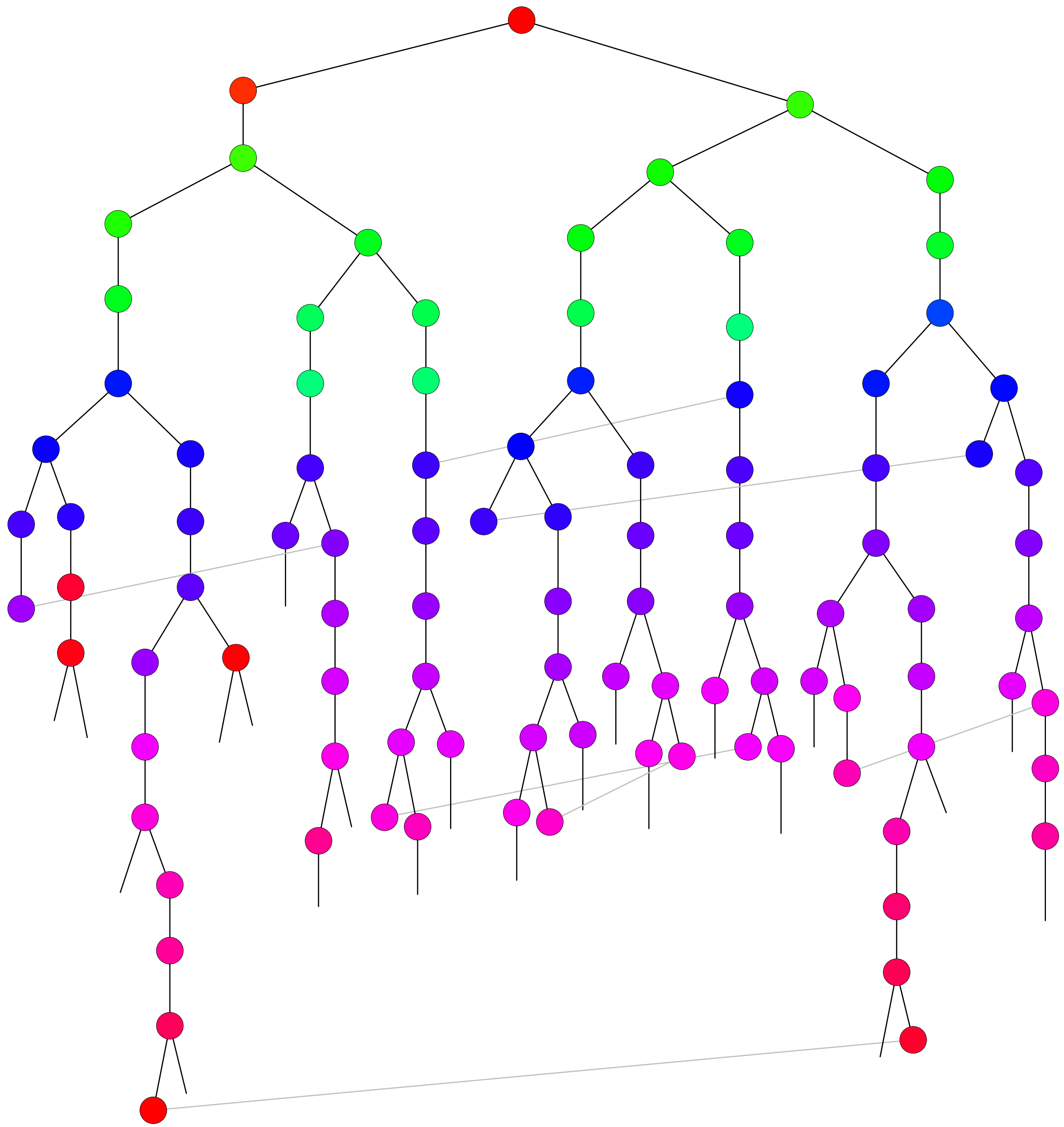
- Want to guarantee exponential growth in expansions



$f = 11$

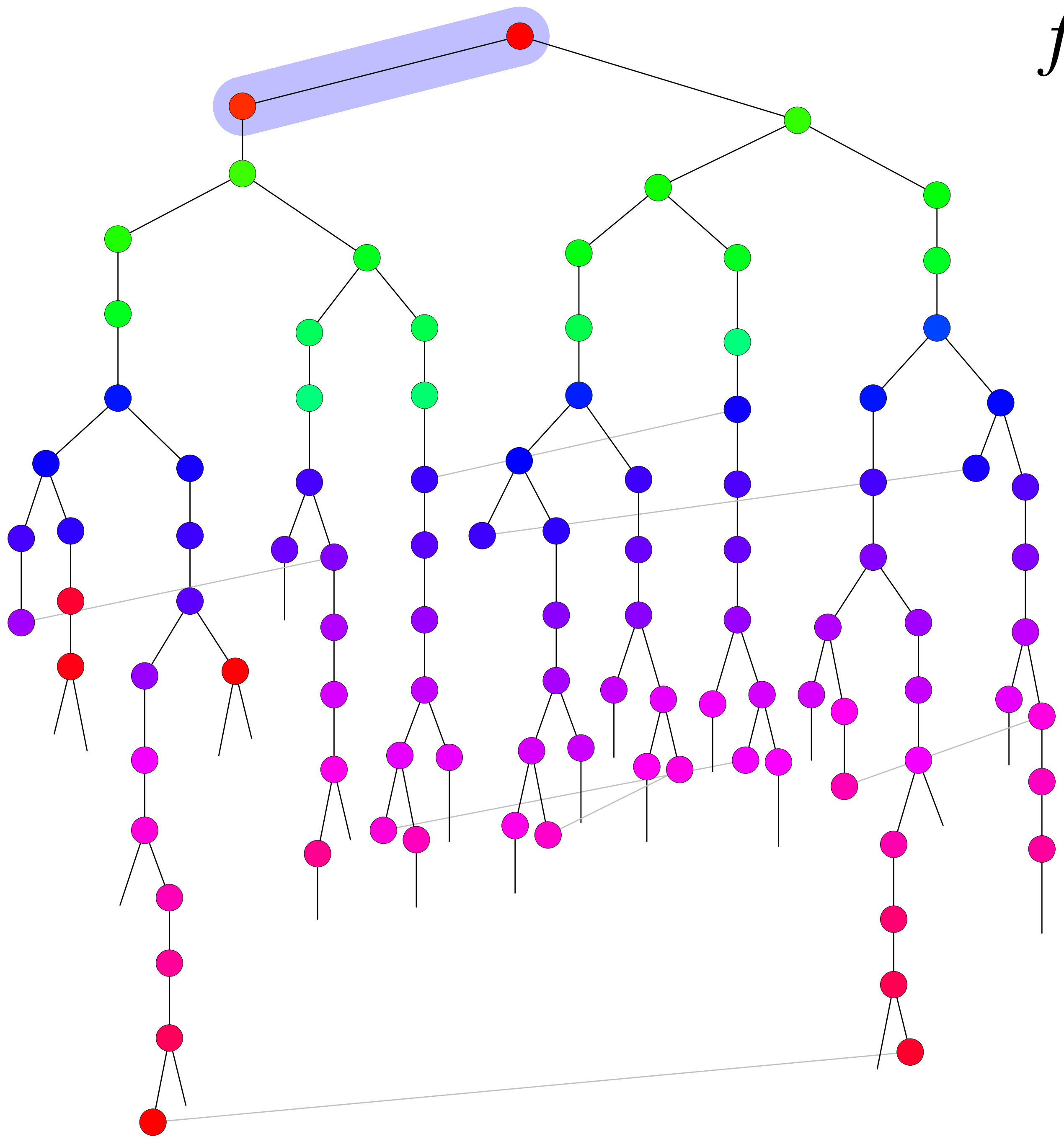
1 node



$f = 11$

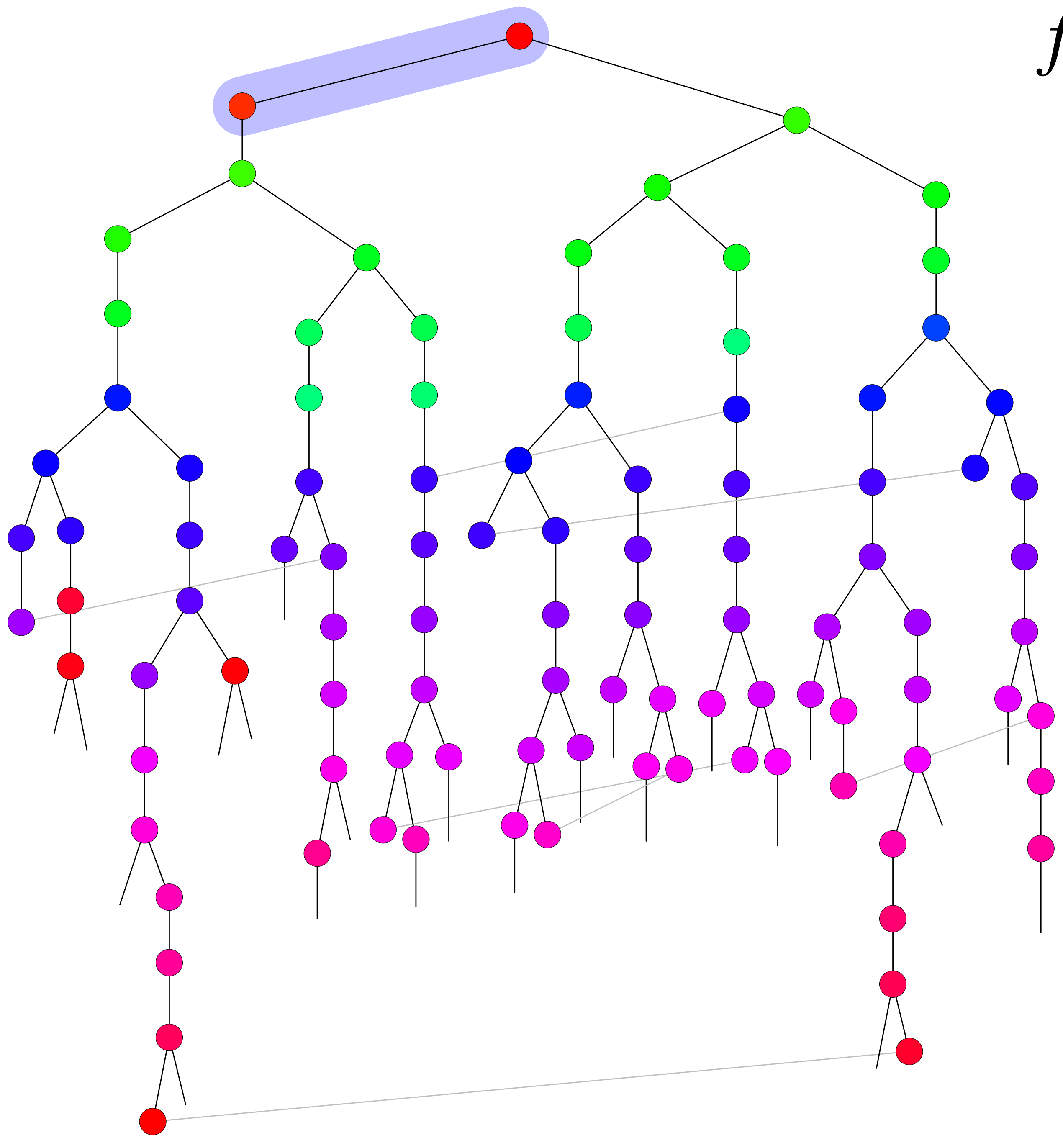
1 node

[2, 8]



$f = 11.25$

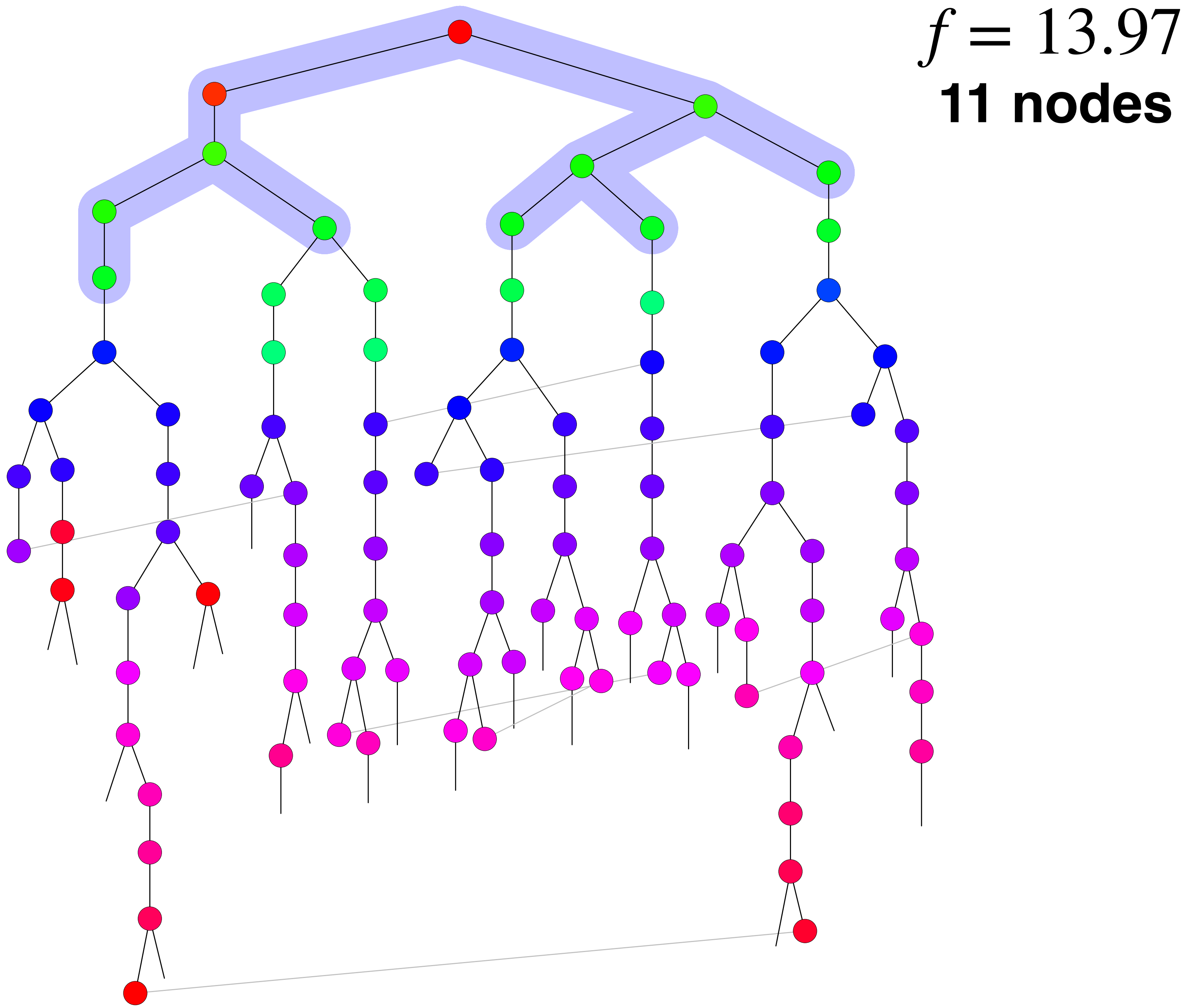
2 nodes

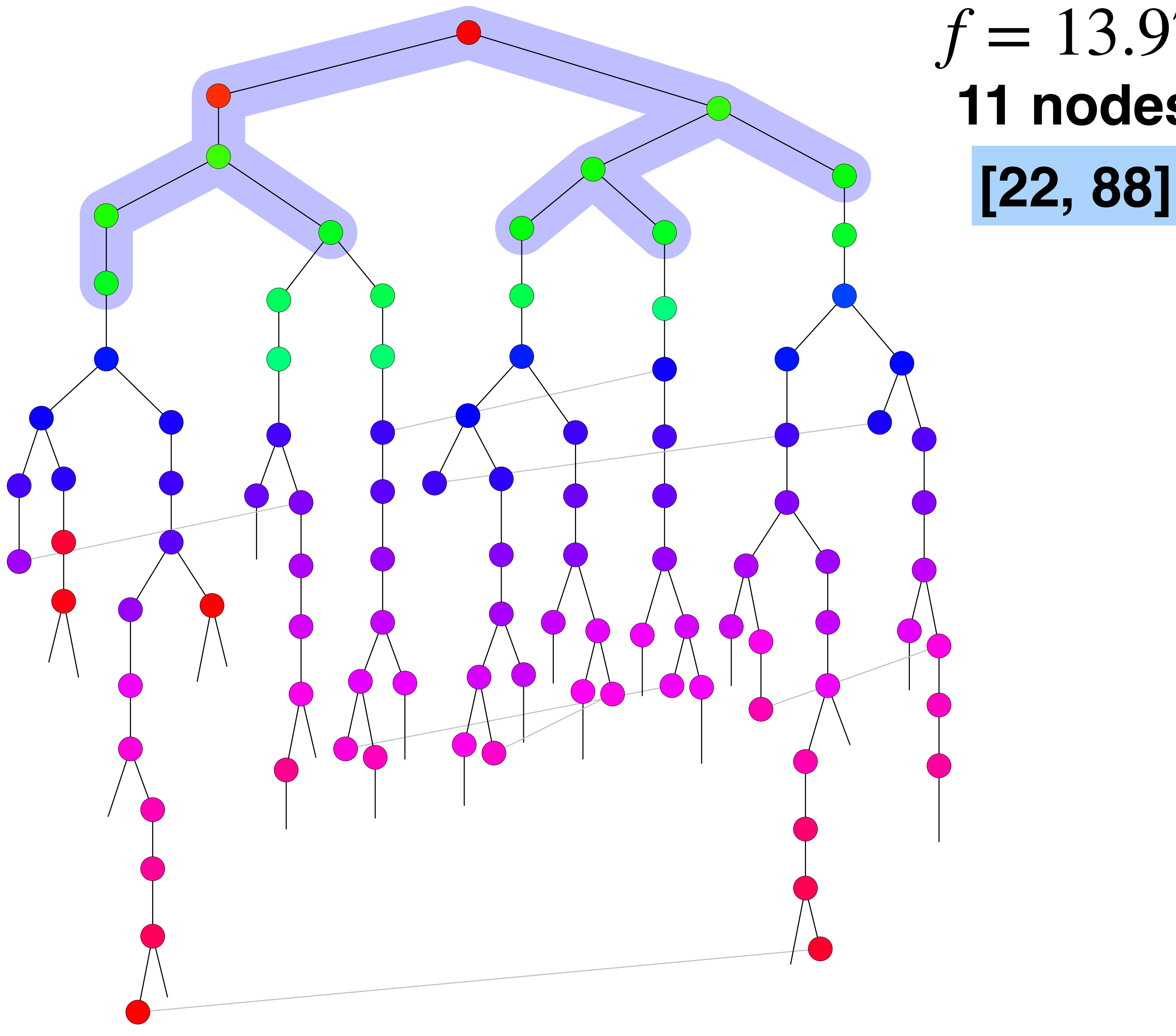


$f = 11.25$

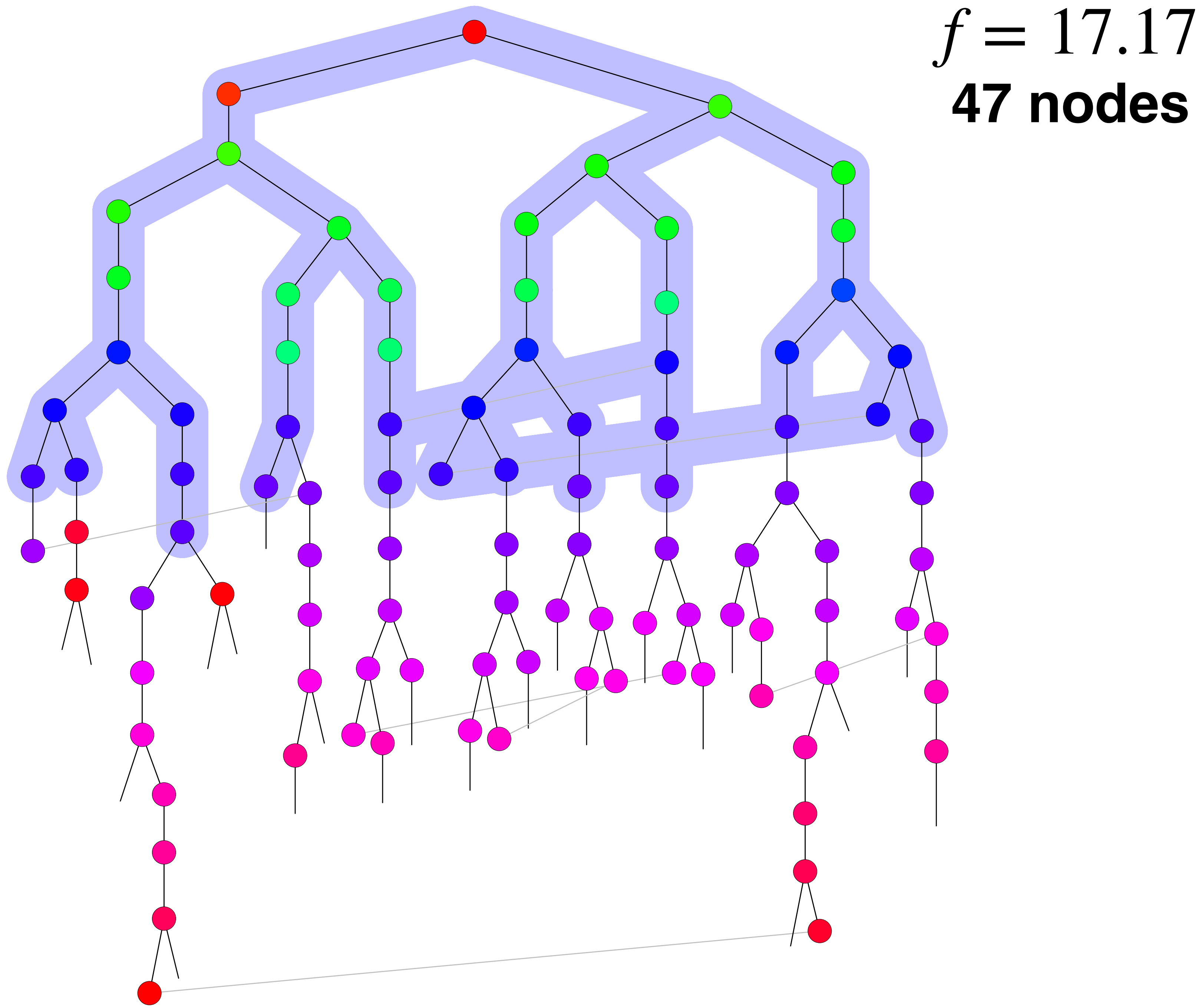
2 nodes

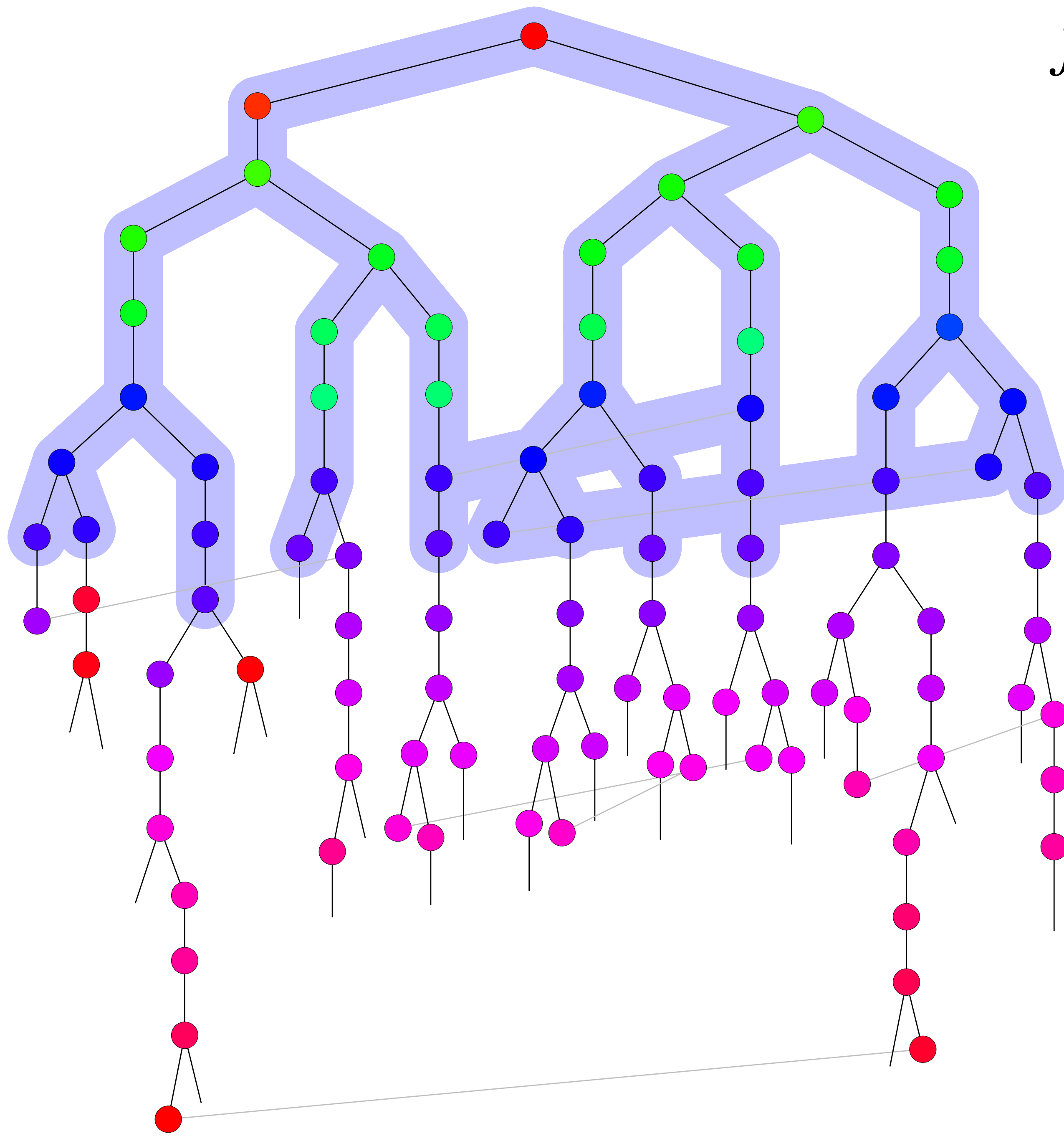
[4, 16]


$f = 13.97$
11 nodes



$f = 13.97$
11 nodes
[22, 88]

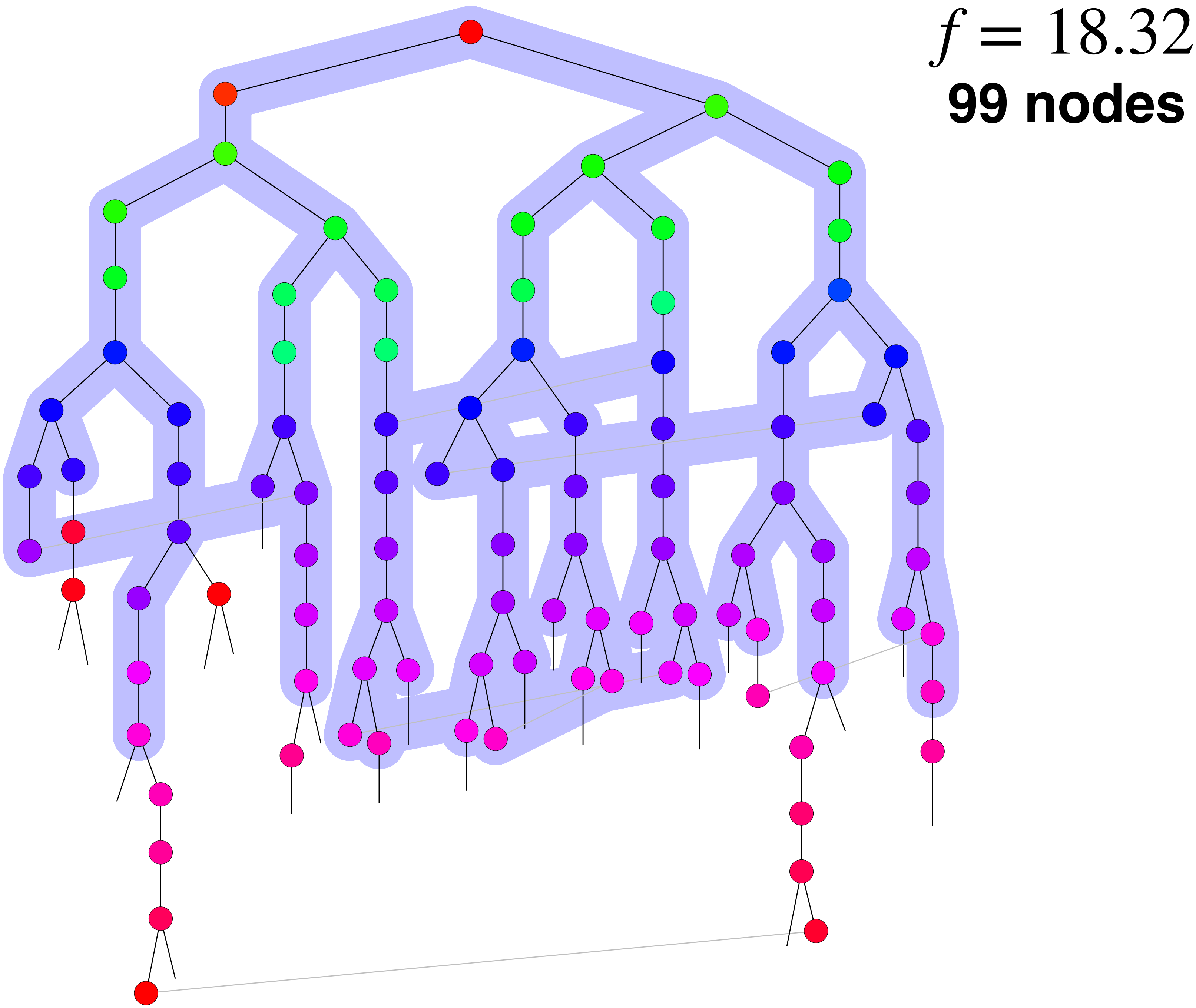

$f = 17.17$
47 nodes

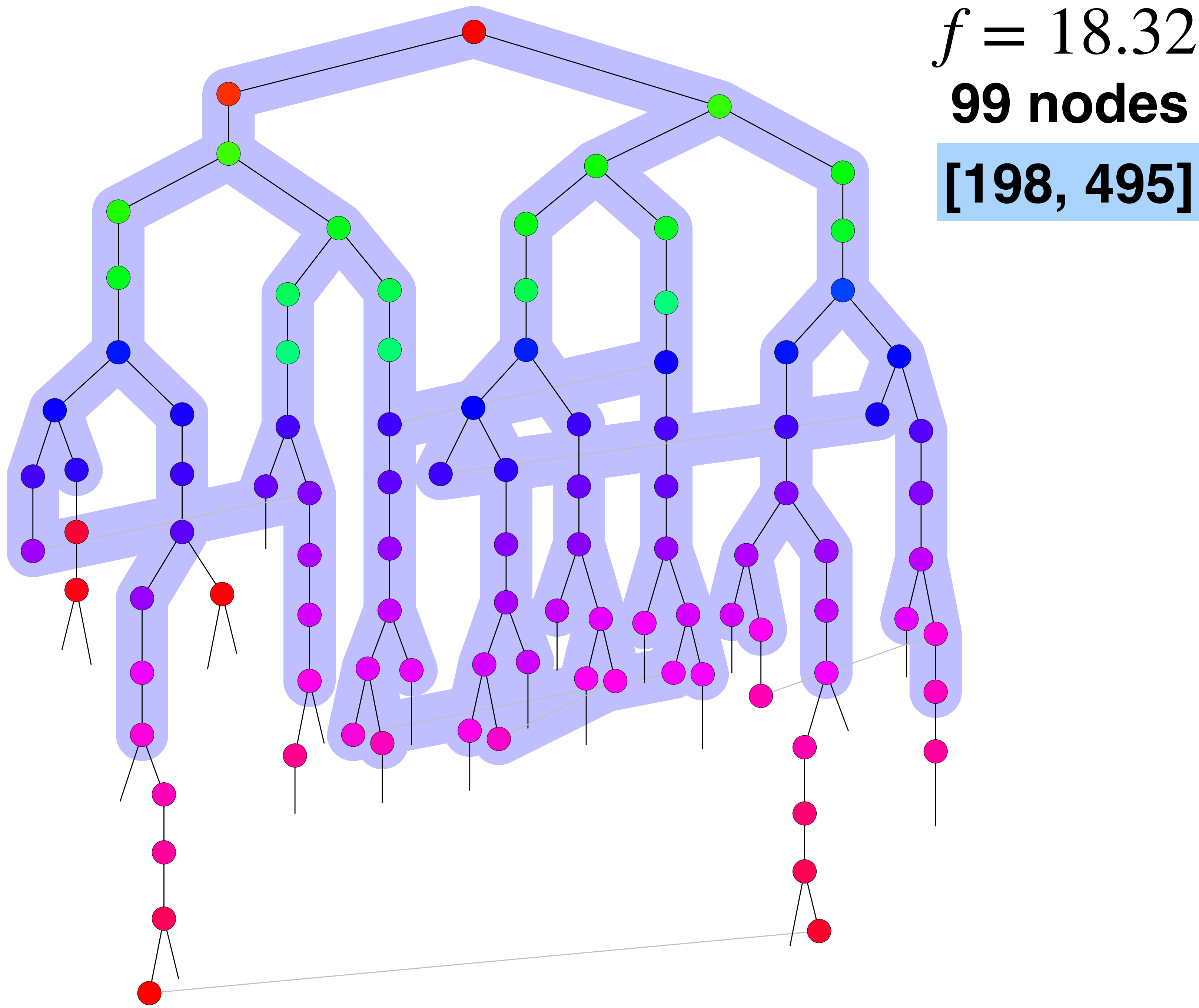


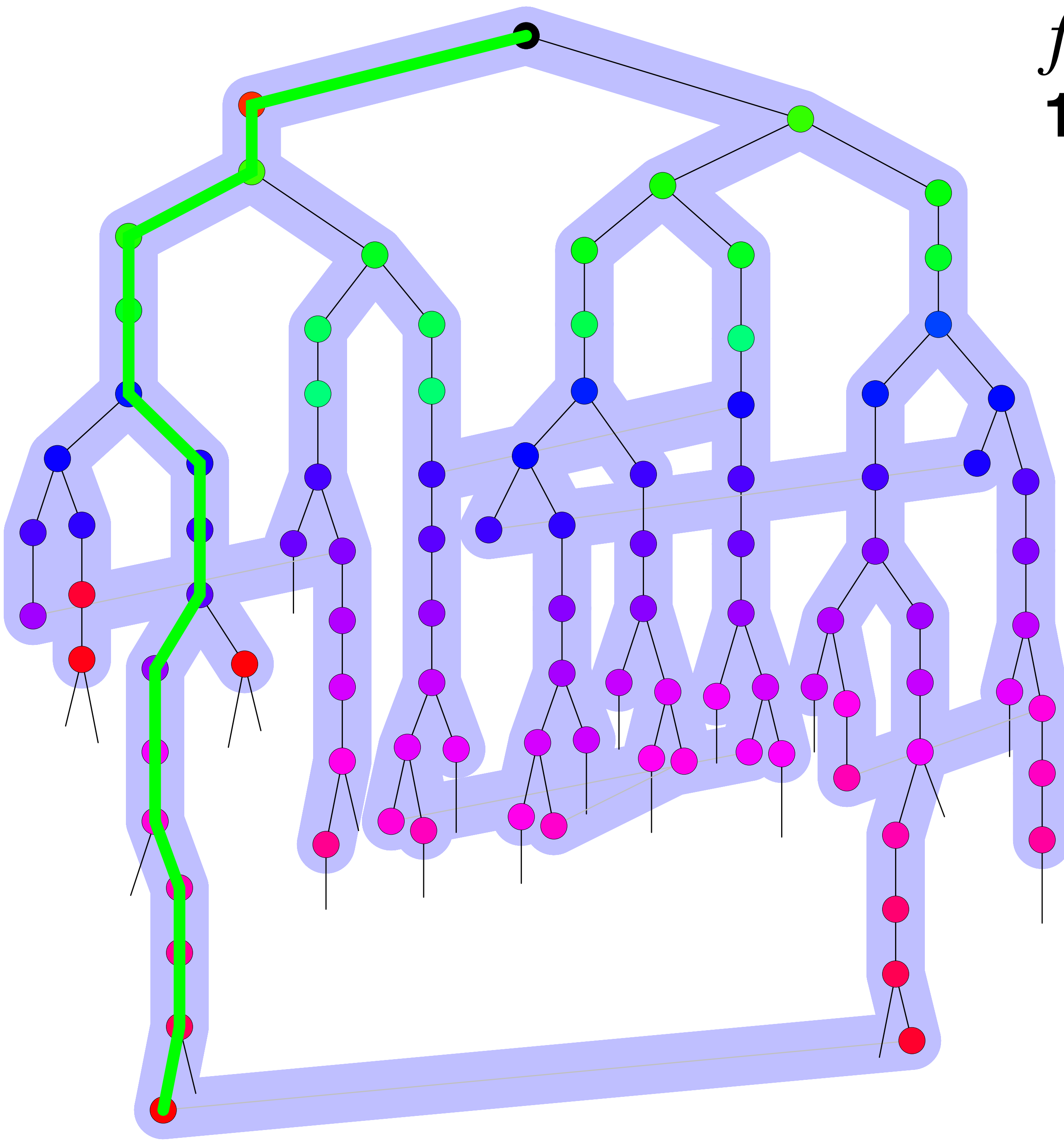
$f = 17.17$

47 nodes

[94, 376]


$f = 18.32$
99 nodes


$f = 18.32$
99 nodes
[198, 495]



$f = 19.35$

117 nodes

Exponential Search

- Bentley and Yao, 1976
- Algorithm for searching sorted/unbounded array

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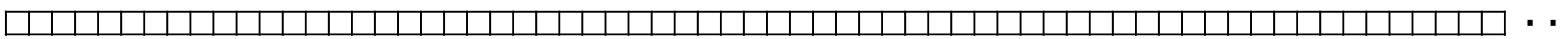

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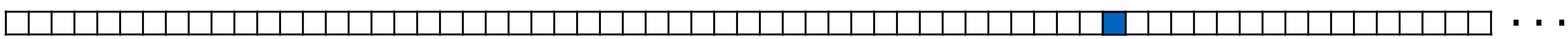

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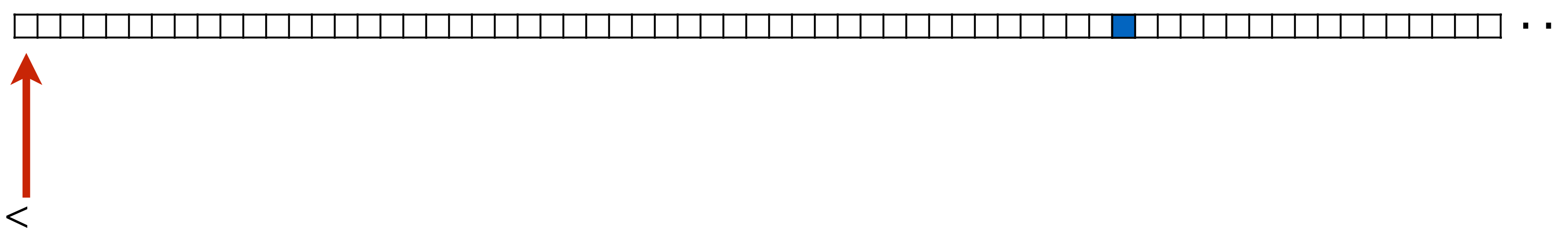

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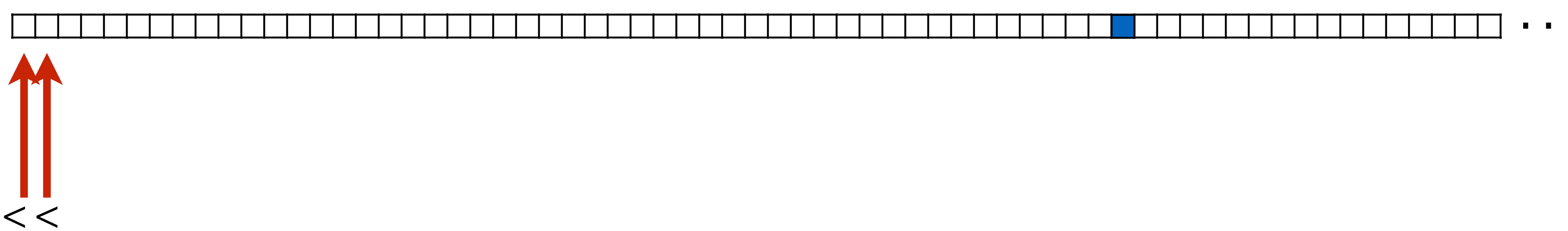

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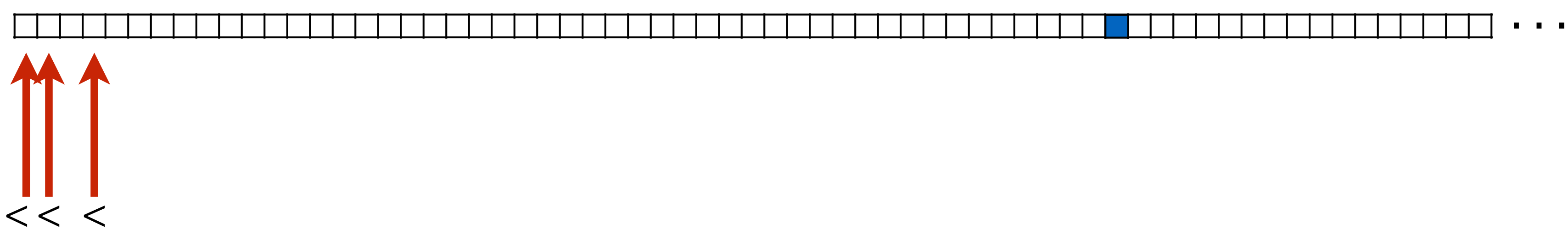

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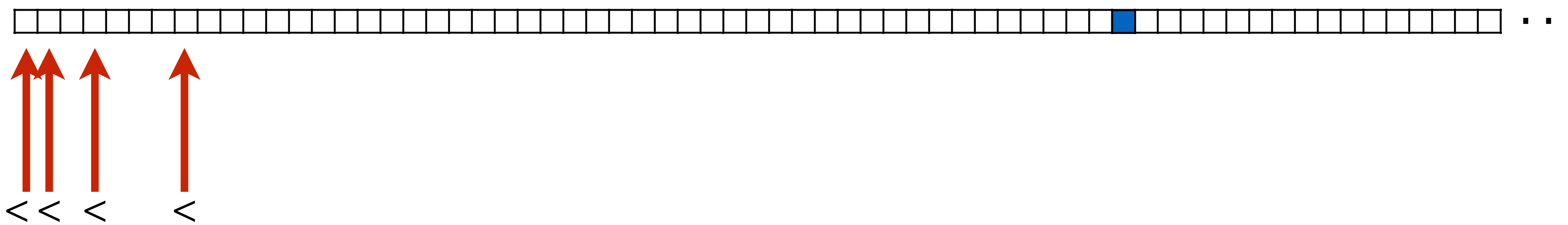

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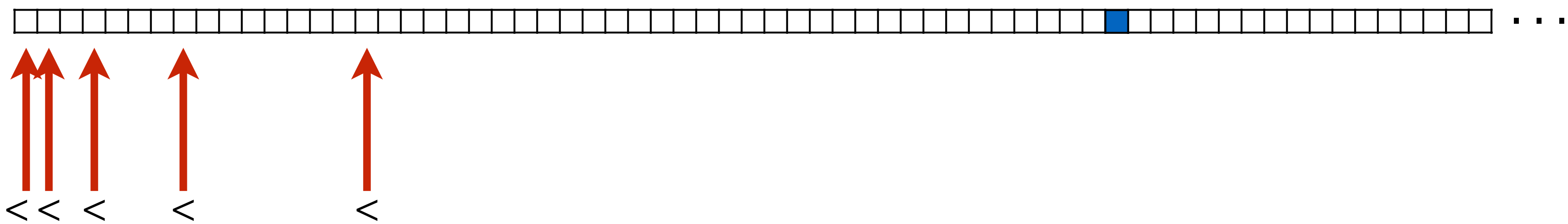

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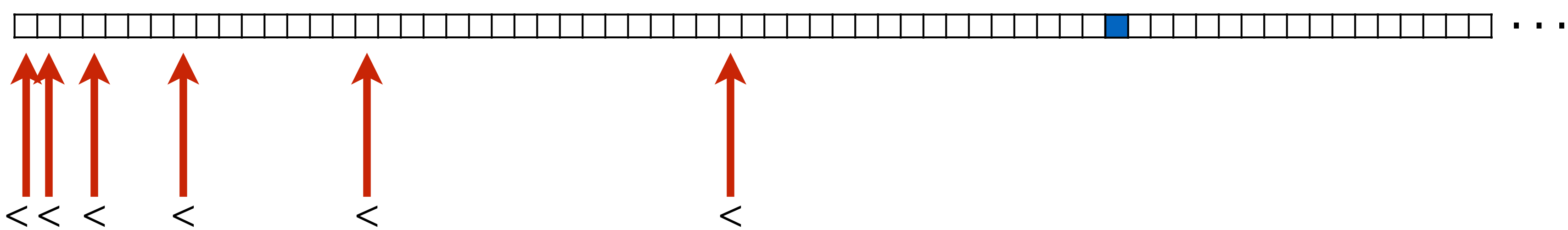

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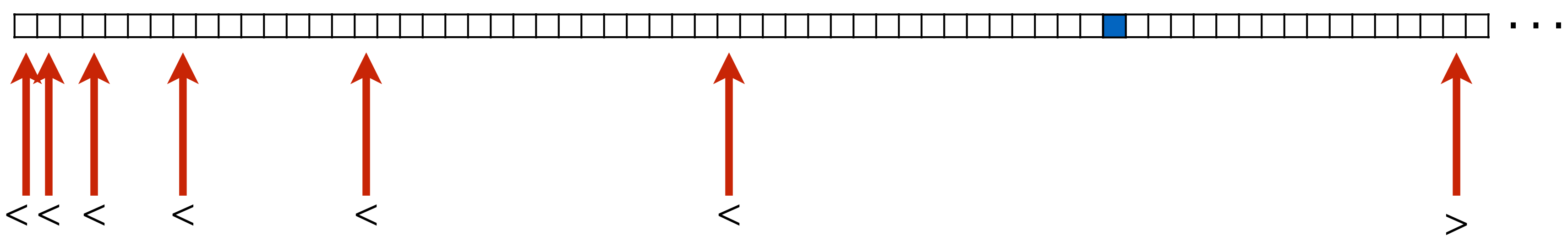

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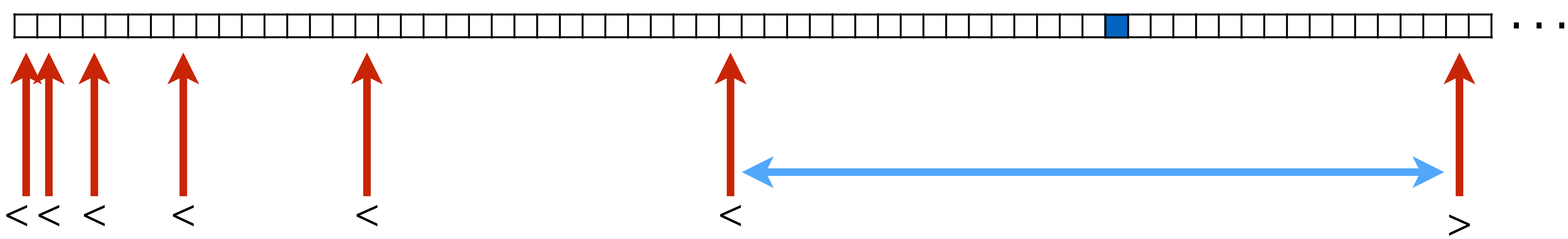

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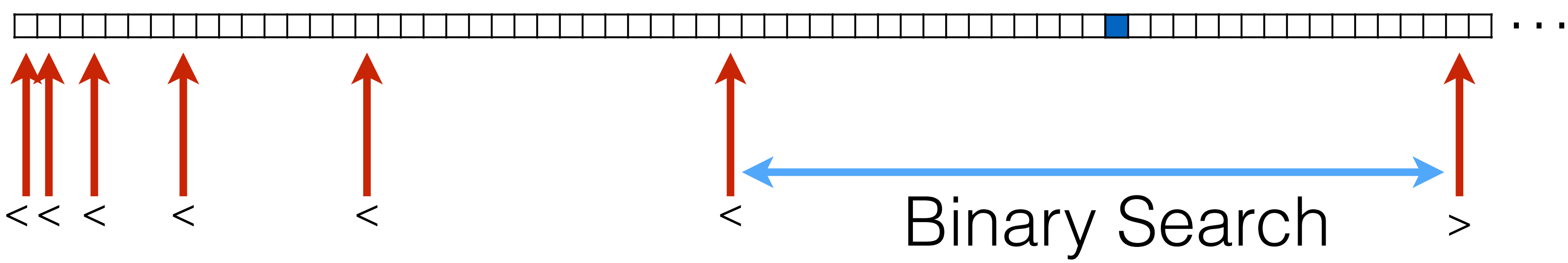

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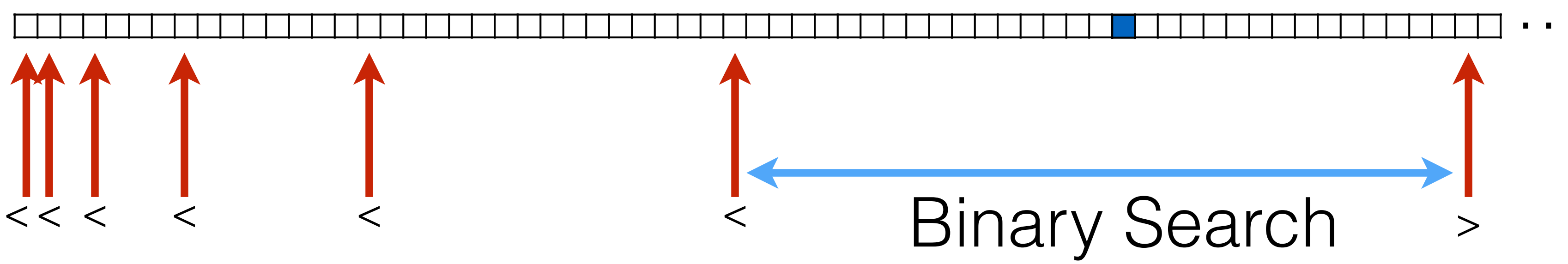

- Running time: $\log(i)$

Nodes and *f*-costs

- Exponential Search:
  - Find *value* in *unbounded sorted array*
- Tree Search:
  - Find (*node expansions*) in (*f-costs*)

- Nodes expansions non-decreasing with *f*-cost

How does BTS work?

Budgeted search

- Like exponential search on f-costs

Budgeted search

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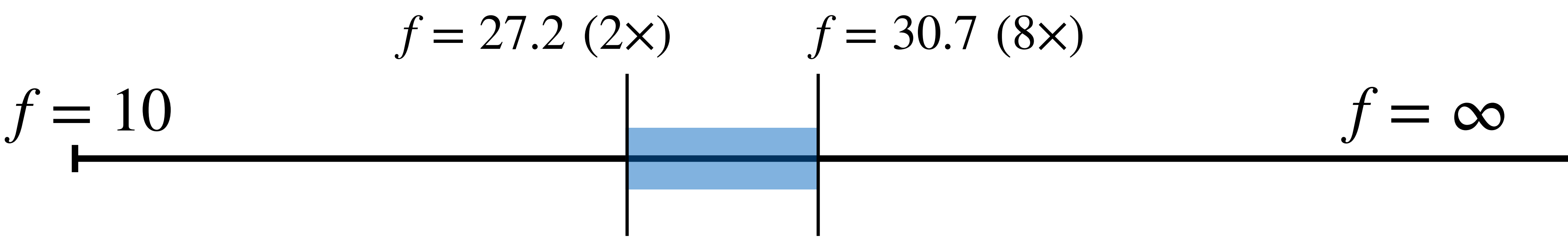
$f = 10$ $f = \infty$

Budgeted search

- Like exponential search on f-costs



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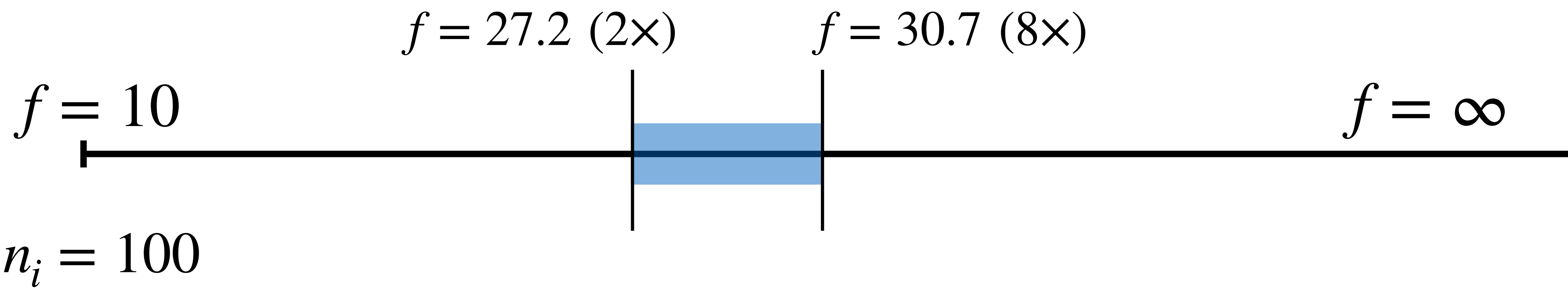

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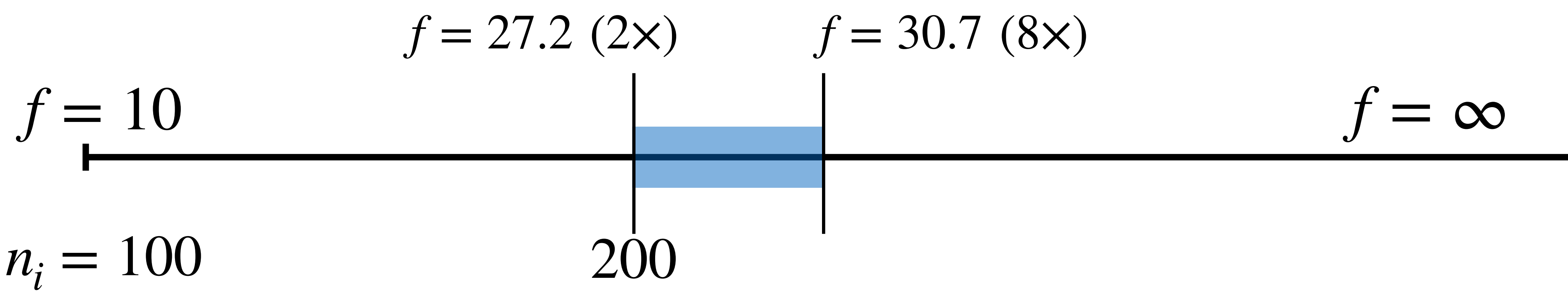

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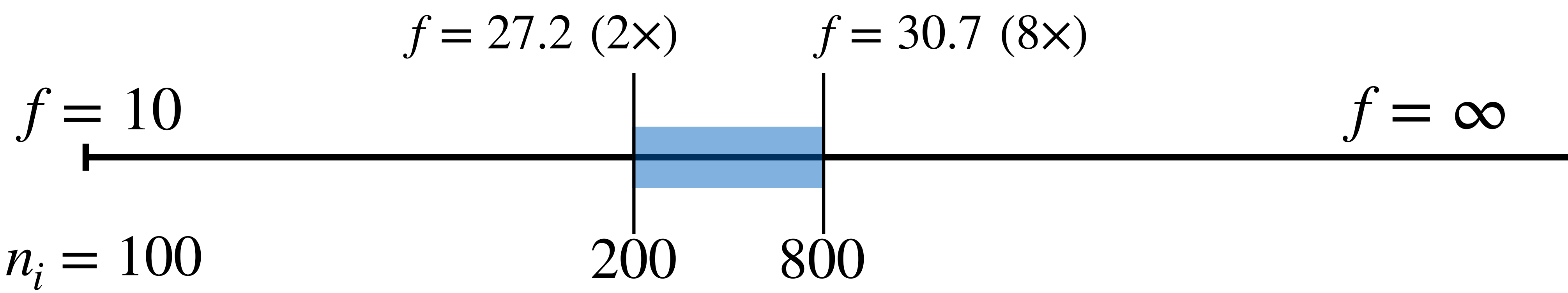

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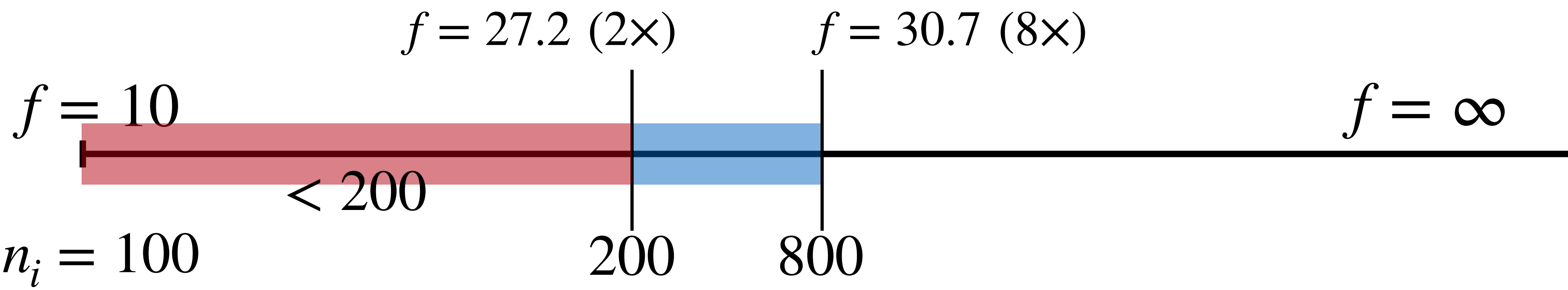

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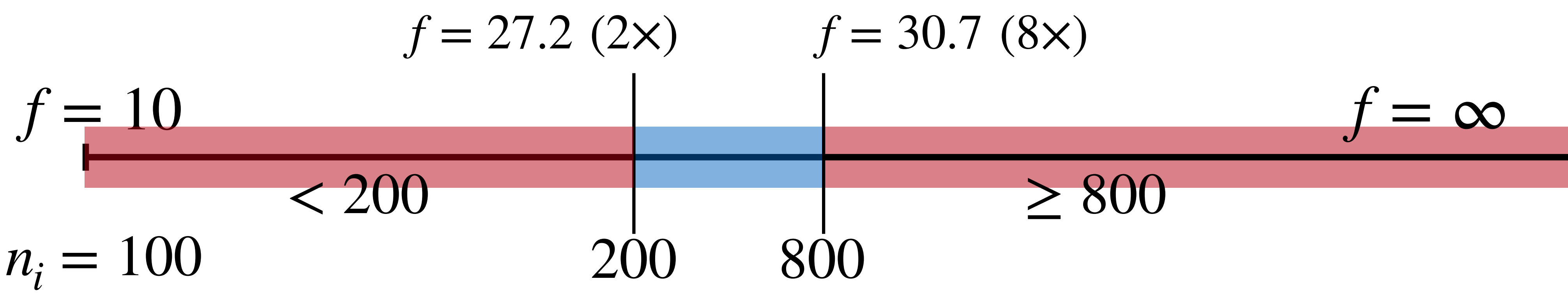

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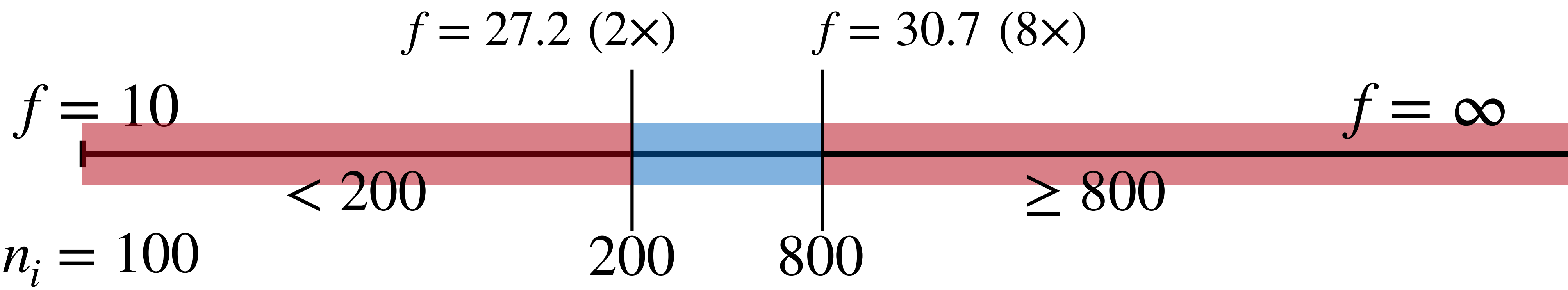

Budget:
Stop when exceeded

Budgeted search

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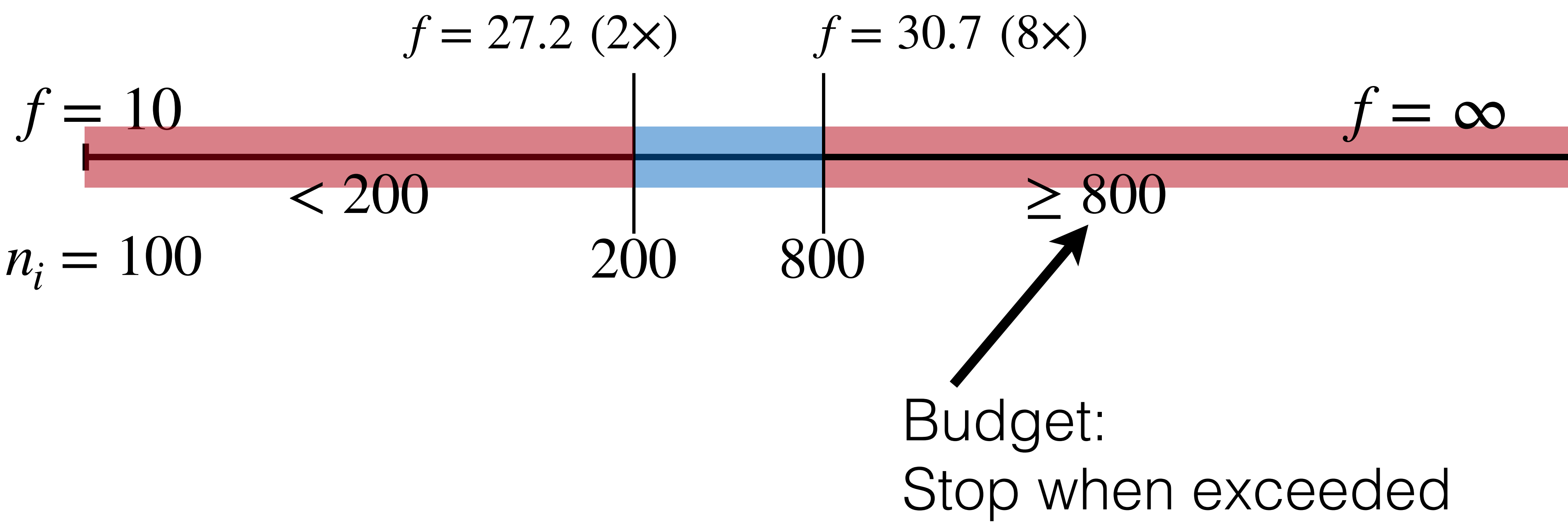

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- Like exponential search on f-costs

$f = 10$

Budgeted search

- Like exponential search on f-costs

$f = 10$

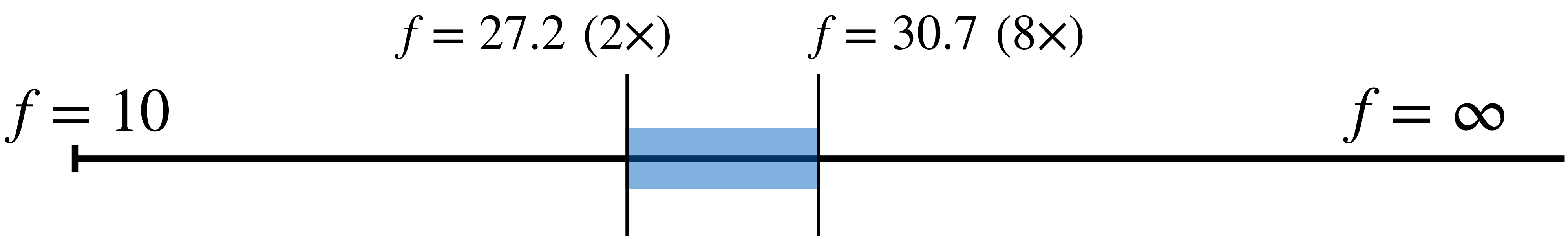
Budgeted search

- Like exponential search on f-costs

$f = 10$ $\qquad\qquad$ $f = \infty$

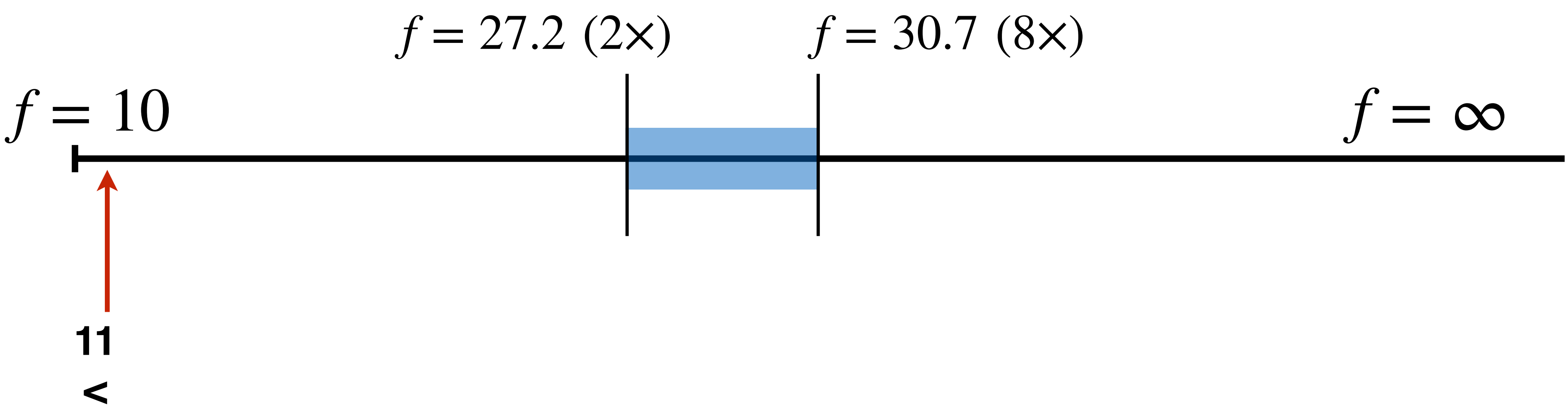
Budgeted search

- Like exponential search on f-costs



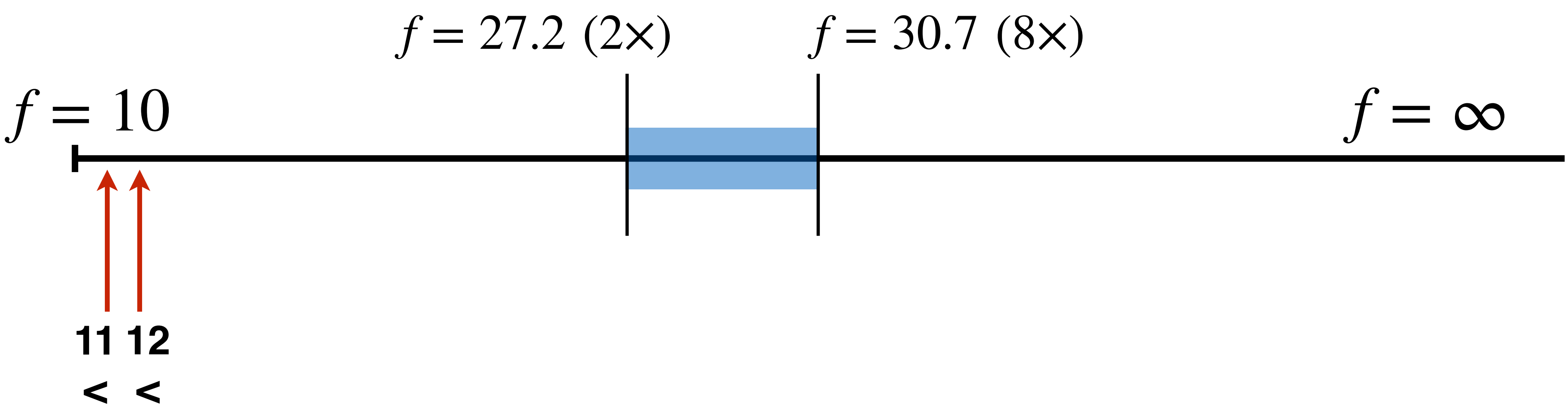
Budgeted search

- Like exponential search on f-costs



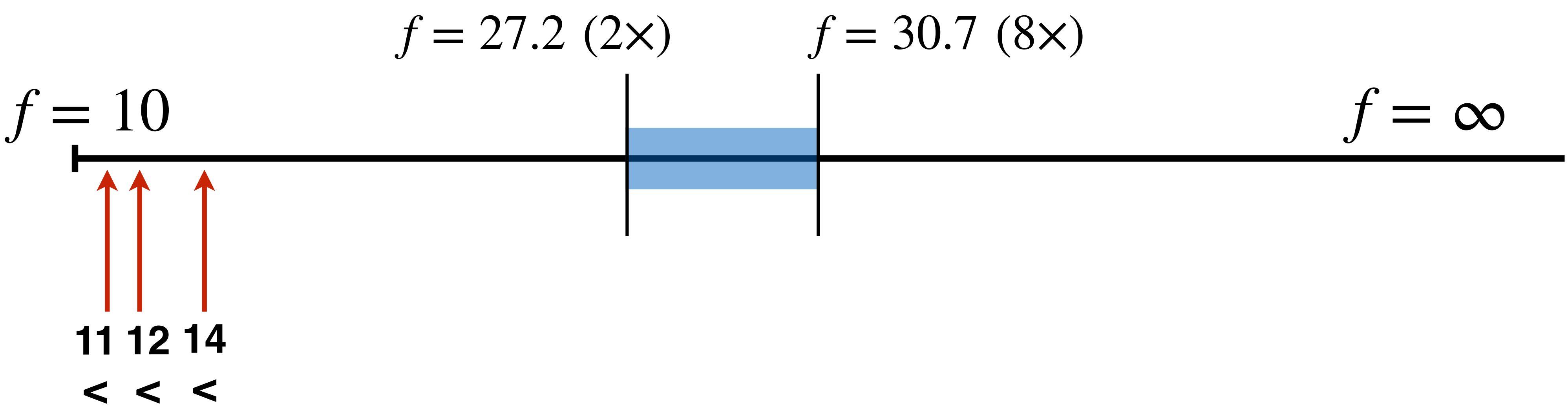
Budgeted search

- Like exponential search on f-costs



Budgeted search

- Like exponential search on f-costs



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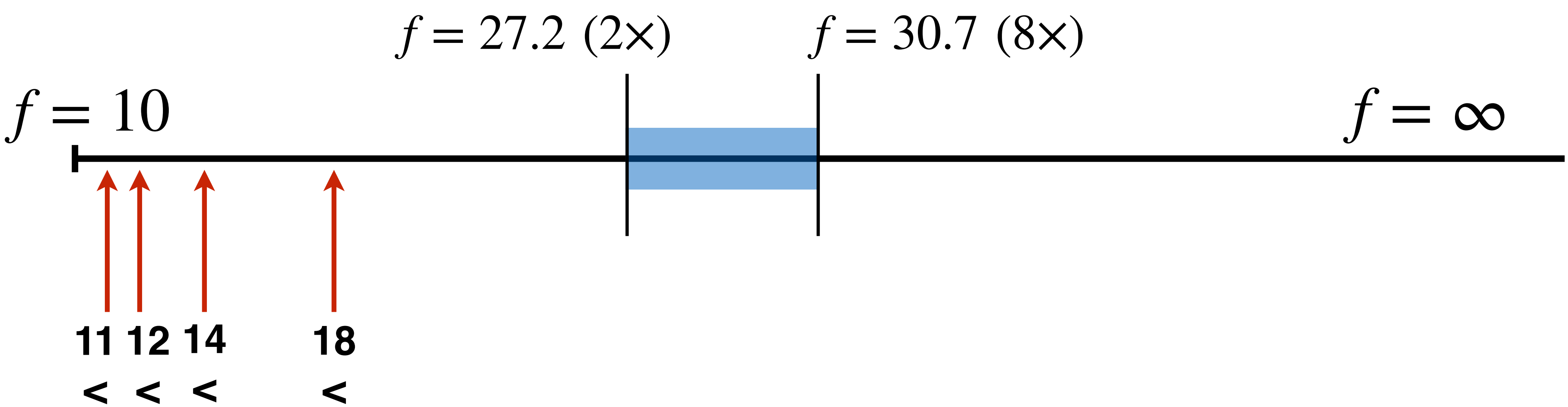
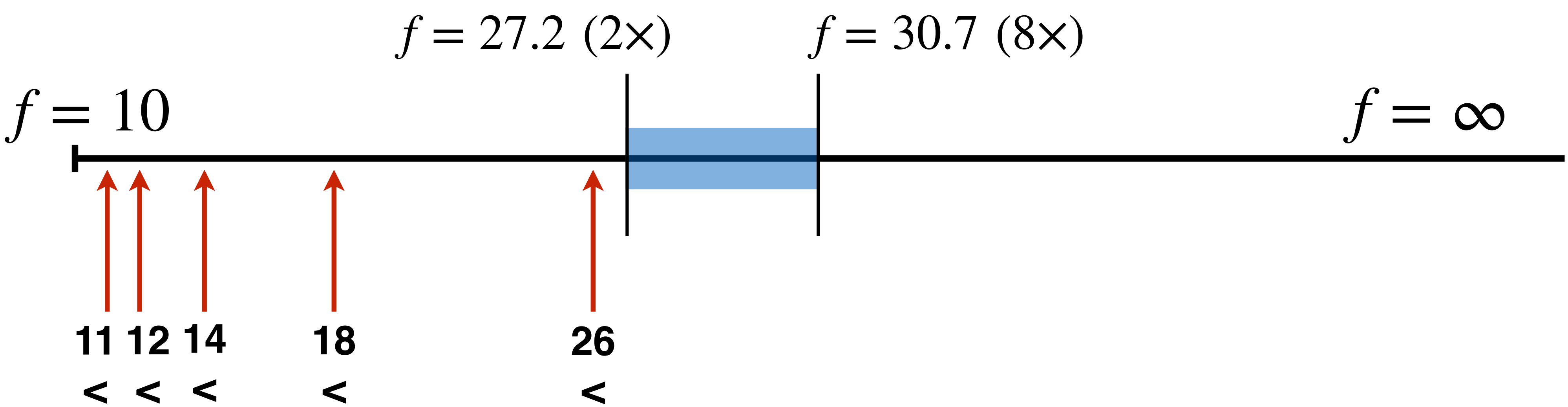


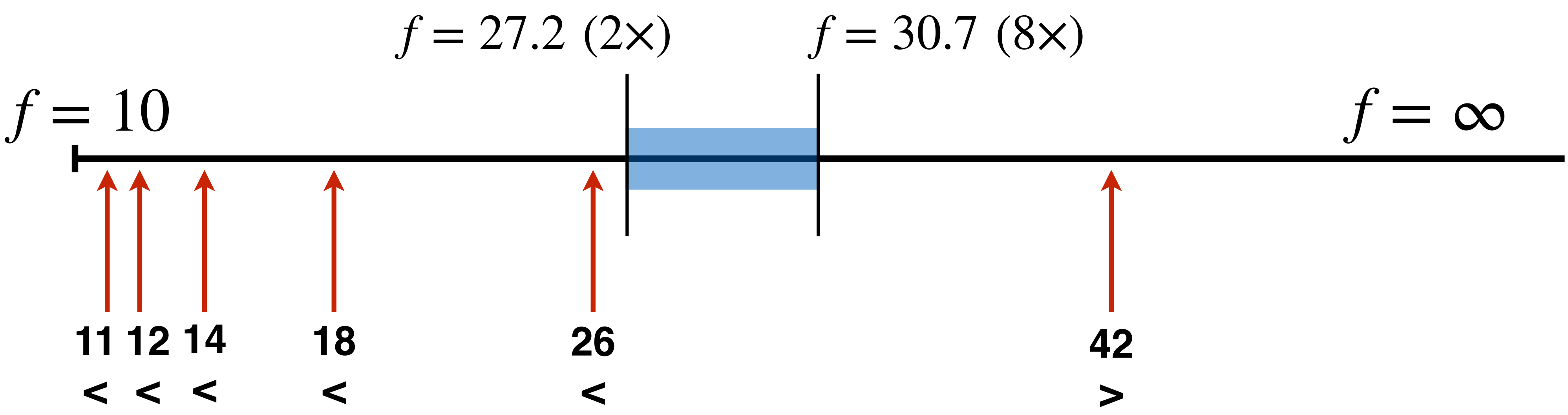
Budgeted search

- Like exponential search on f-costs



Budgeted search

- Like exponential search on f-costs



Budgeted search

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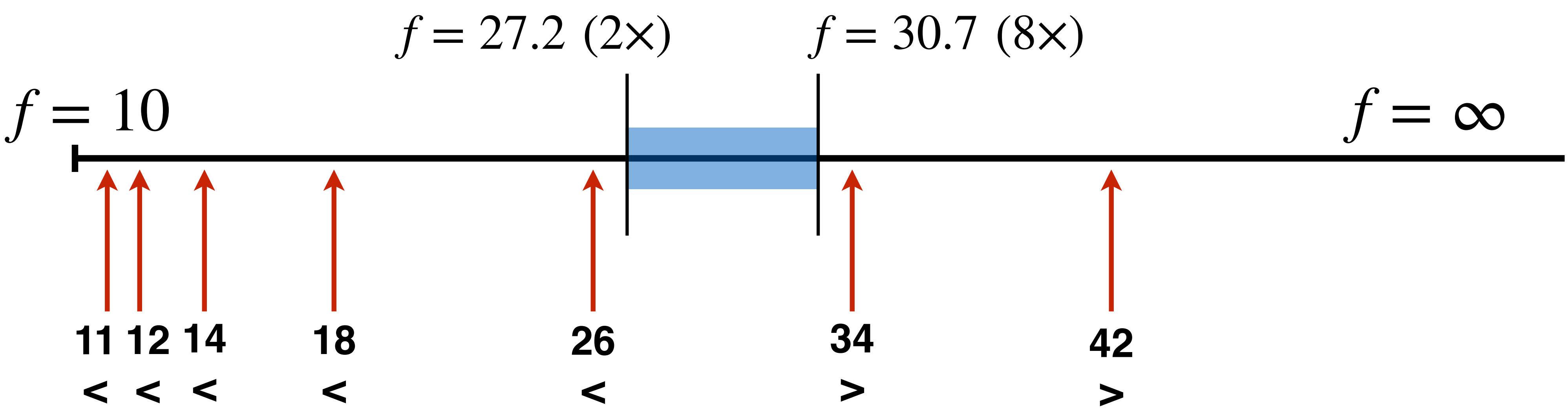

Budgeted search

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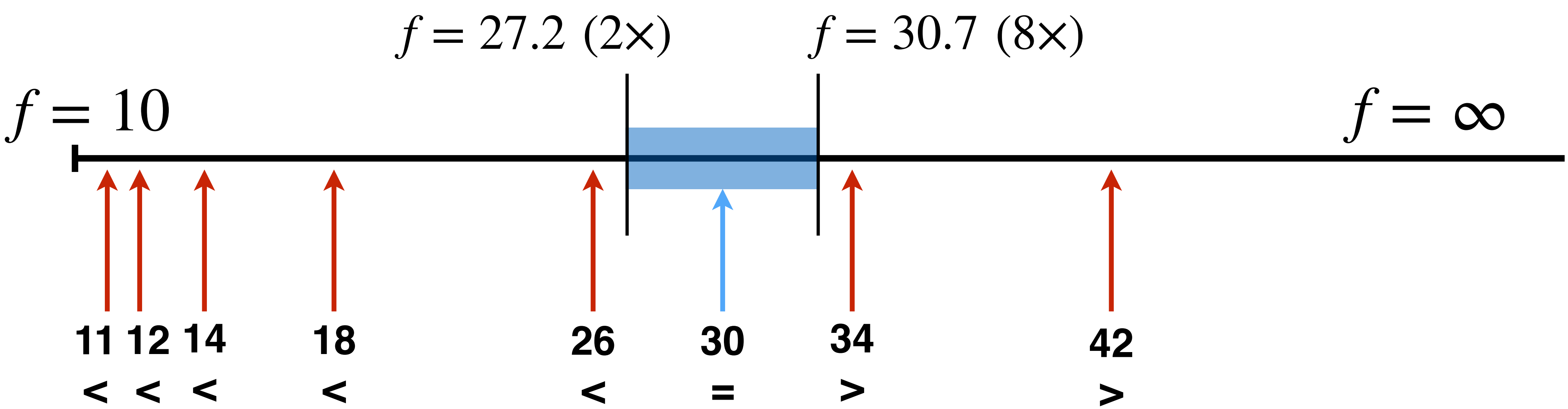
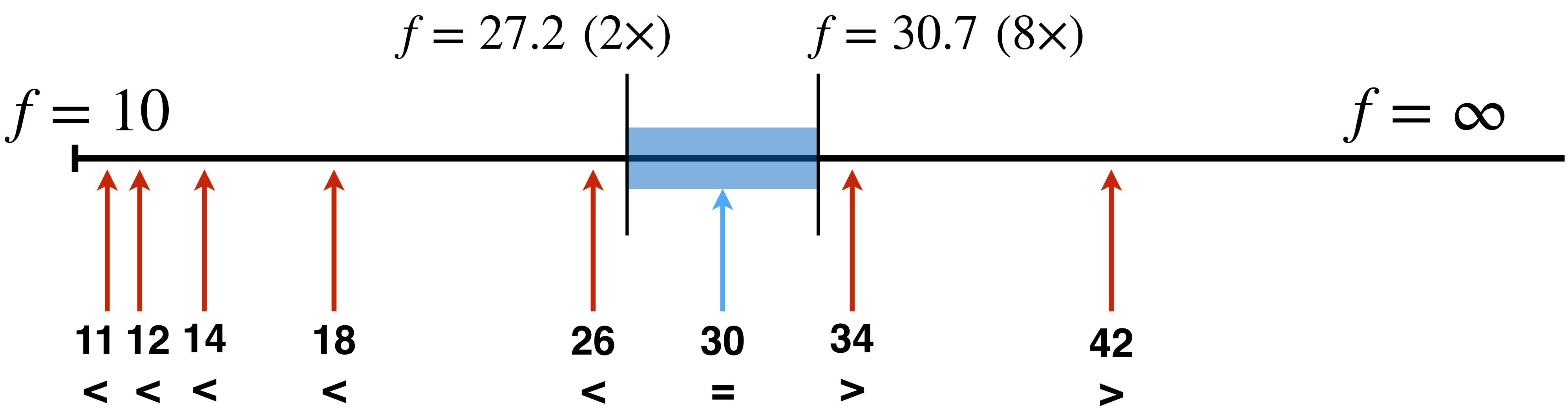


Budgeted search

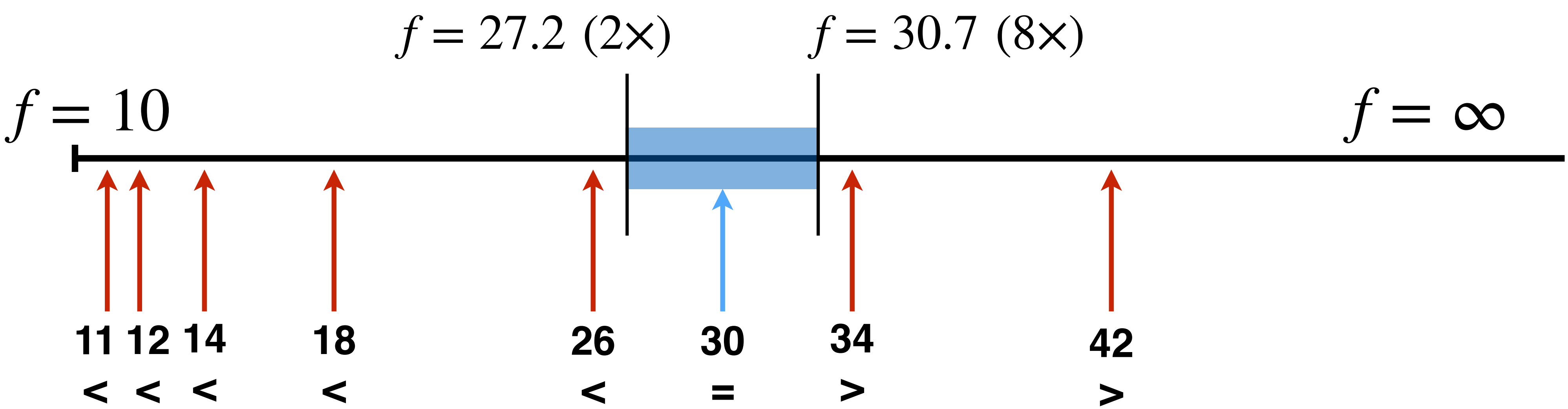
- Like exponential search on f-costs



$n_i \log(f_i)$

Budgeted search

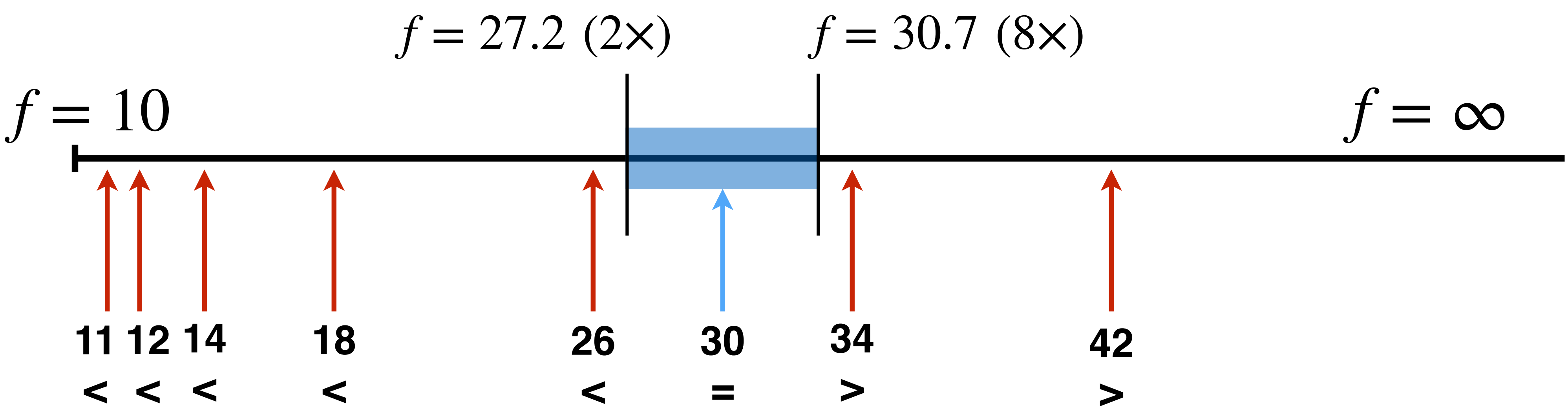
- Like exponential search on f-costs



$$\sum_{0}^{i} n_i \log(f_i)$$

Budgeted search

- Like exponential search on f-costs



$$\sum_{0}^{i} n_i \log(f_i) < \left(\sum_{0}^{i} n_i\right) \log(C^*) \approx N \log(C^*)$$

BTS Phases

Find next *f*-cost bound:

BTS Phases

Find next *f*-cost bound:

1. Search with conservative *f*, ∞ budget (IDA*)

BTS Phases

Find next f-cost bound:

1. Search with conservative f, ∞ budget (IDA*)
2. Grow f exponentially, constant budget

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Find next *f*-cost bound:

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2. Grow *f* exponentially, constant budget
3. Do binary search on *f*, constant budget

BTS Phases

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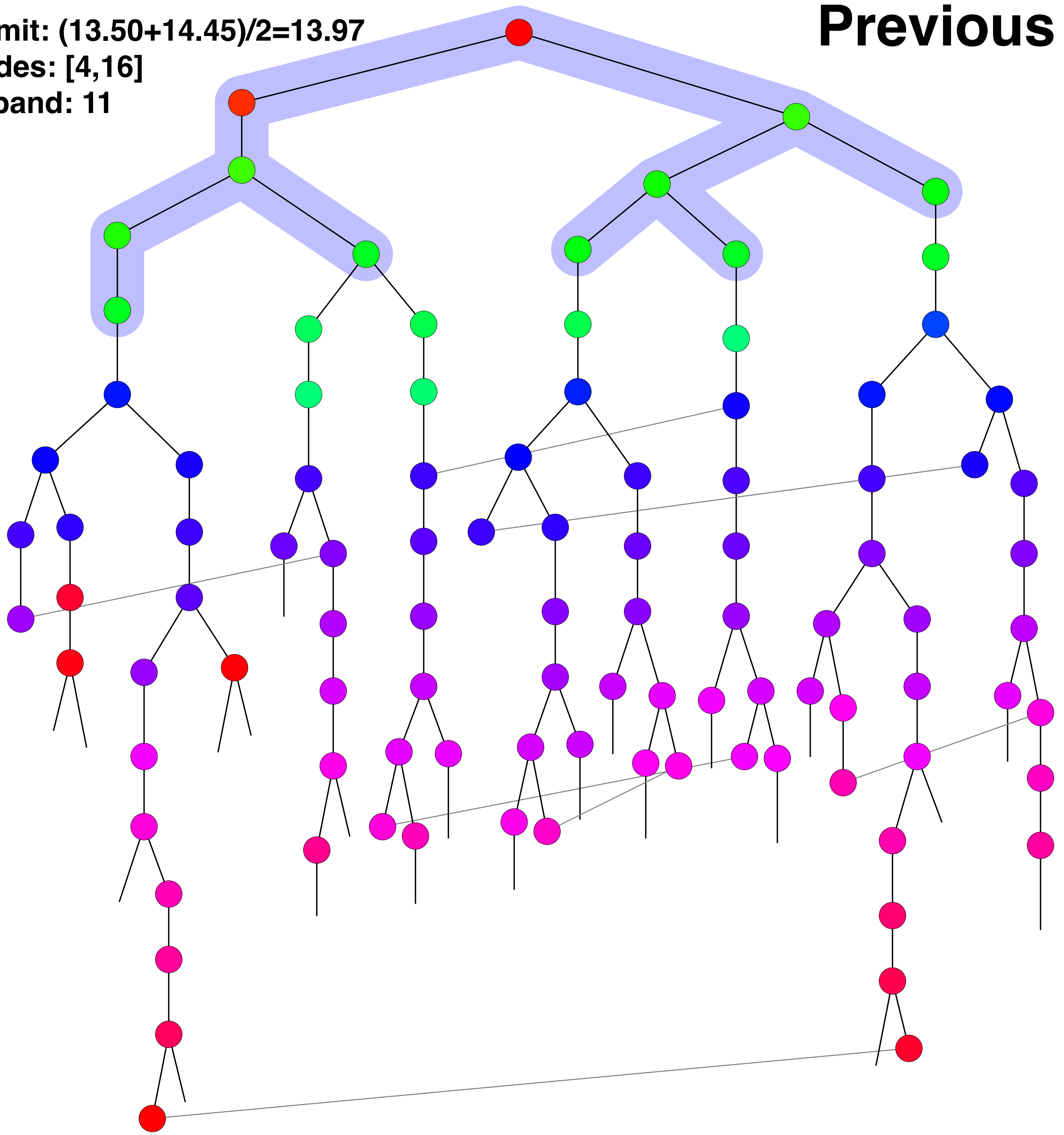
1. Search with conservative *f*, ∞ budget (IDA*)
2. Grow *f* exponentially, constant budget
3. Do binary search on *f*, constant budget

Previous Iteration

f-limit: (13.50+14.45)/2=13.97
nodes: [4,16]
expand: 11



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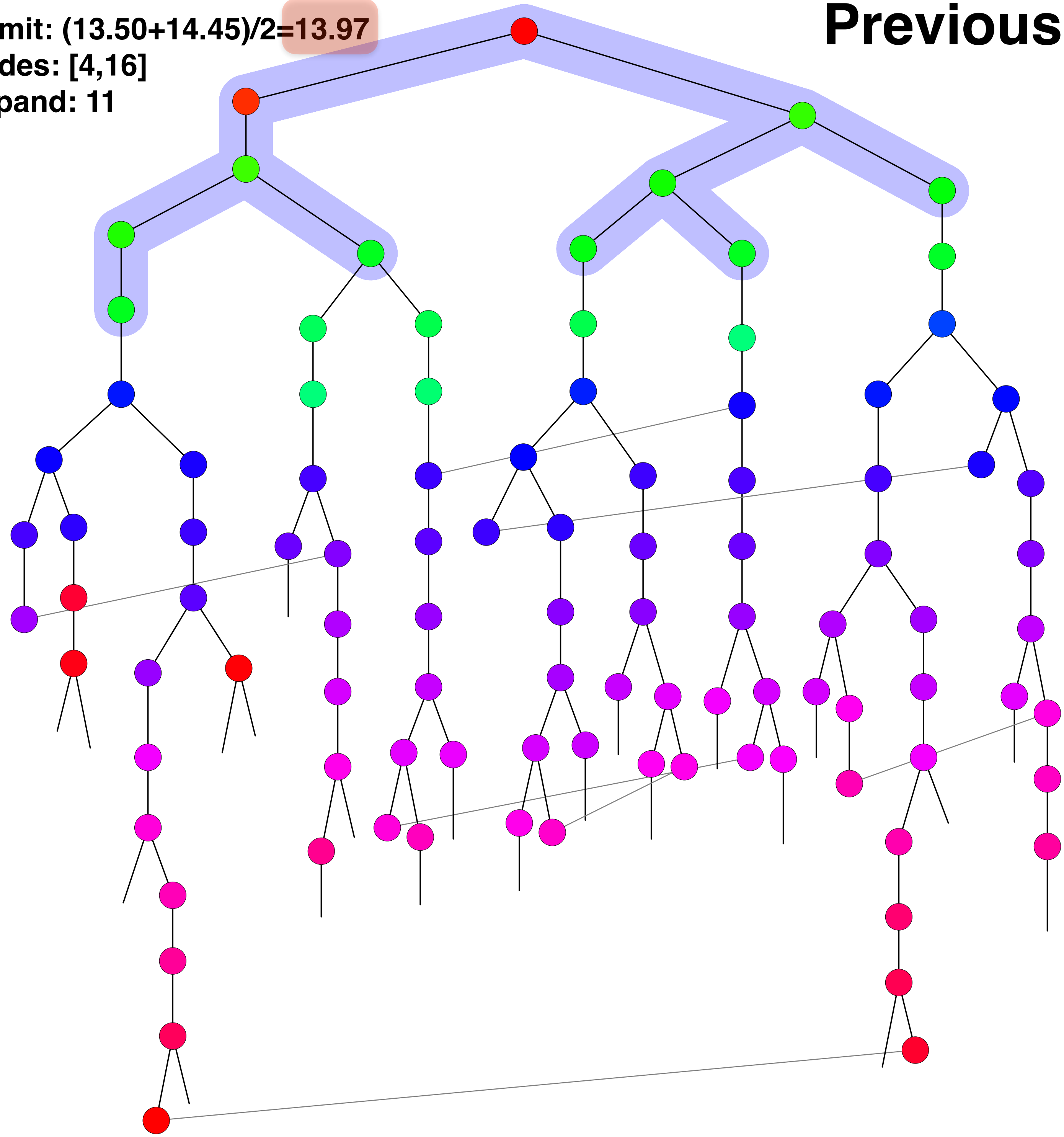

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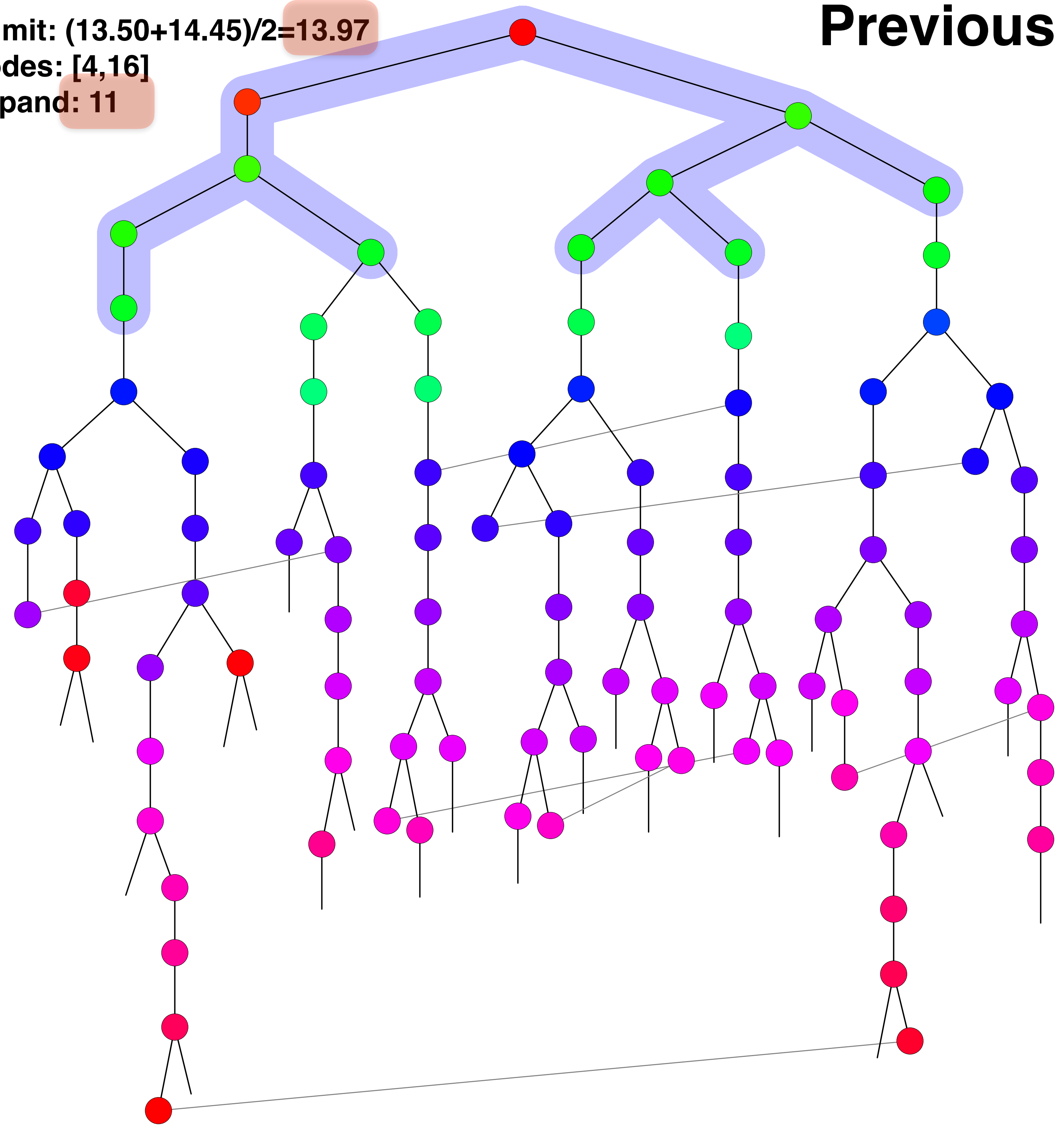
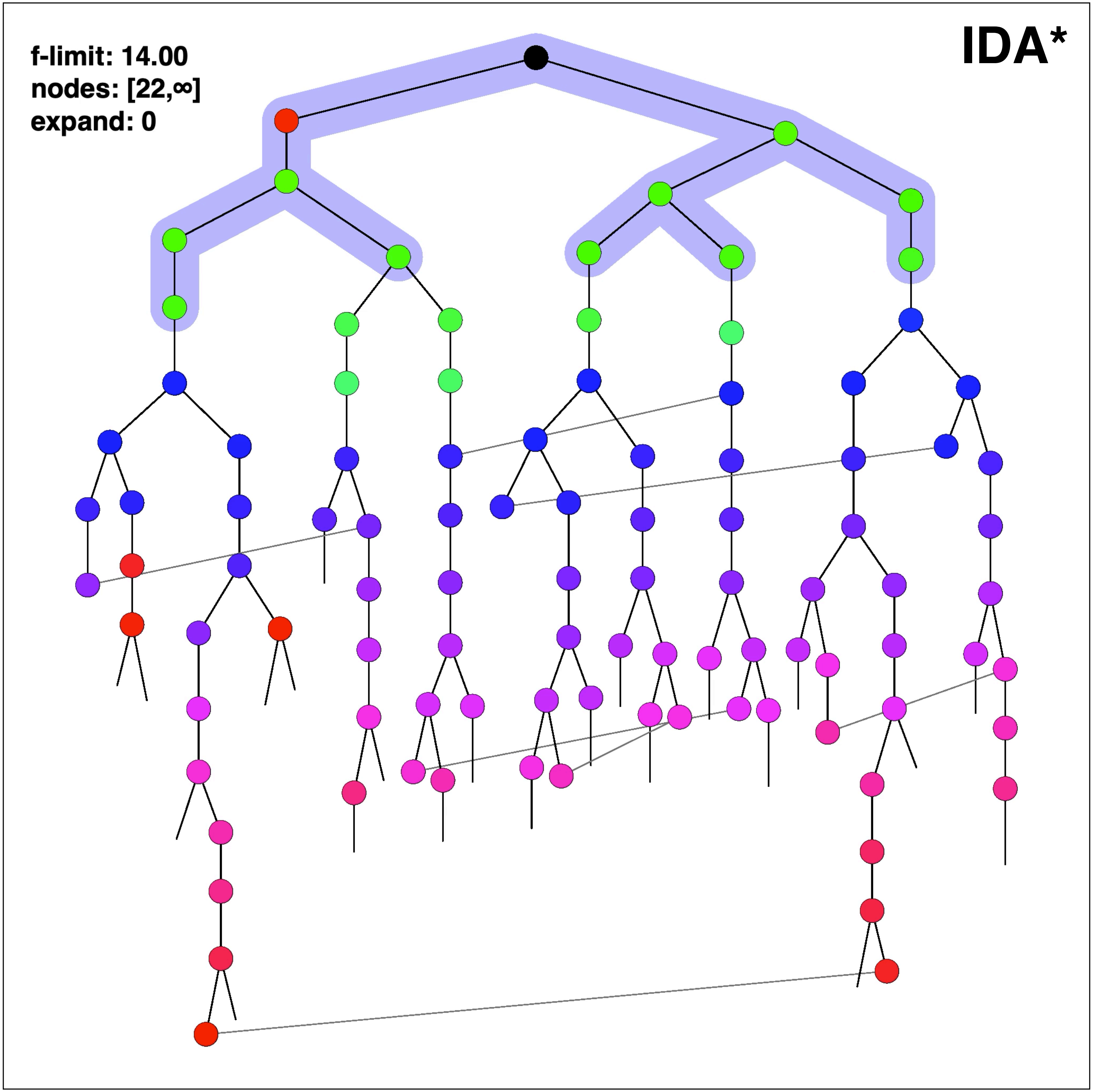
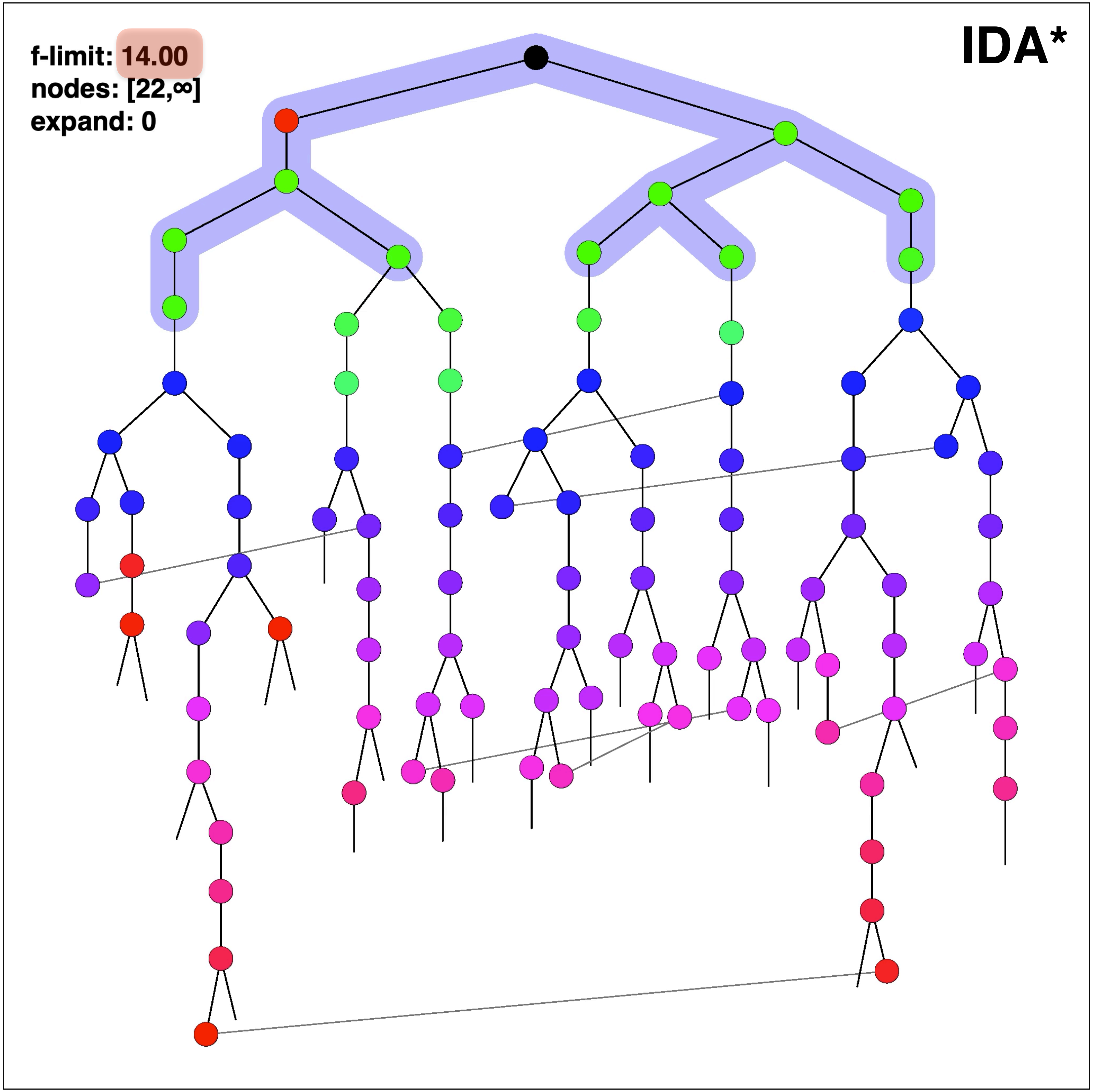
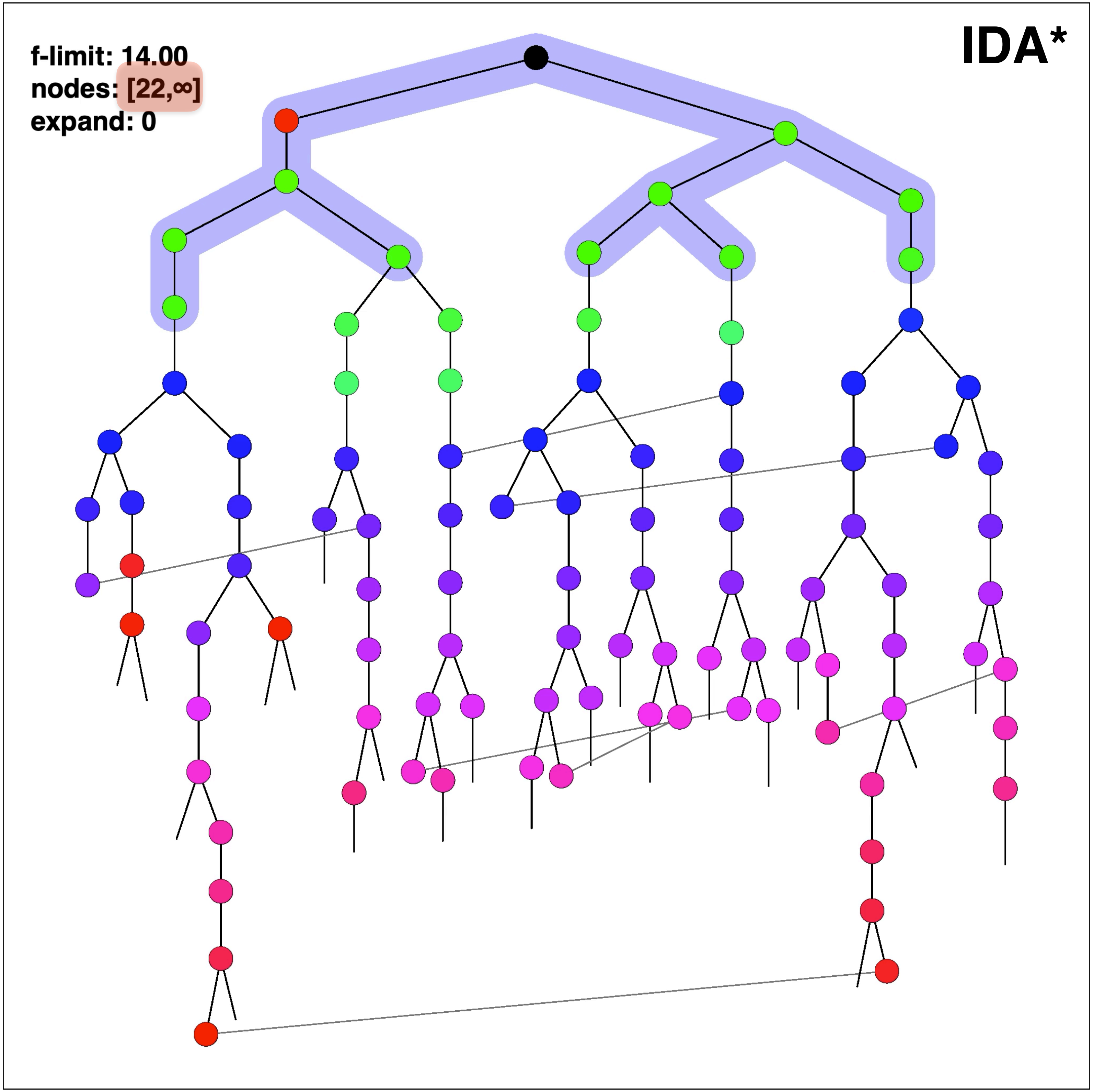



f-limit: 14.00
nodes: [22,∞]
expand: 0
IDA*


f-limit: 14.00
nodes: [22,∞]
expand: 0
IDA*



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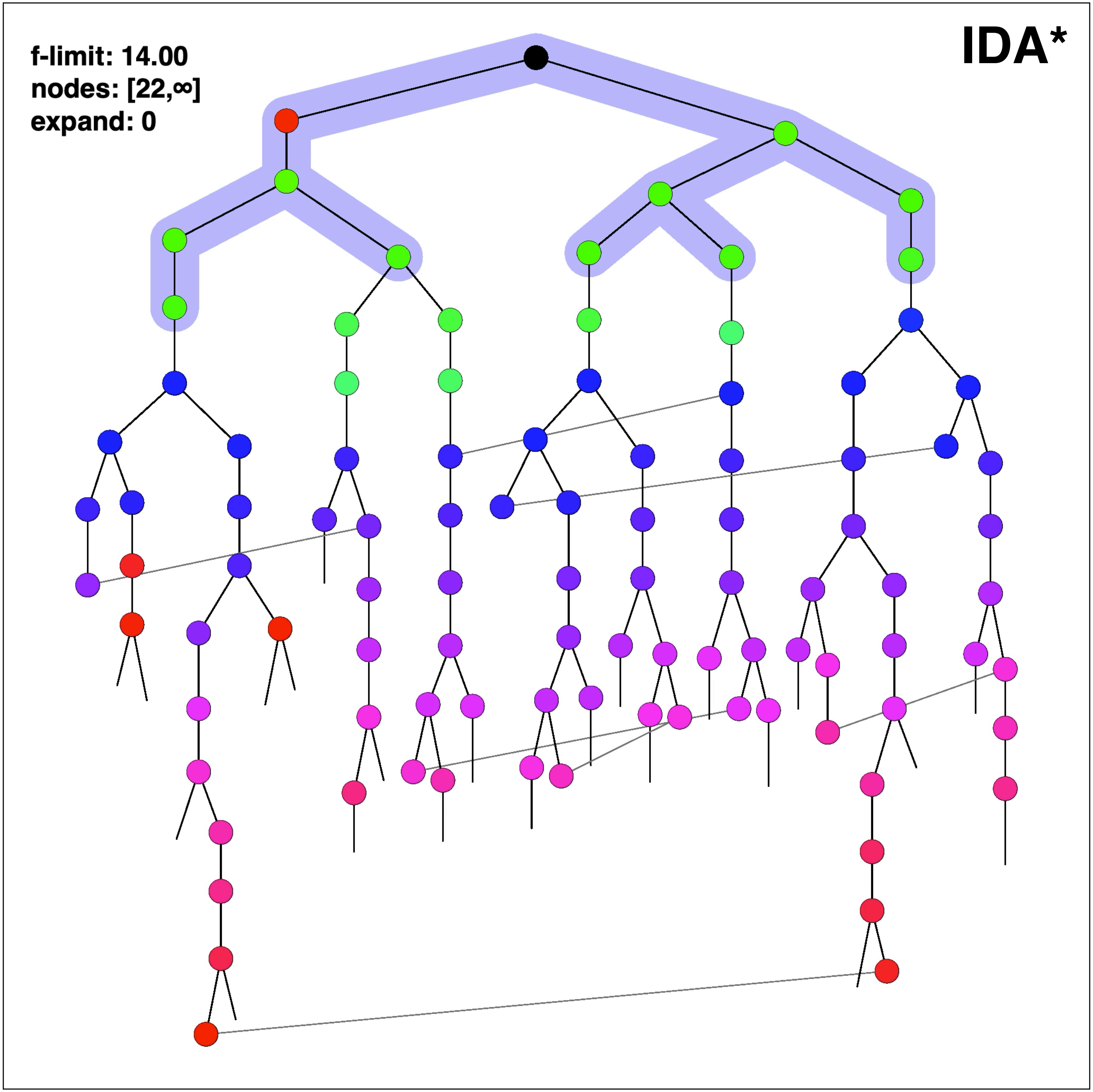
IDA*



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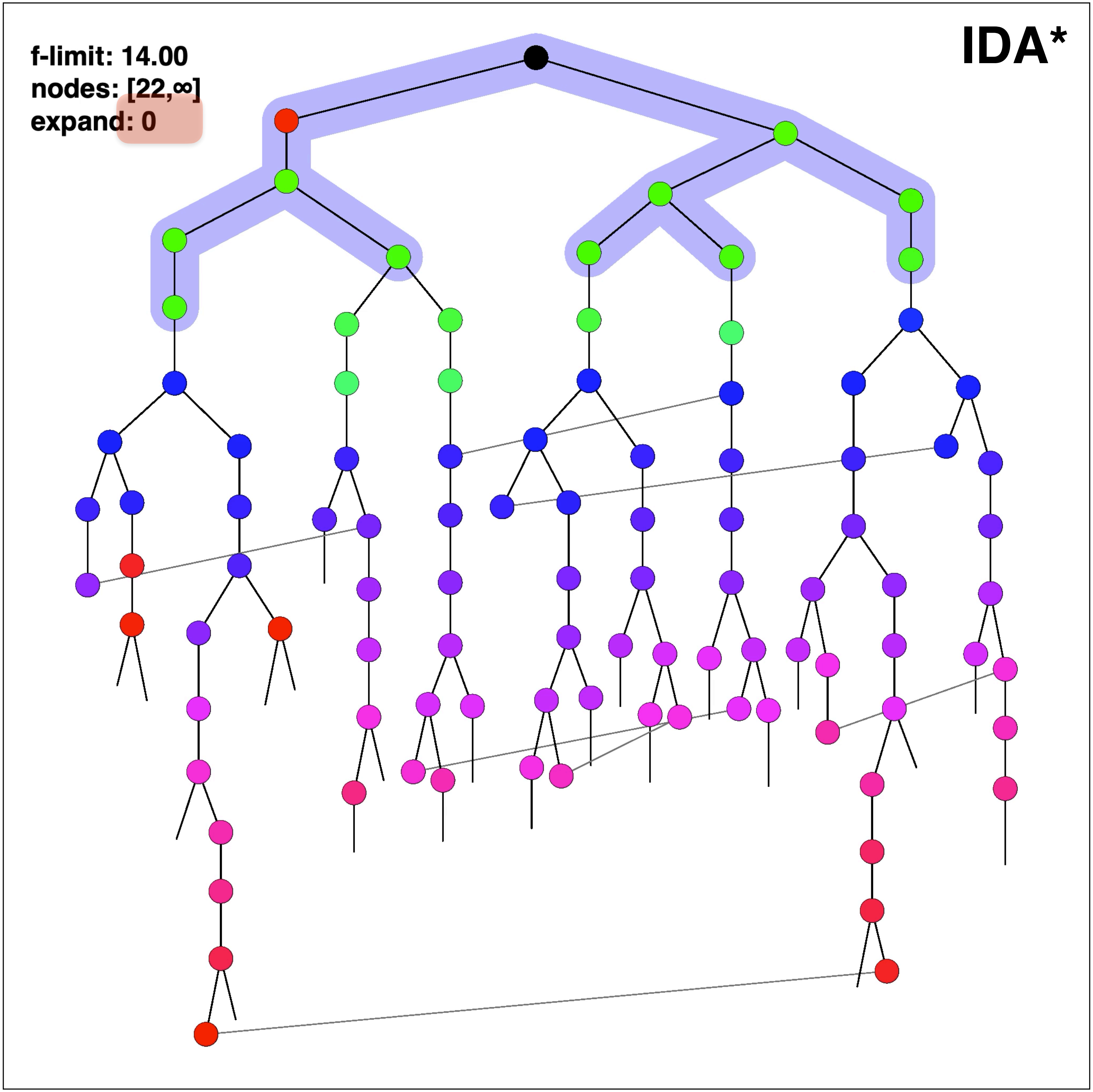
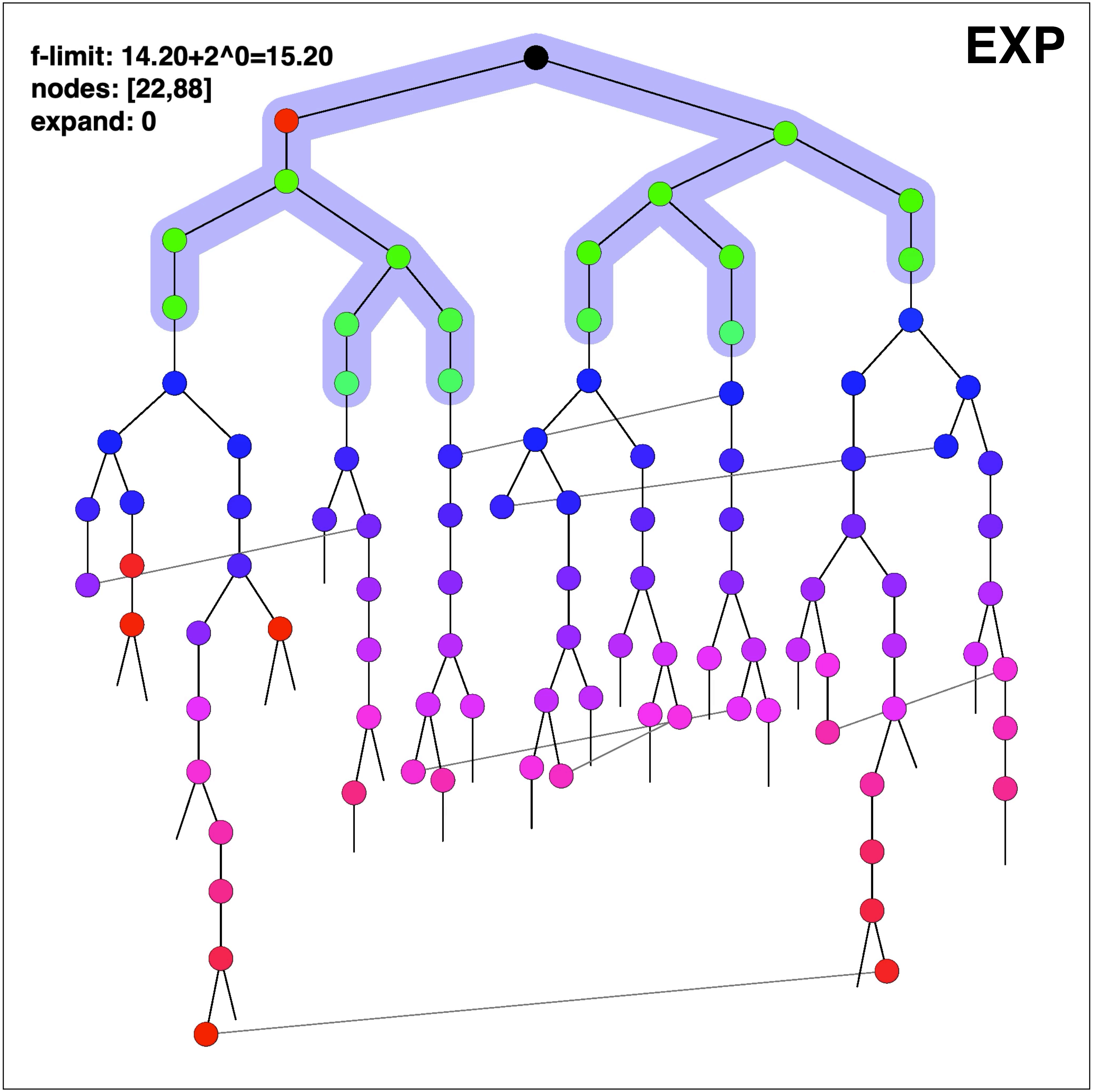
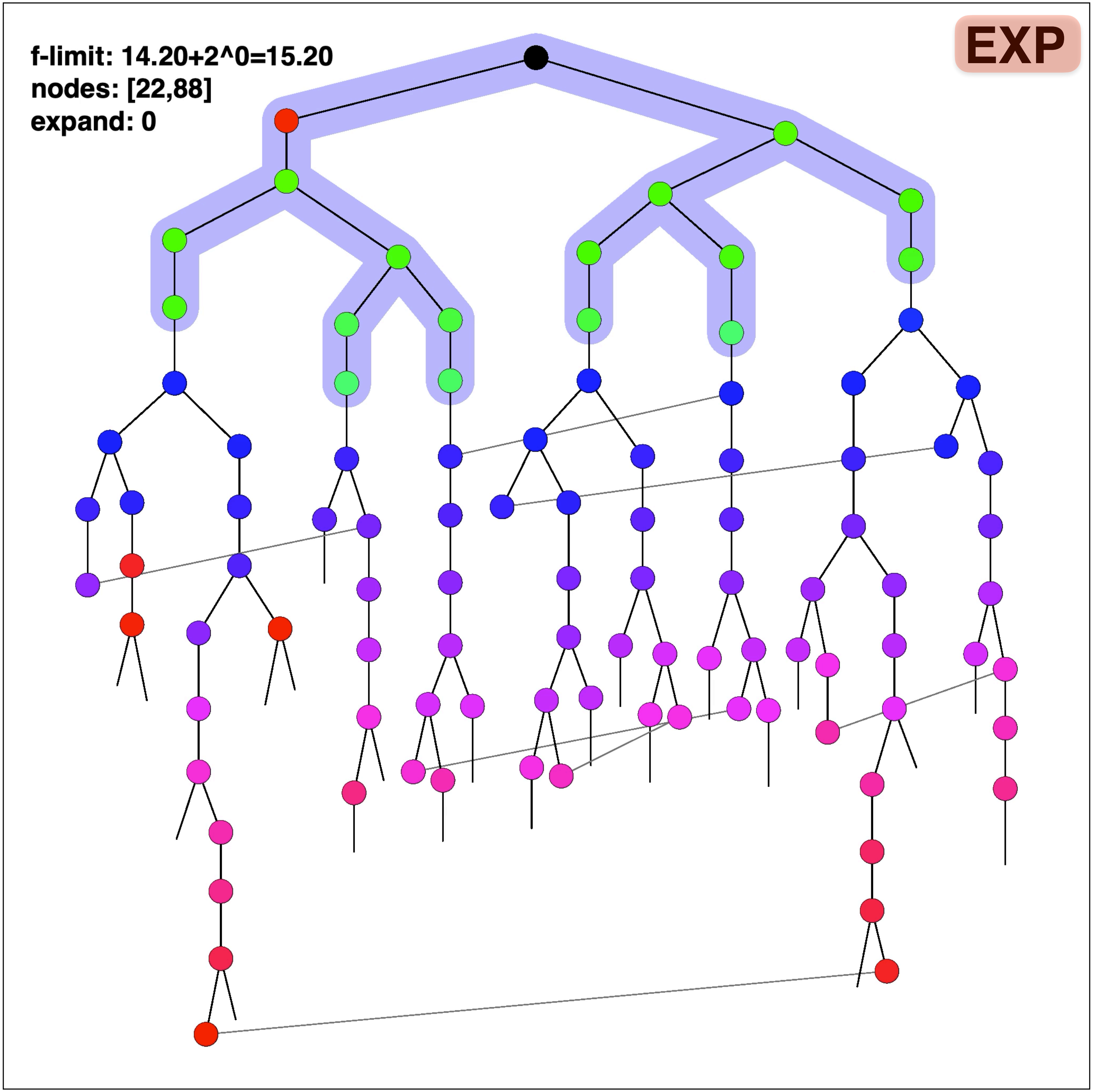
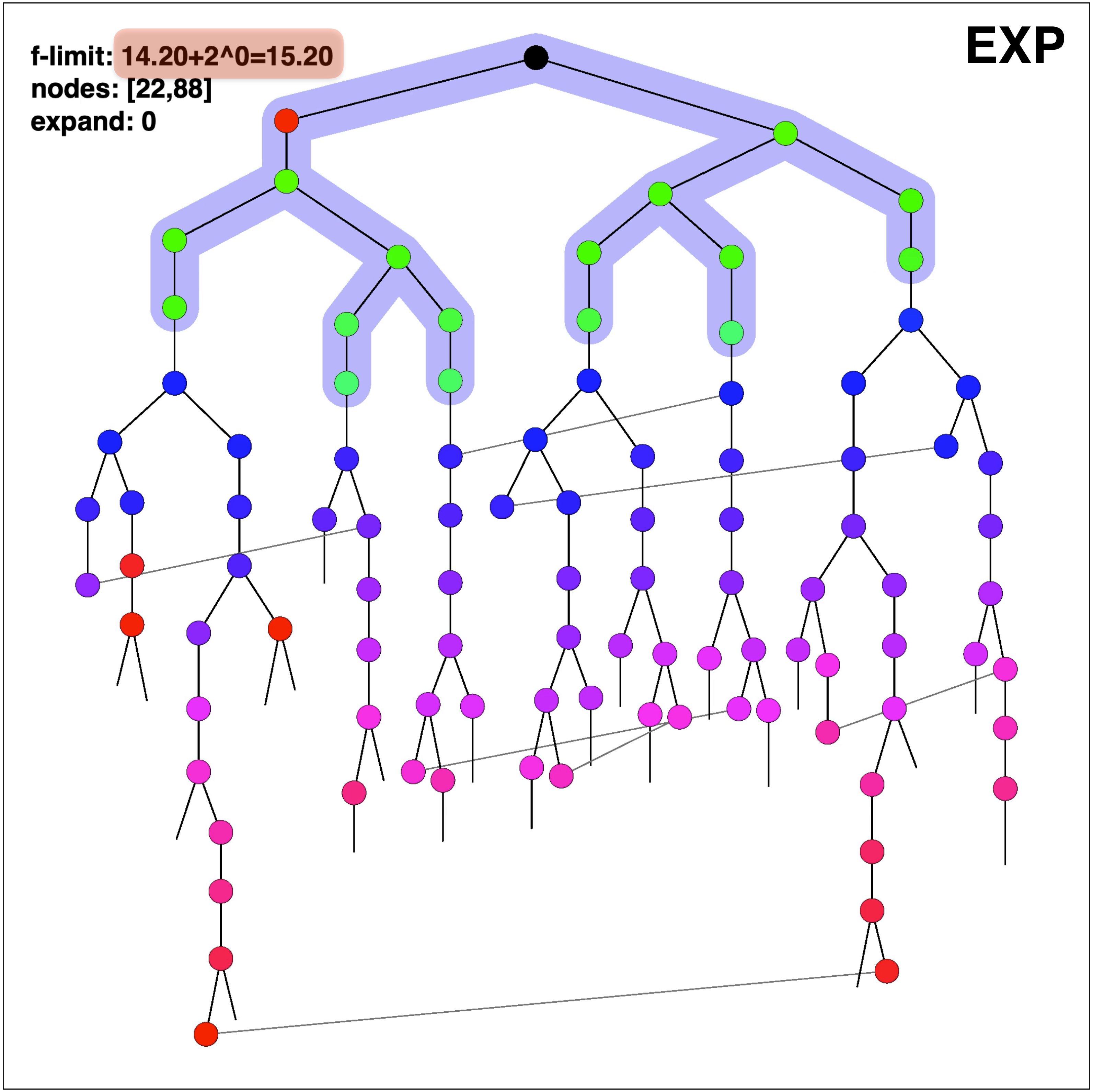
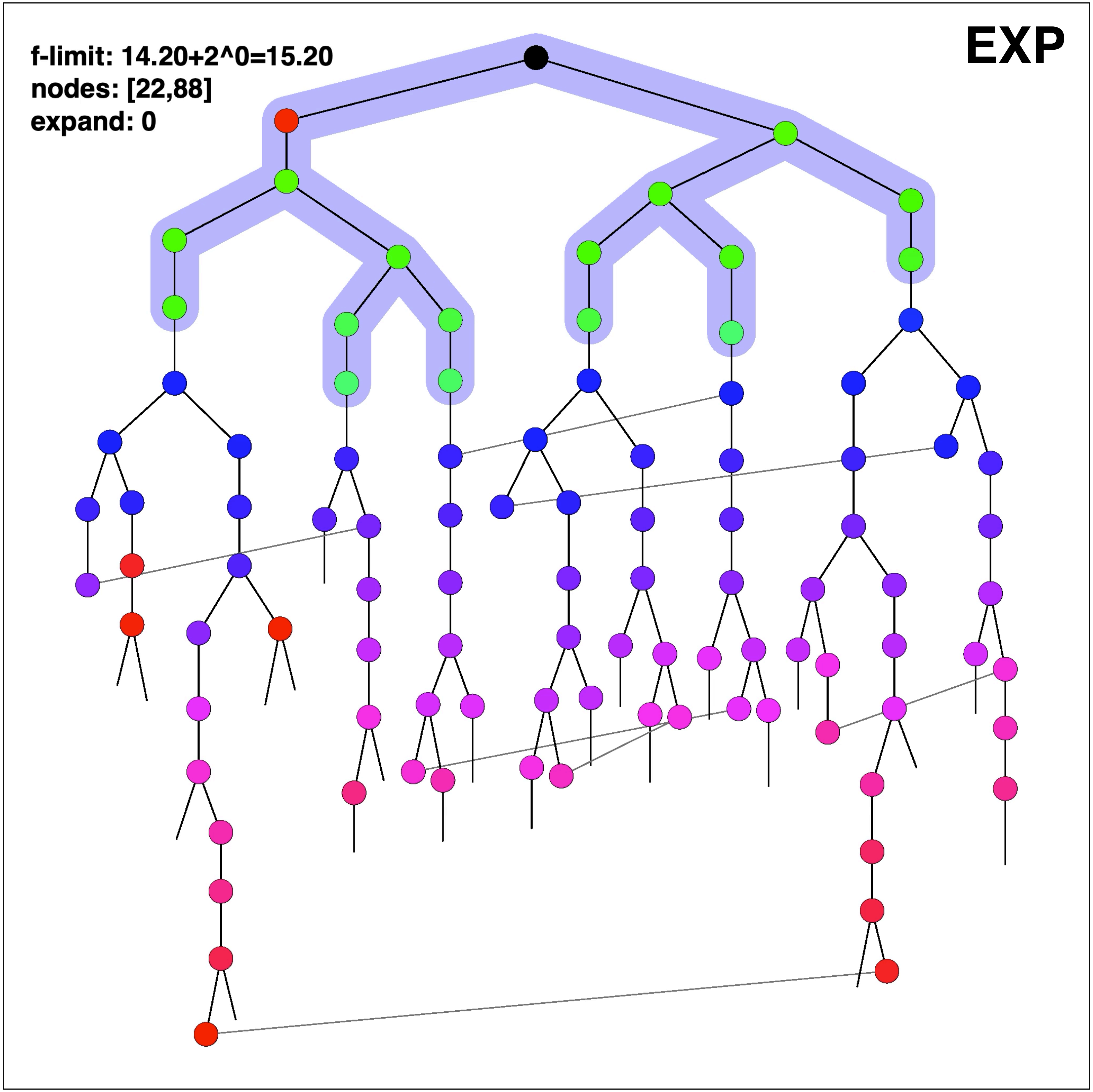


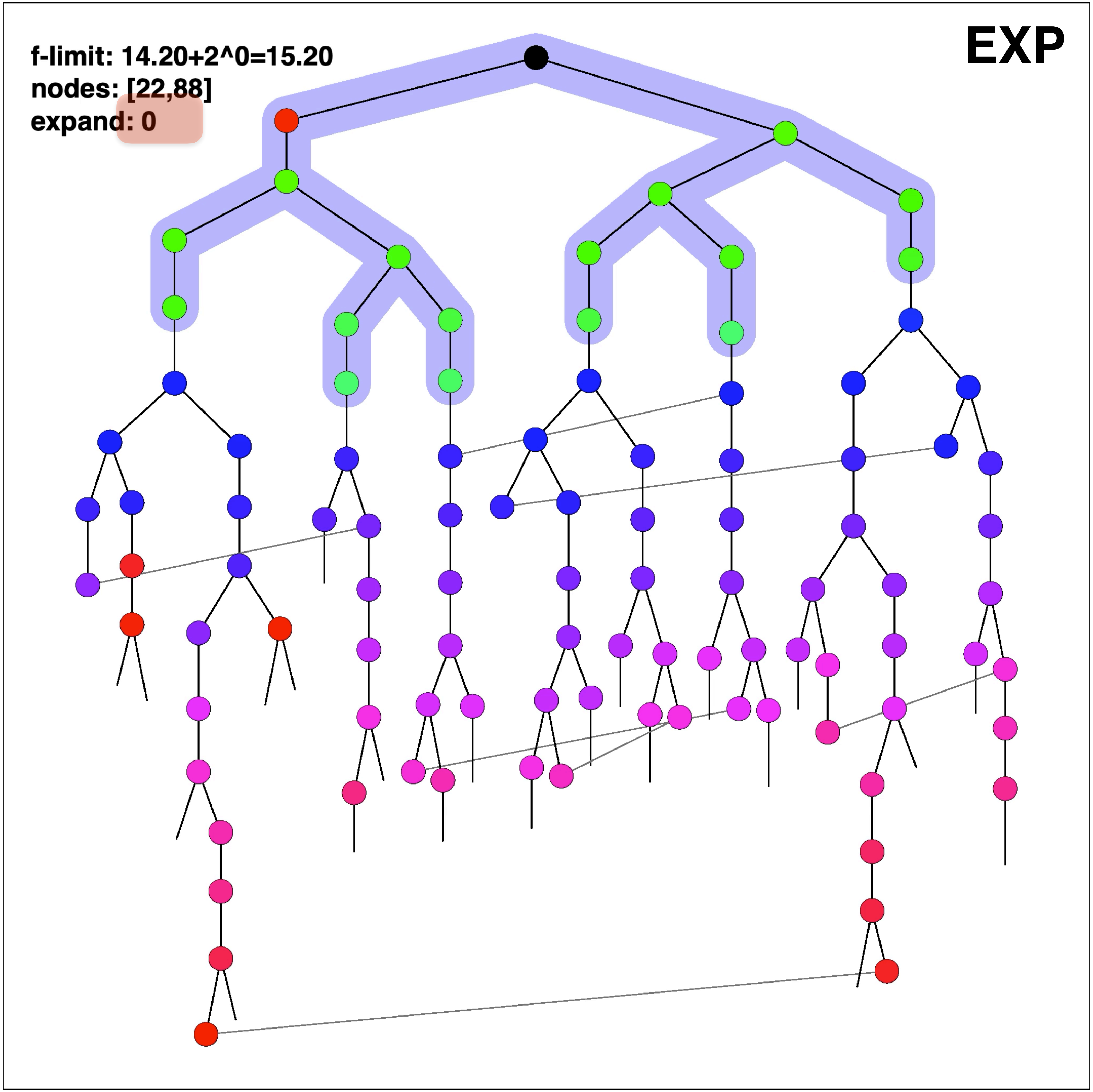

f-limit: 14.20+2^0=15.20
nodes: [22,88]
expand: 0
EXP

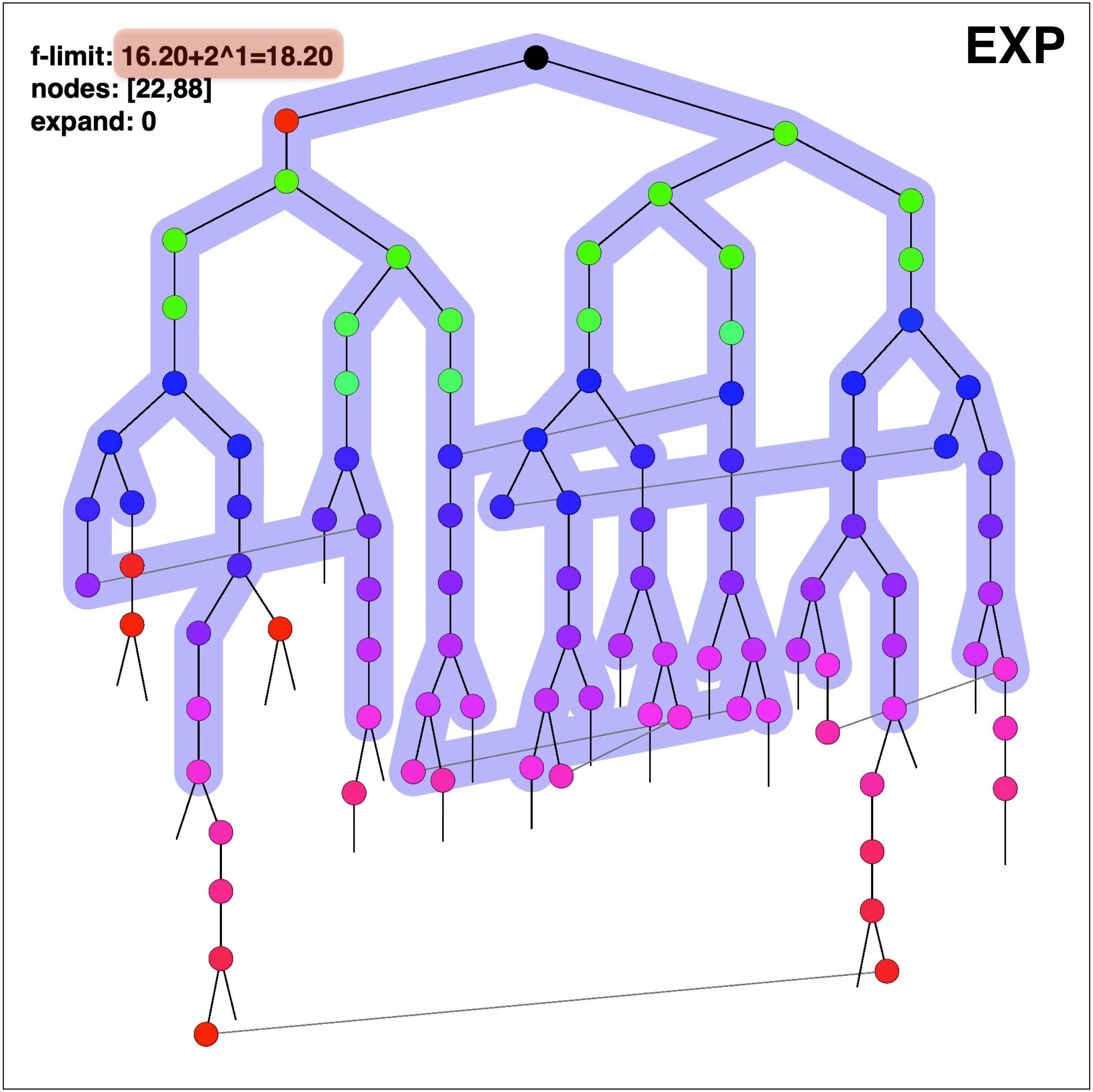

f-limit: 14.20+2^0=15.20
nodes: [22,88]
expand: 0
EXP

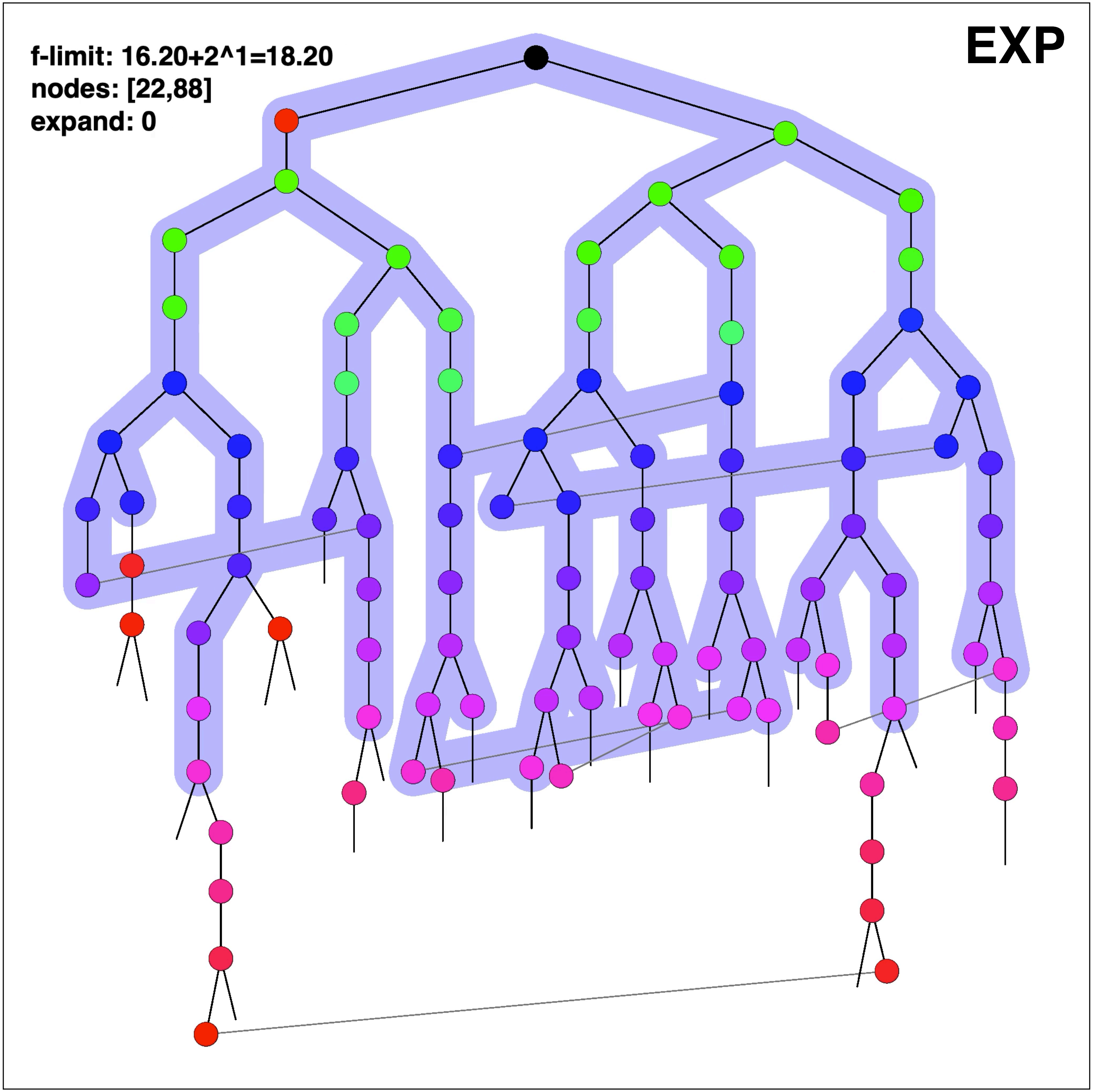

f-limit: 14.20+2^0=15.20
nodes: [22,88]
expand: 0
EXP

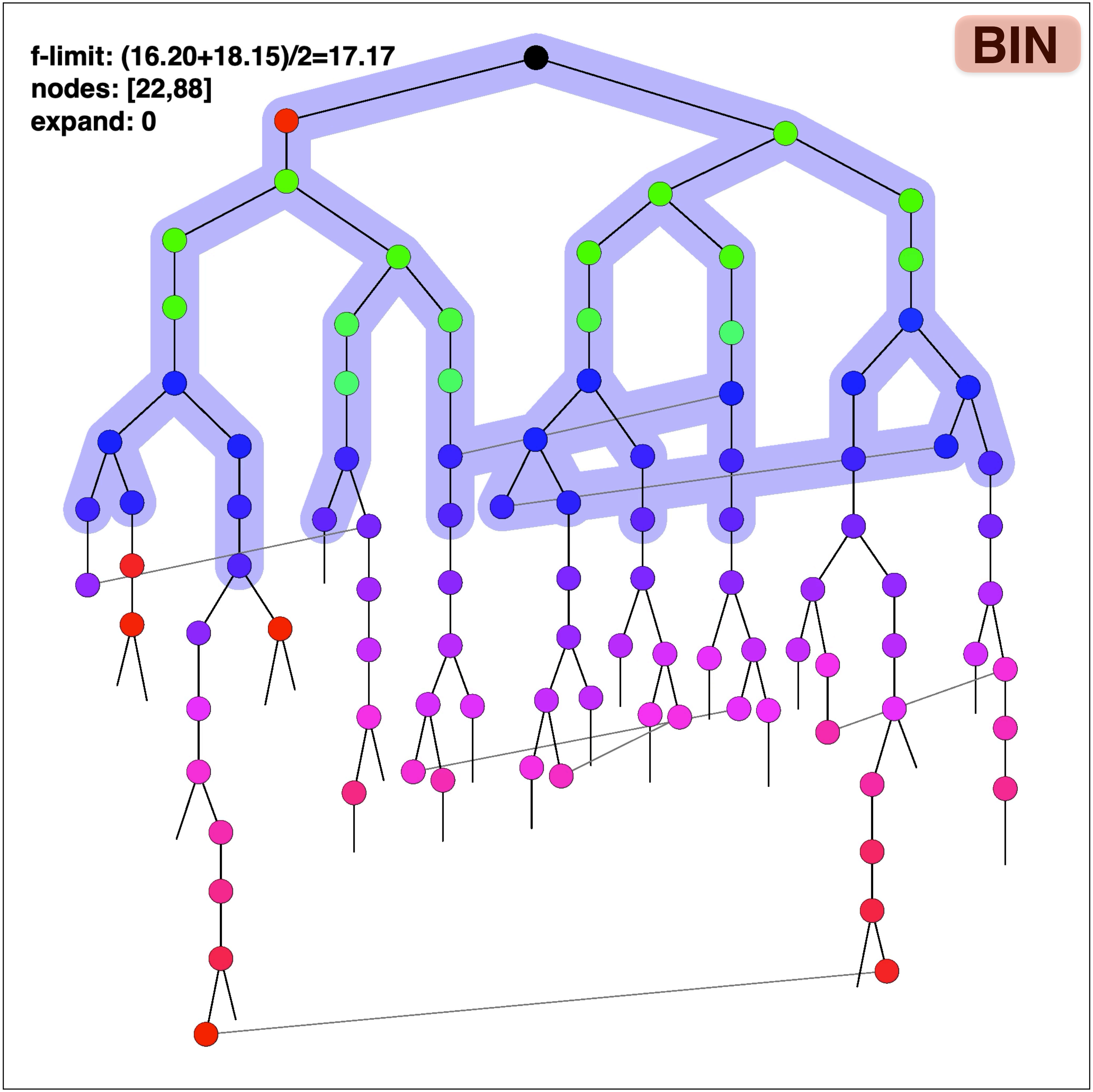


f-limit: 14.20+2^0=15.20
nodes: [22,88]
expand: 0
EXP


EXP
f-limit: 14.20+2^0=15.20
nodes: [22,88]
expand: 0


f-limit: 16.20+2^1=18.20
nodes: [22,88]
expand: 0
EXP

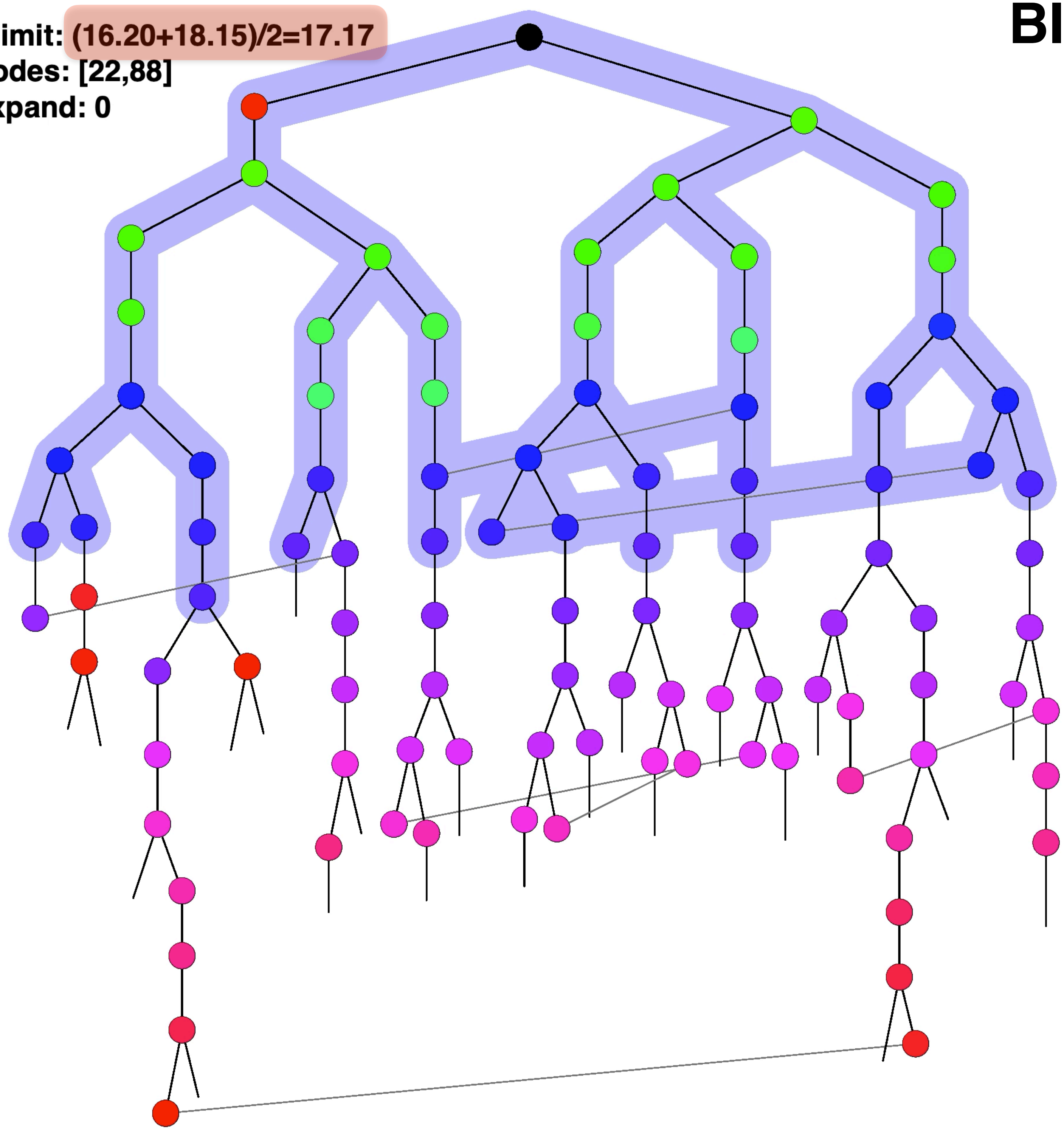

f-limit: 16.20+2^1=18.20
nodes: [22,88]
expand: 0
EXP

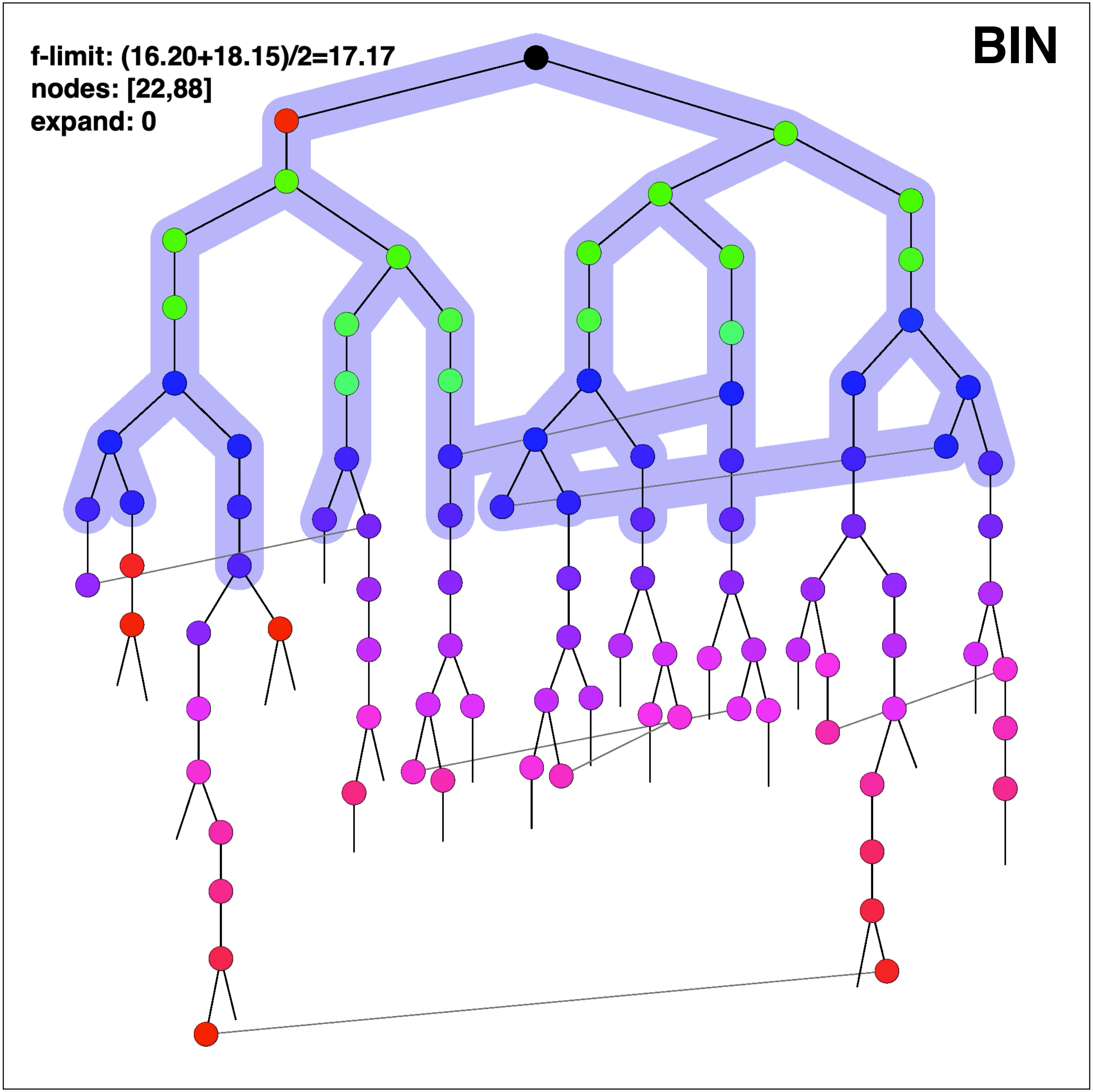


f-limit: (16.20+18.15)/2=17.17
nodes: [22,88]
expand: 0
BIN

f-limit: (16.20+18.15)/2=17.17
nodes: [22,88]
expand: 0

BIN





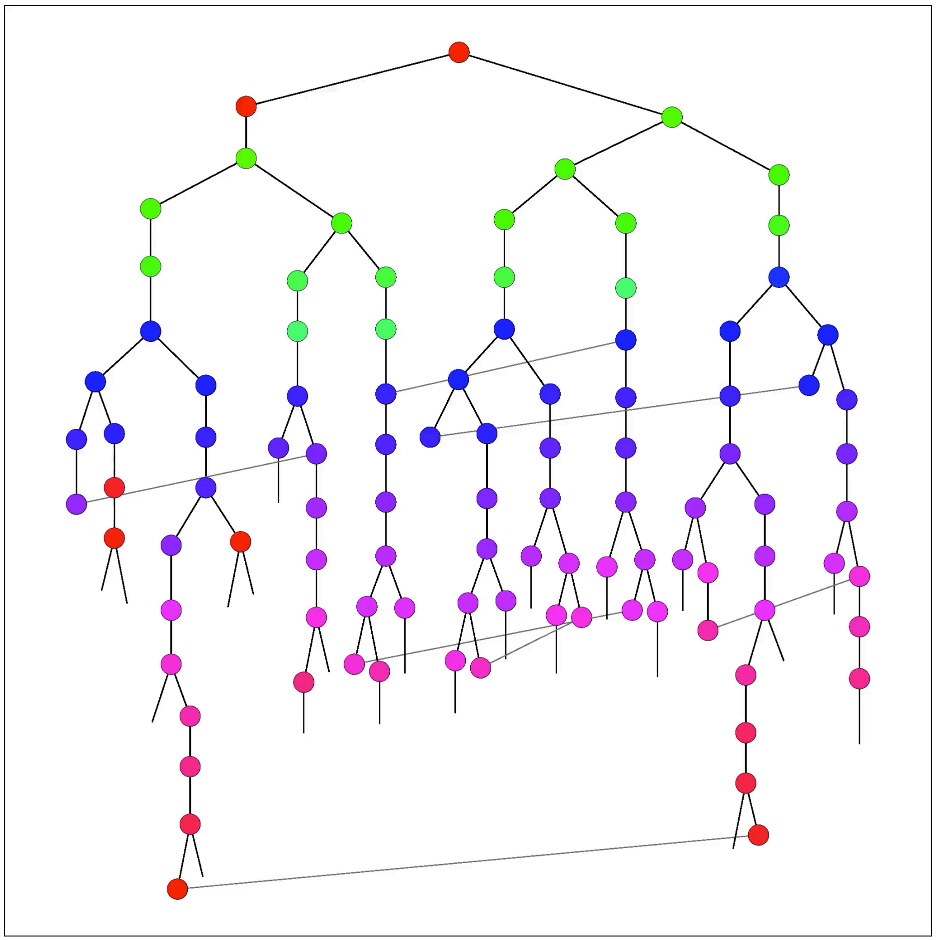
f-limit: (16.20+18.15)/2=17.17
nodes: [22,88]
expand: 0
BIN

How does BTS work?

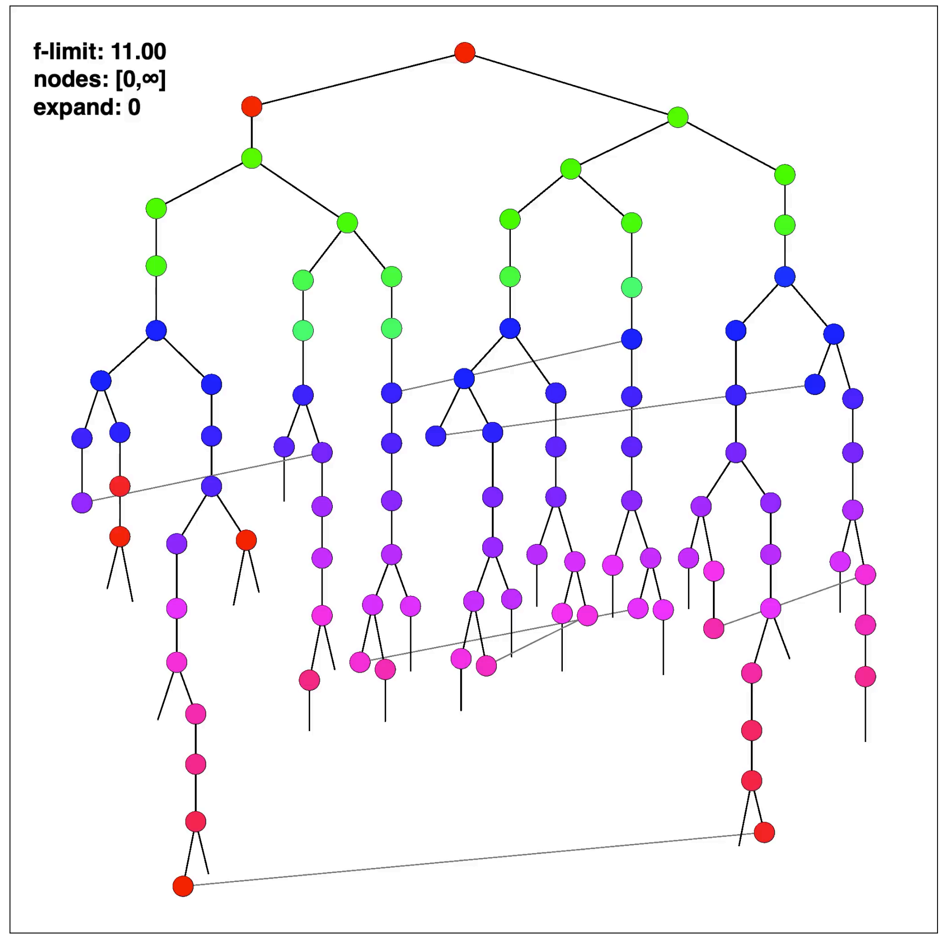
IDA*

With budget $\left\{ \begin{array}{l} \text{Exponential Search} \\ \text{Binary Search} \end{array} \right.$

IDA*

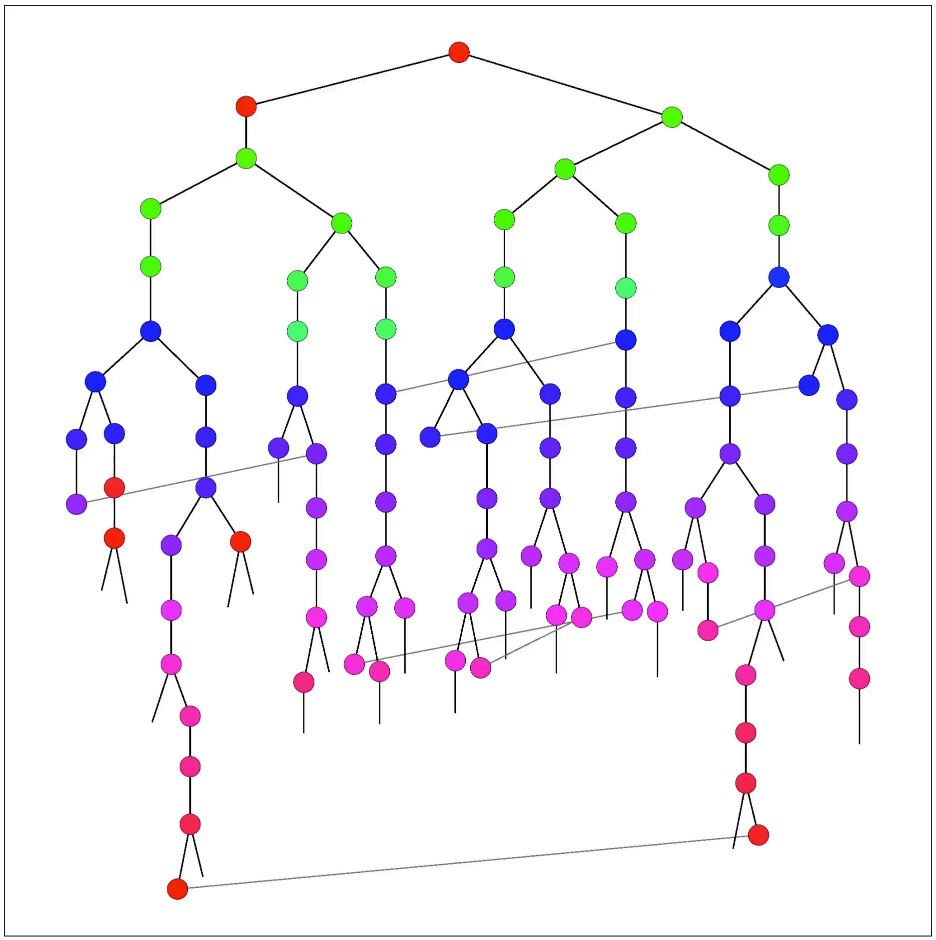


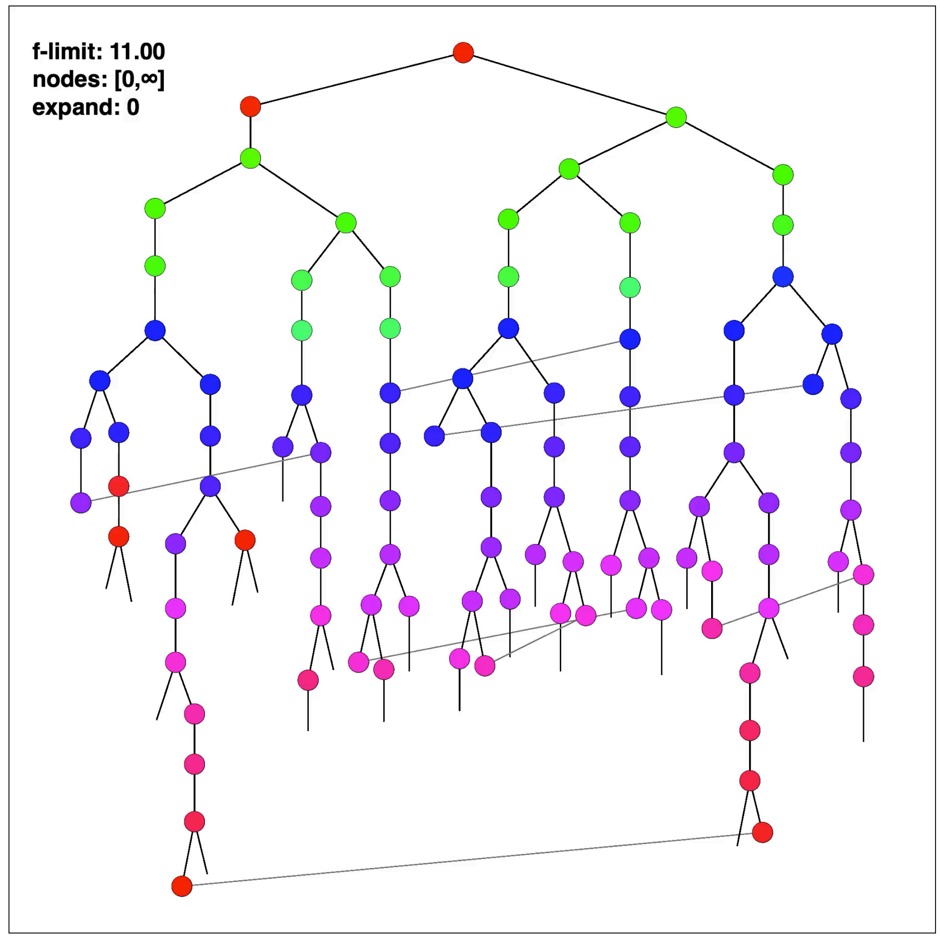
BTS*



IDA*

BTS*





Conclusions

- BTS reduces worst case of IDA*
  - Same performance as IDA* if tree grows exponentially
- IBEX solves similar problems in different contexts

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- Demos & videos online:
  - https://www.movingai.com/SAS/
  - https://www.movingai.com/SAS/BTS/