Last-Branch and Speculative Pruning Algorithms for Max^n

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UCLA Computer Science Department*

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Problem Overview

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- We'd like to develop programs to play multi-player games well

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- Implemented with alpha-beta pruning
  - In best case, alpha-beta reduces tree size from b^d to $b^{d/2}$
  - Approach best case in practice by ordering nodes well

Max^n

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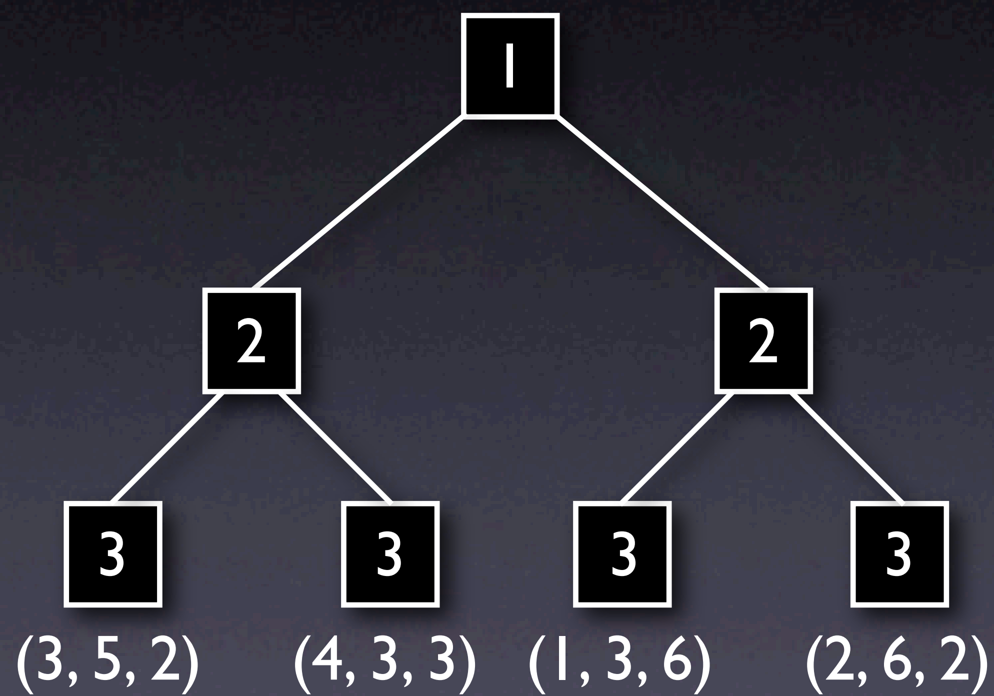
Max^n

- Generalization of minimax to *n* players
  - Luckhardt and Irani, 1986
- Static evaluation returns *n*-tuple of scores
  - Each player tries to maximize their own component of the *n*-tuple

Max^n Decision Rule



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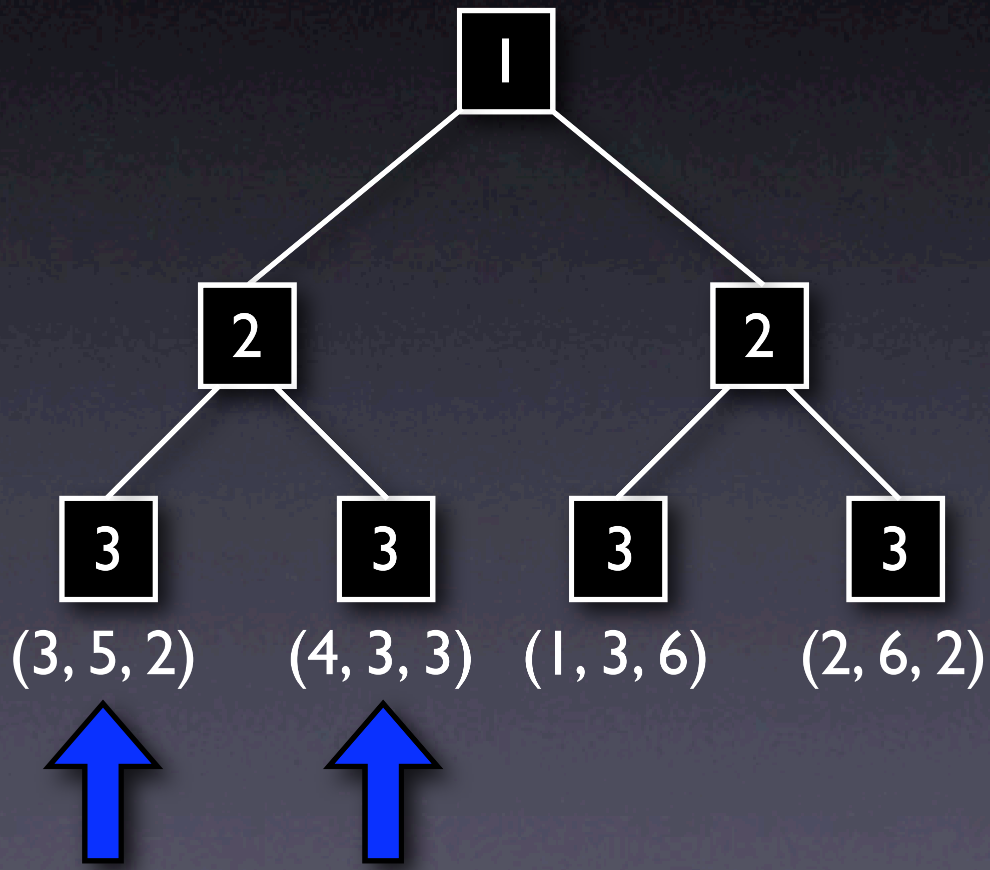

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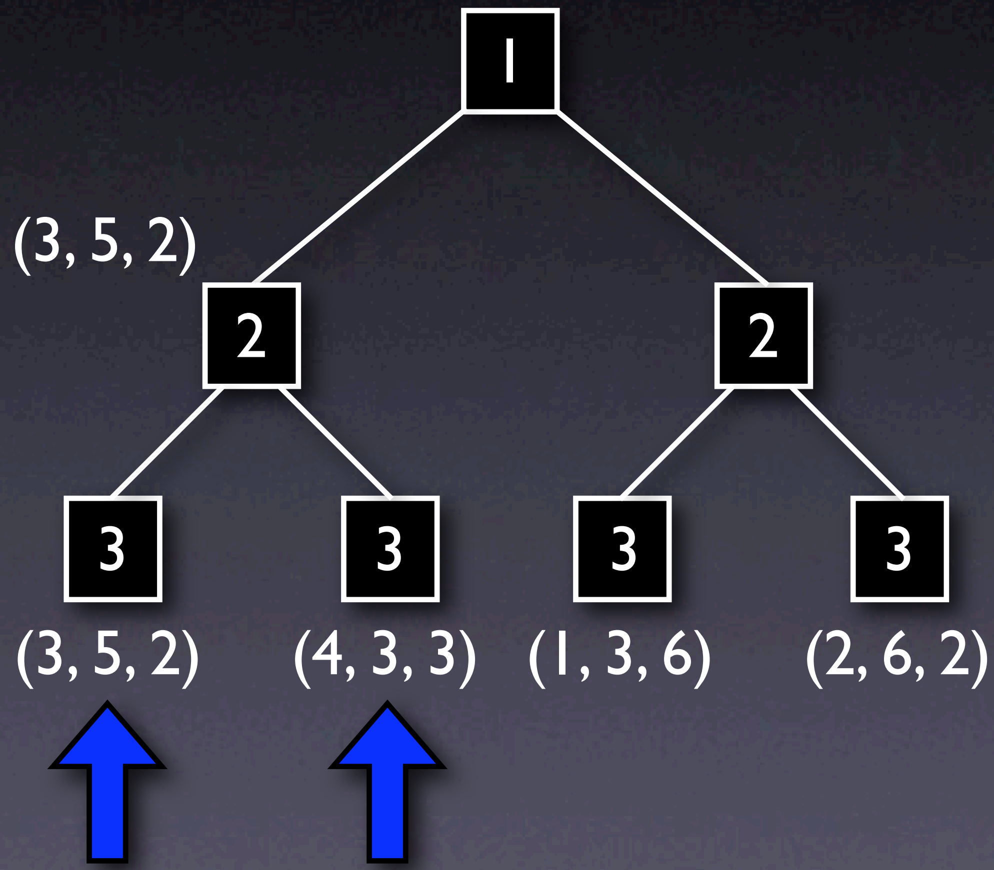

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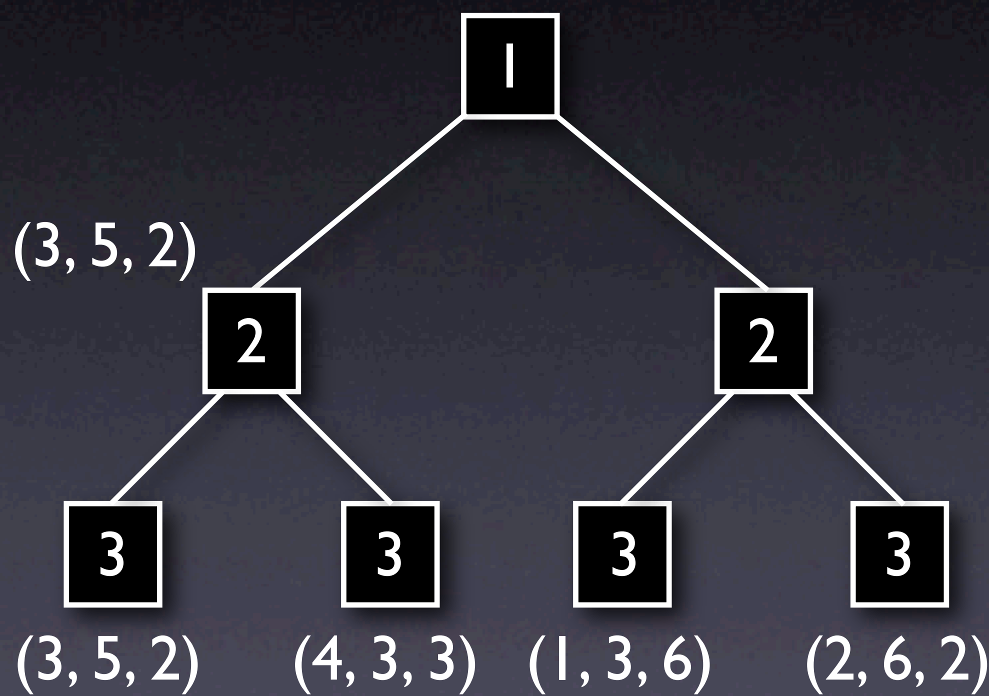

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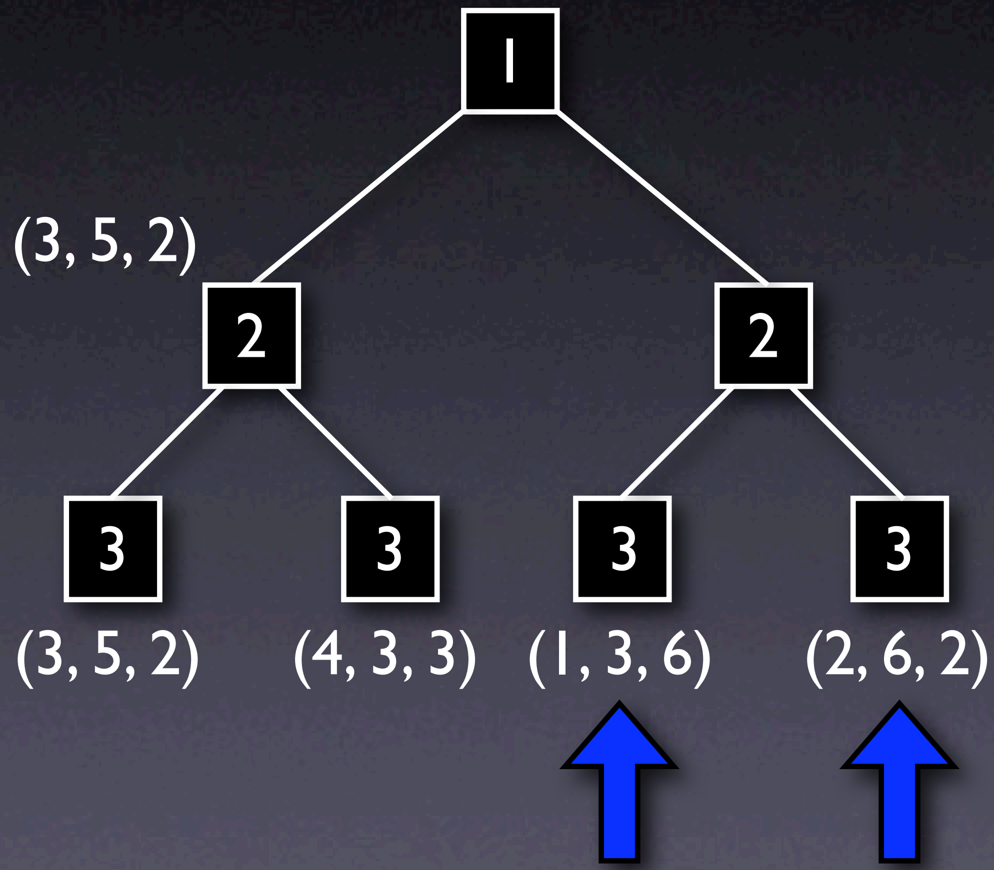

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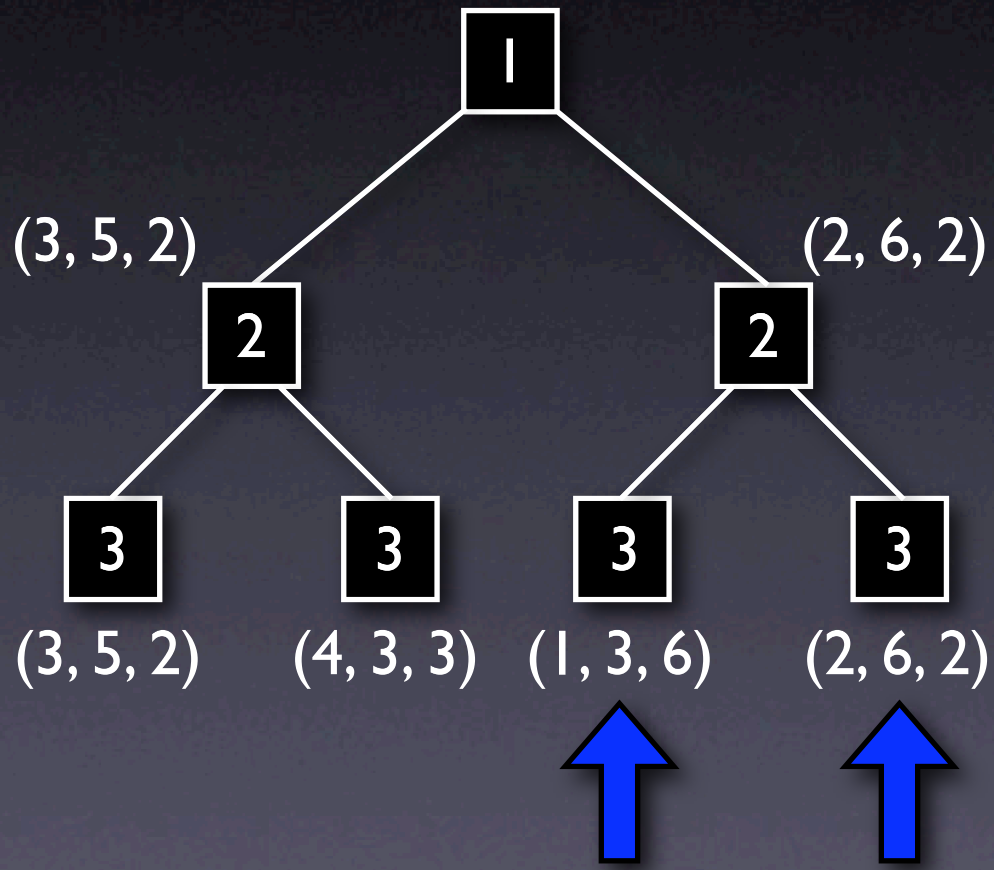

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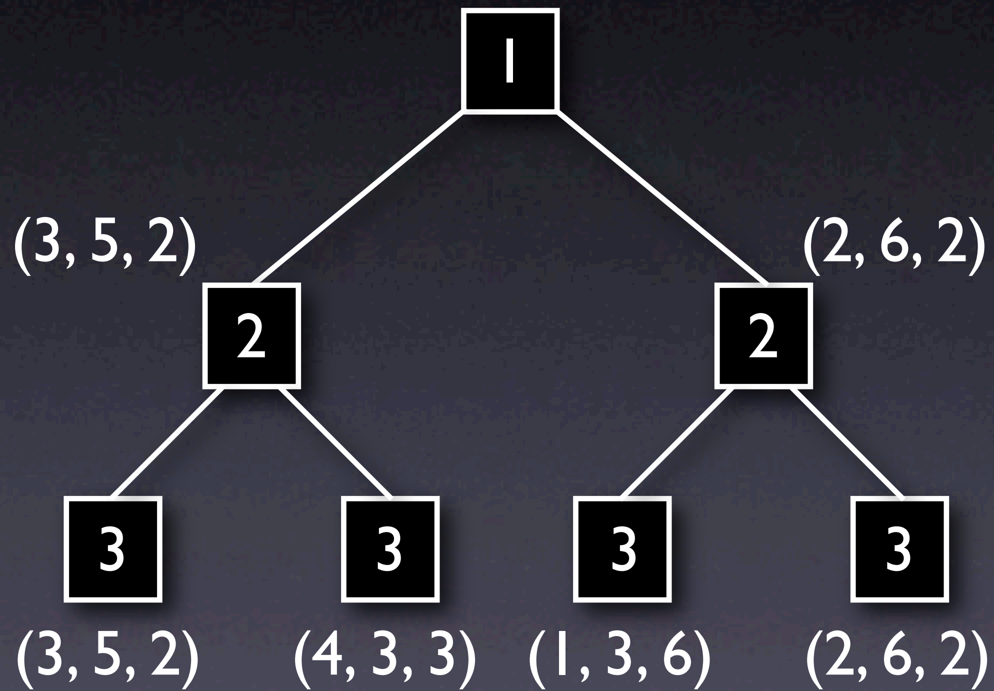

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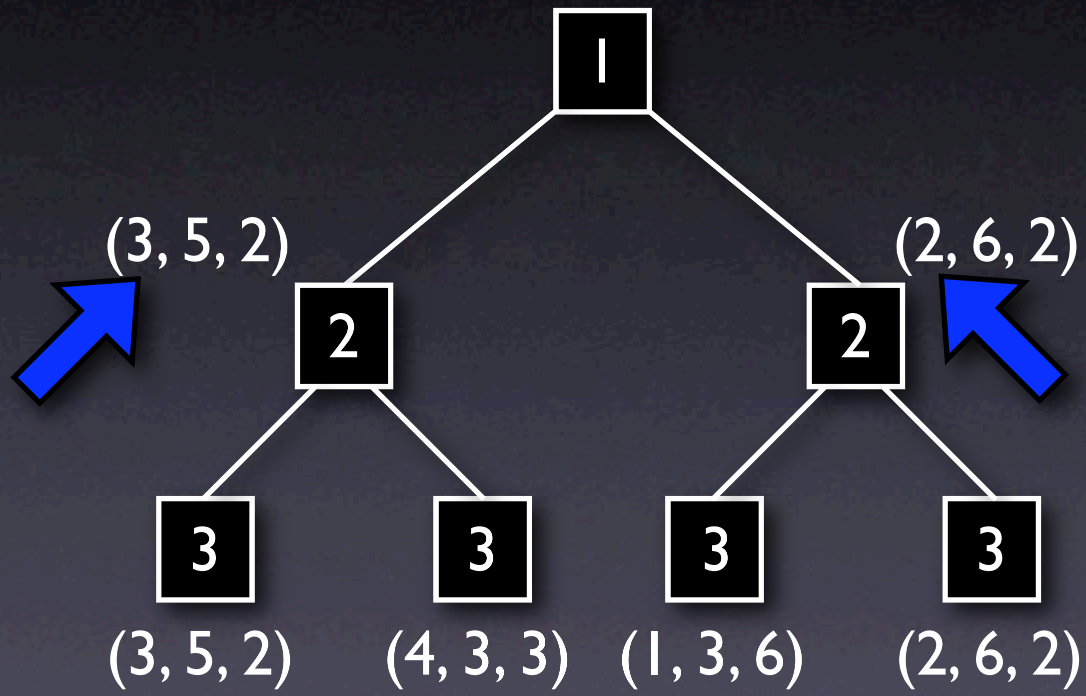

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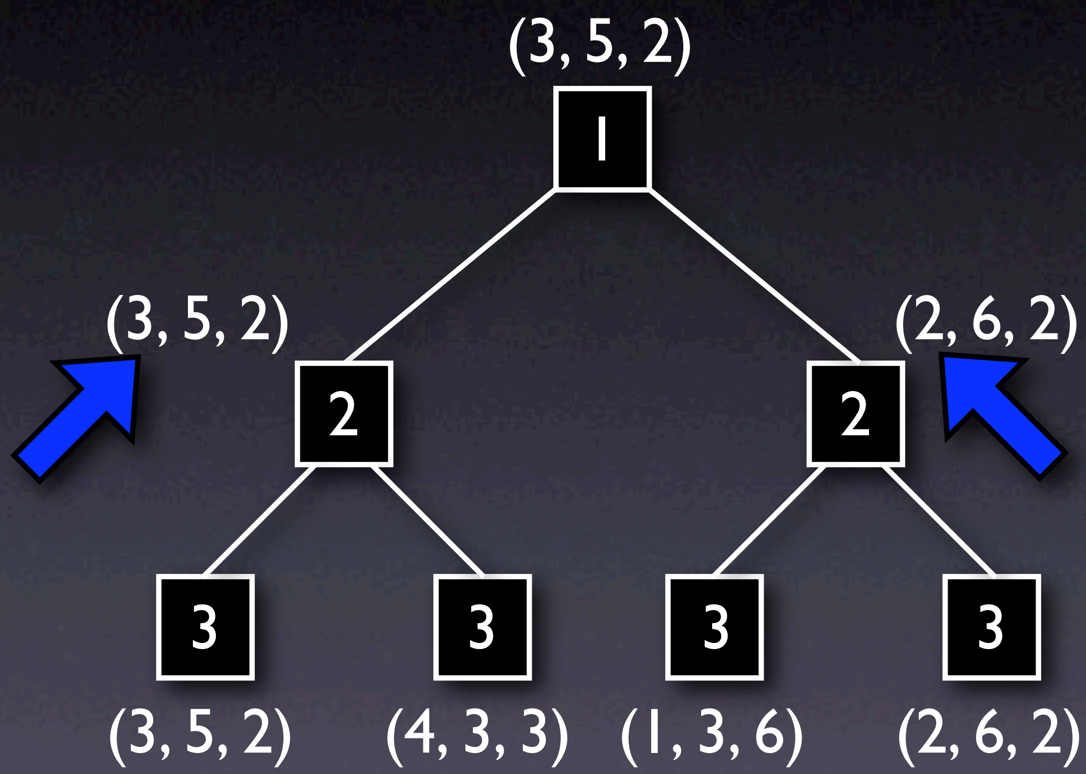

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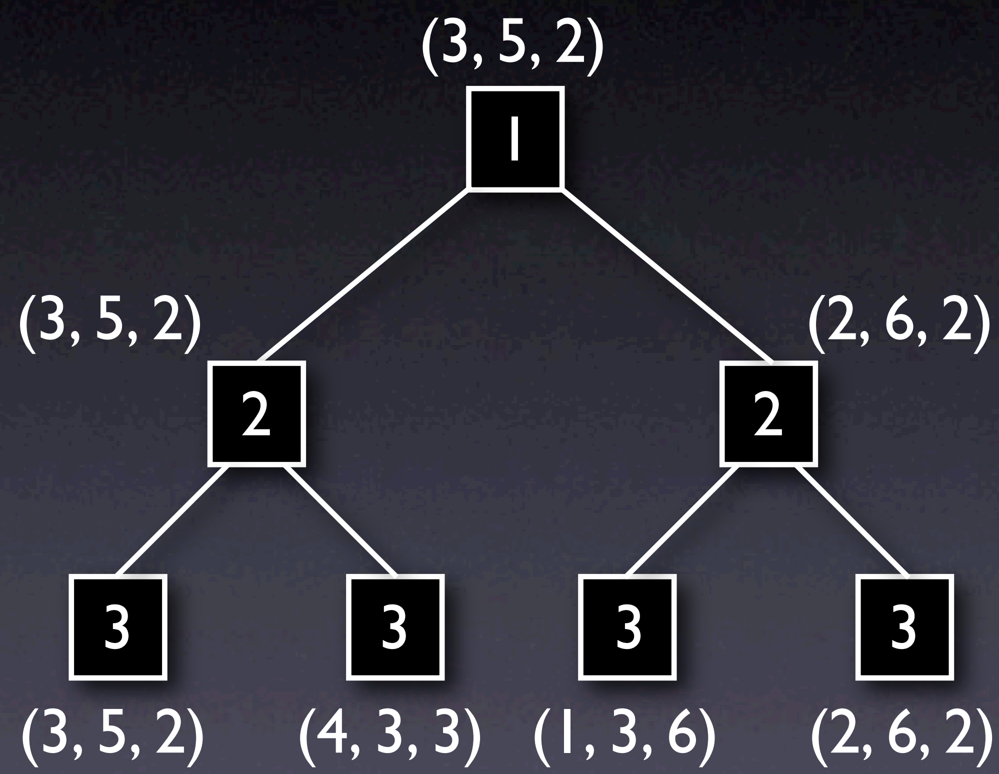

Outline

- Max^n Decision Rule
- Max^n Pruning Techniques
- Experimental Results
- Conclusions

Previous Max^n Pruning

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- Shallow Pruning (Korf, 1991)

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Previous Max^n Pruning

- Shallow Pruning (Korf, 1991)
- Alpha-Beta Branch and Bound Pruning (Sturtevant and Korf, 2000)
- Are not always effective and/or applicable
  - Effectiveness depends on *both* node ordering and static evaluation function

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- Assume at least:

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- Assume at least:
  - Lower bound on each score (0)
  - Upper bound on sum of all scores (*maxsum*)
- If the sum of scores always equals *maxsum*, the game is *constant-sum*

Shallow Pruning

maxsum = 10

1

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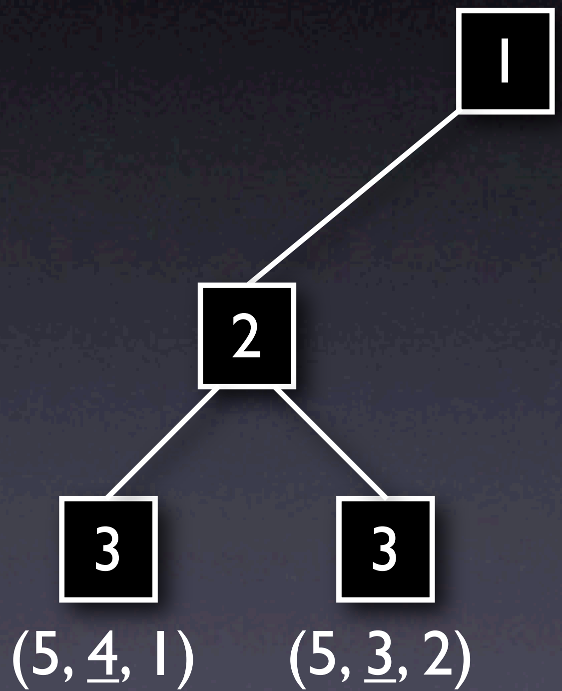

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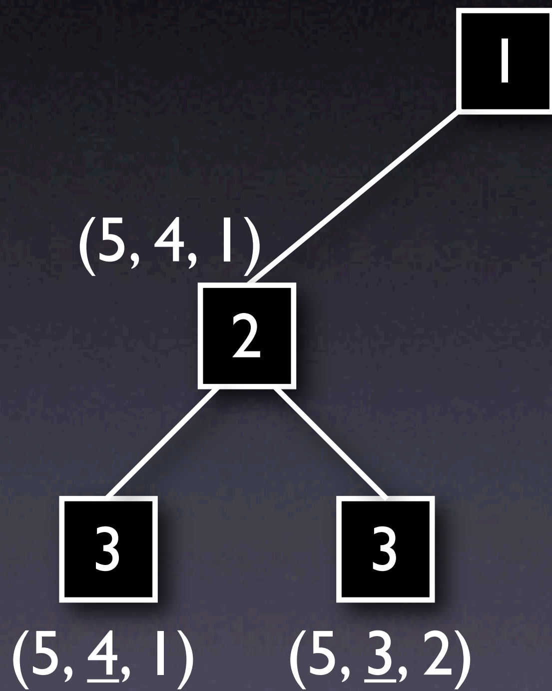

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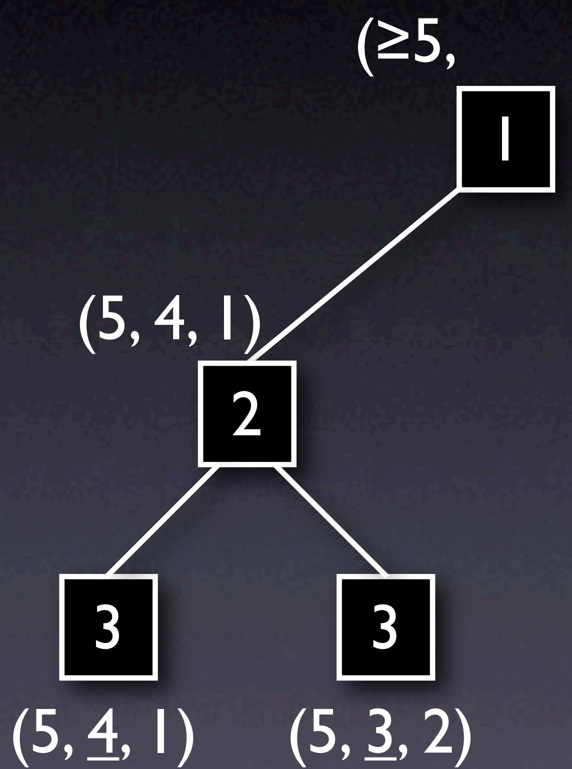

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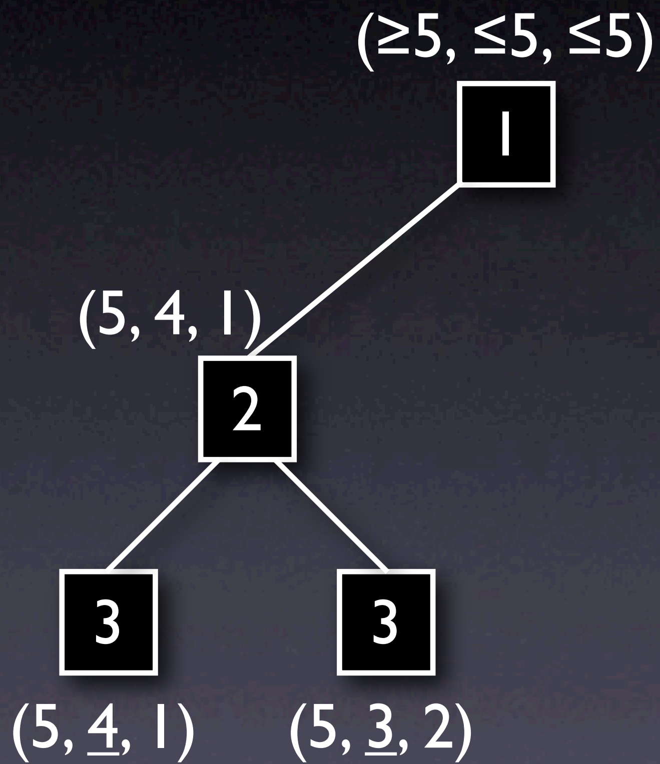

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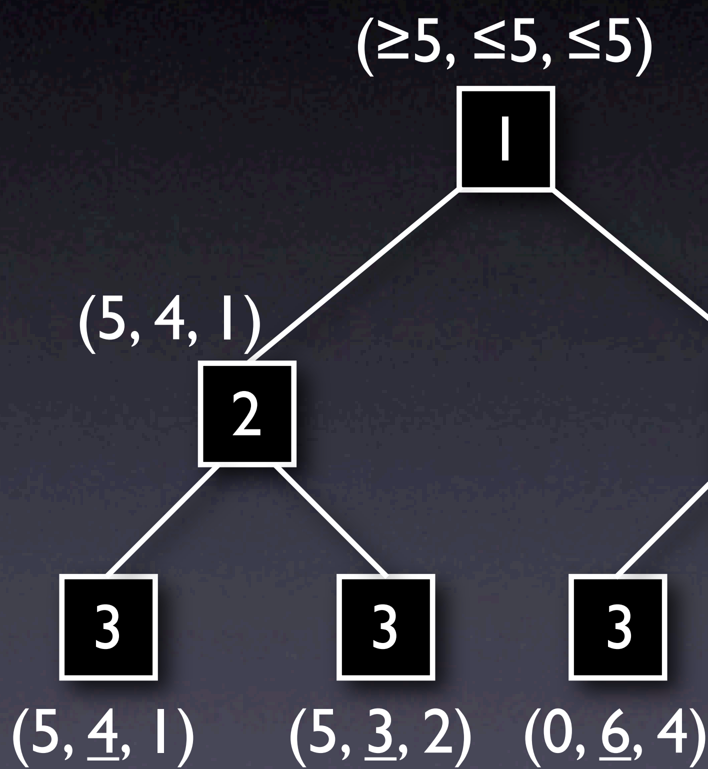

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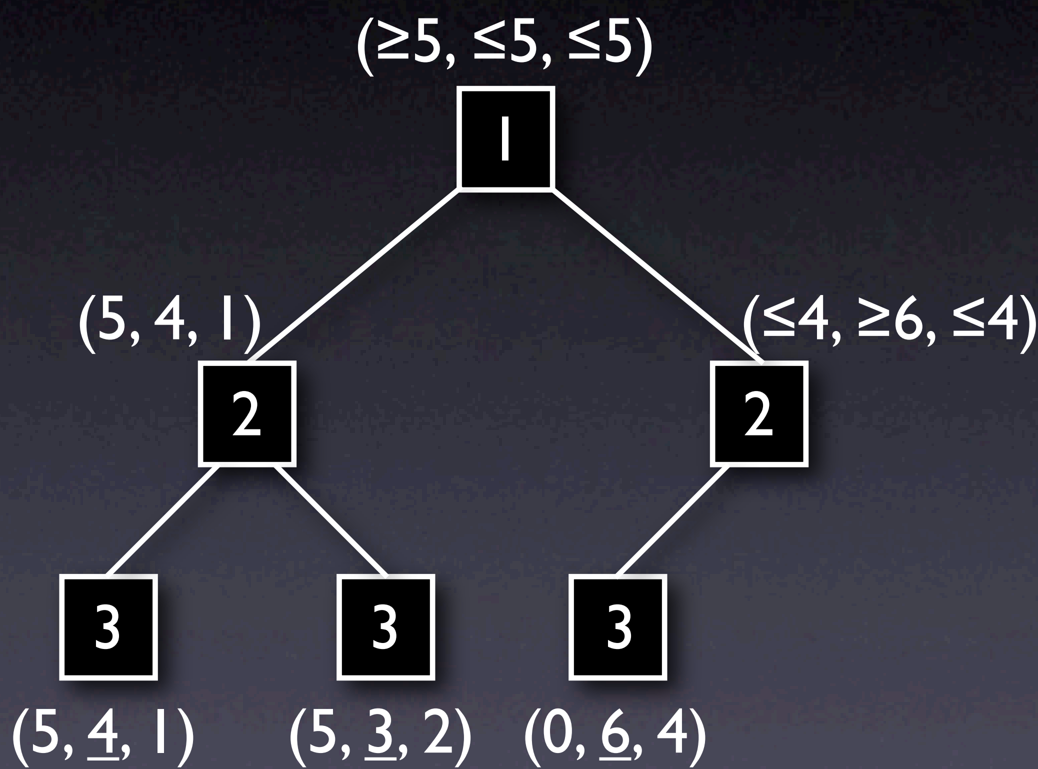

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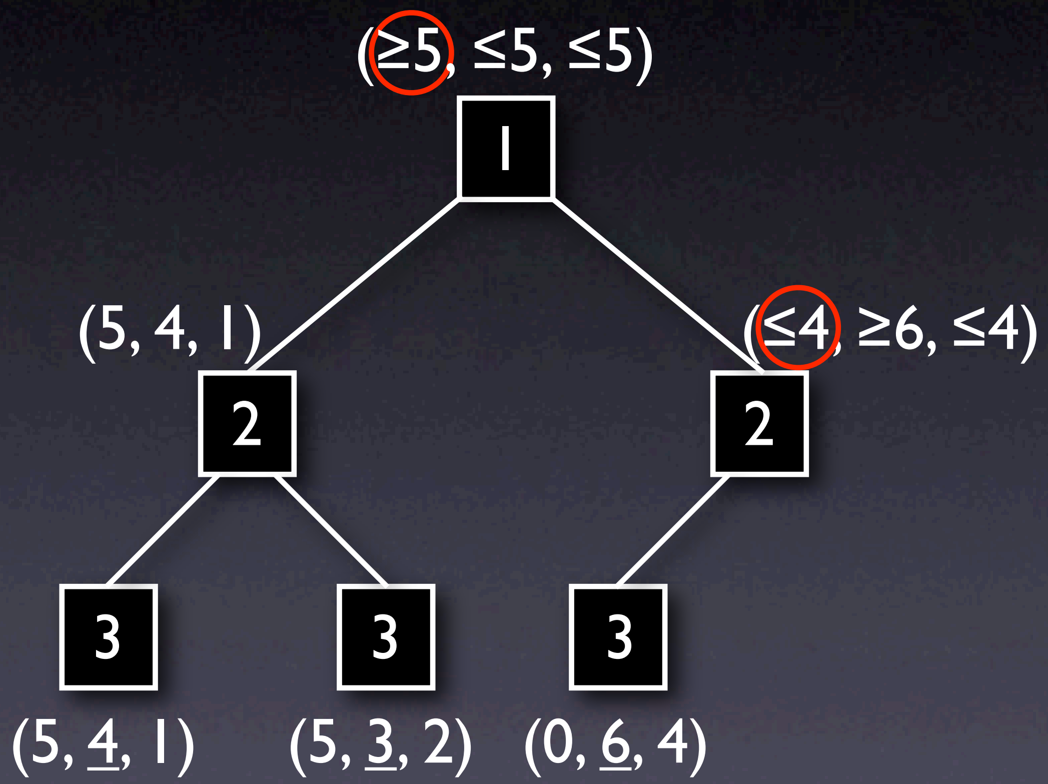

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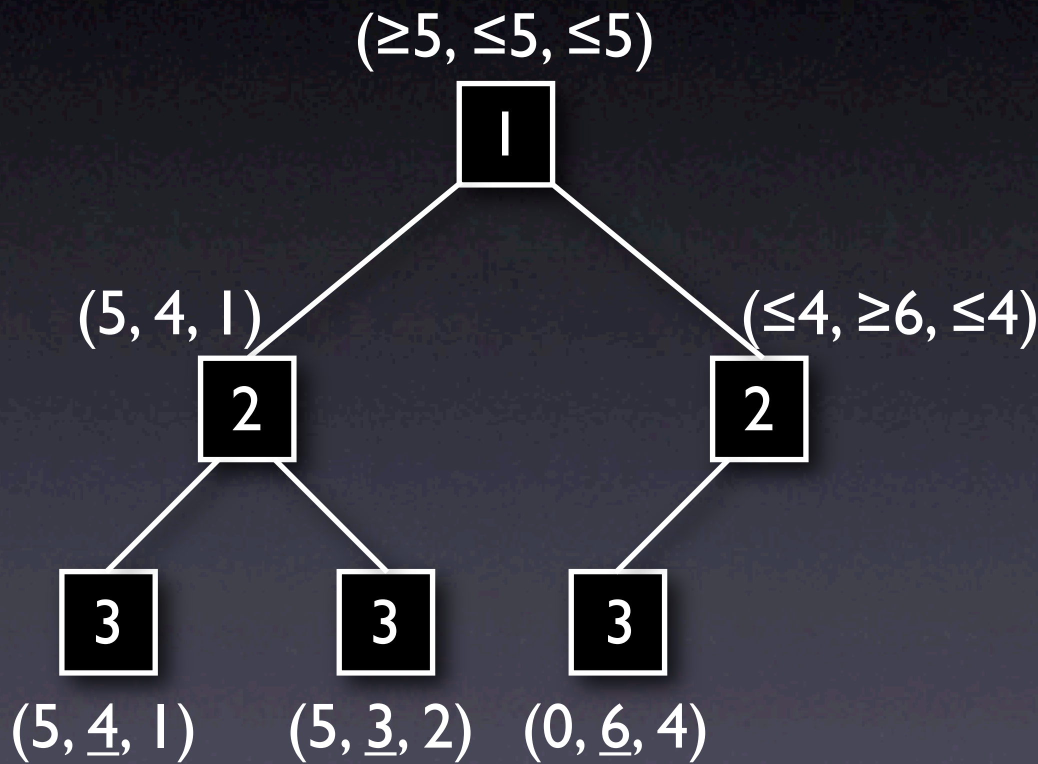

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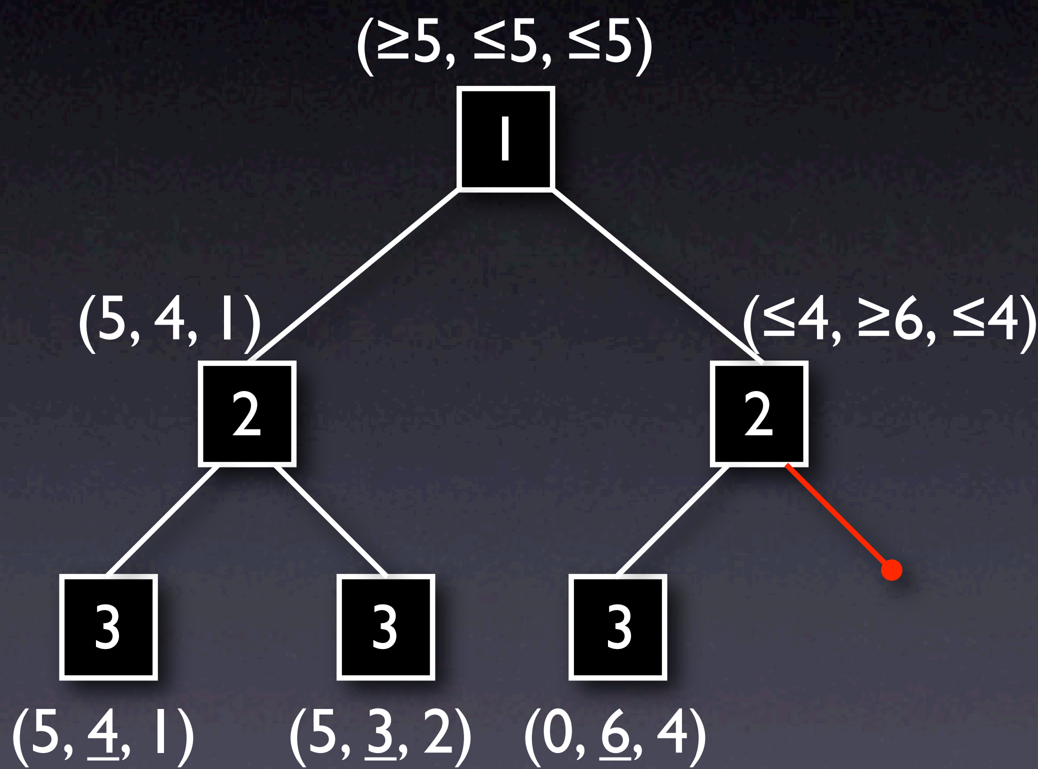

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- Average case model predicts no asymptotic gain (Korf, 1991)
- Dependent on properties of evaluation function
  - For some games, natural evaluation functions allow no pruning

Deep Pruning

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- Can we prune deeper in the tree based on the same bound?

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  - Not directly valid in multi-player games

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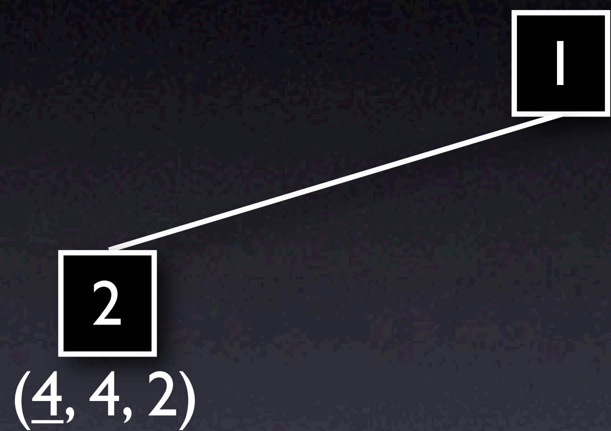

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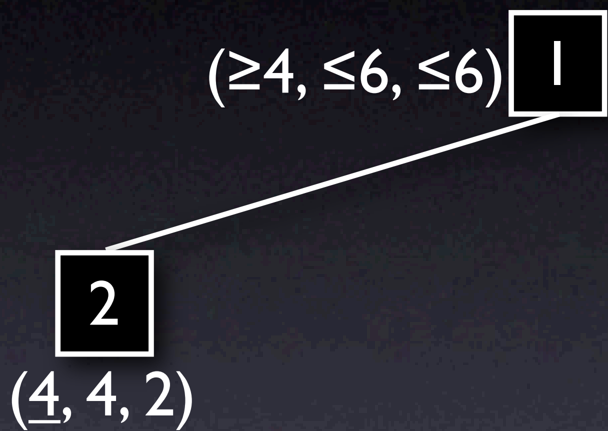

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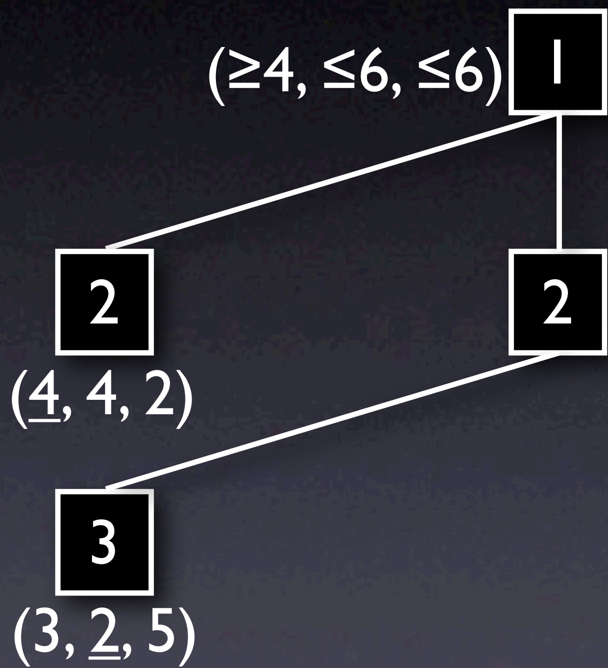

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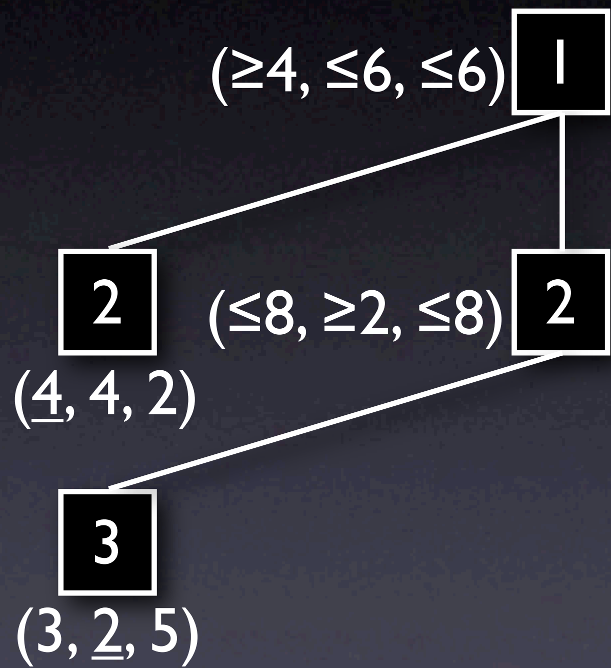

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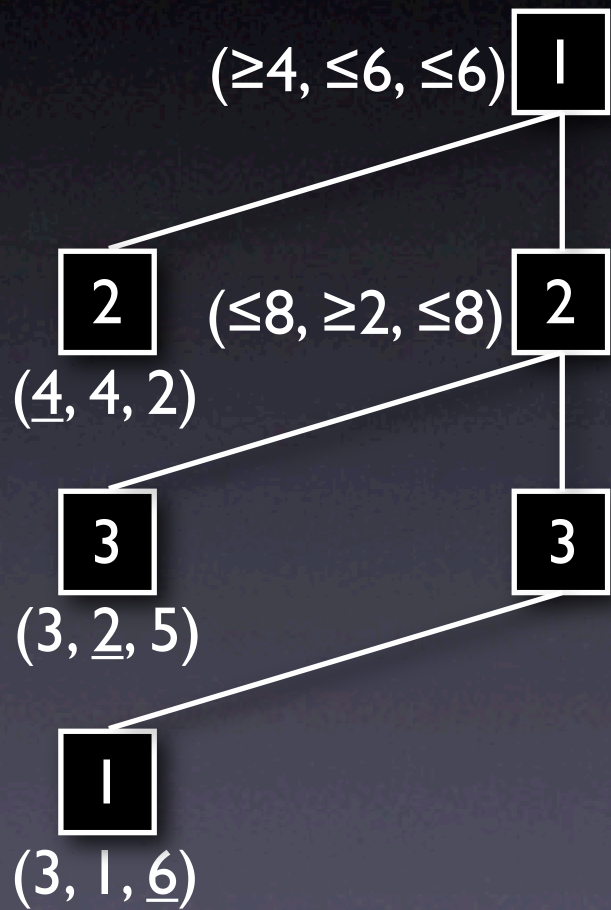

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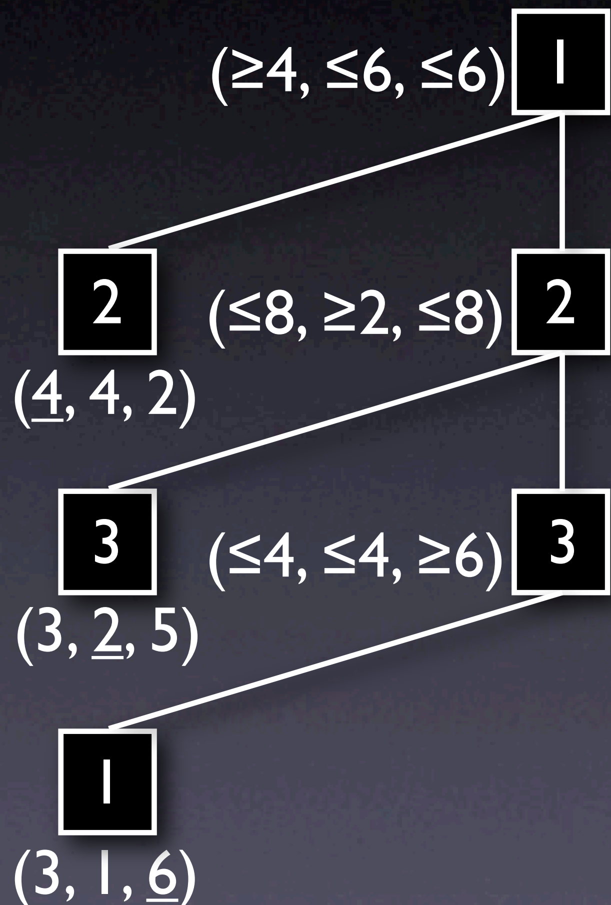

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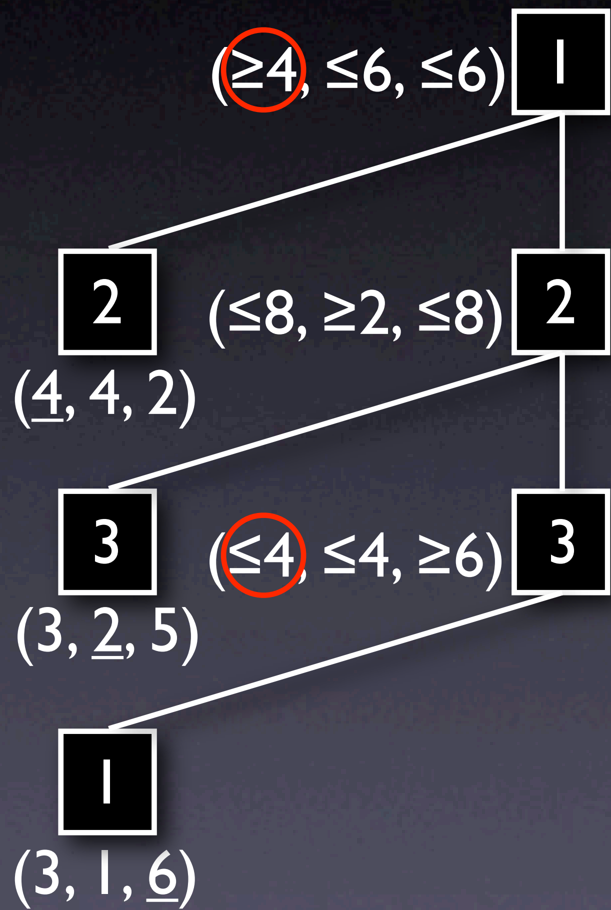

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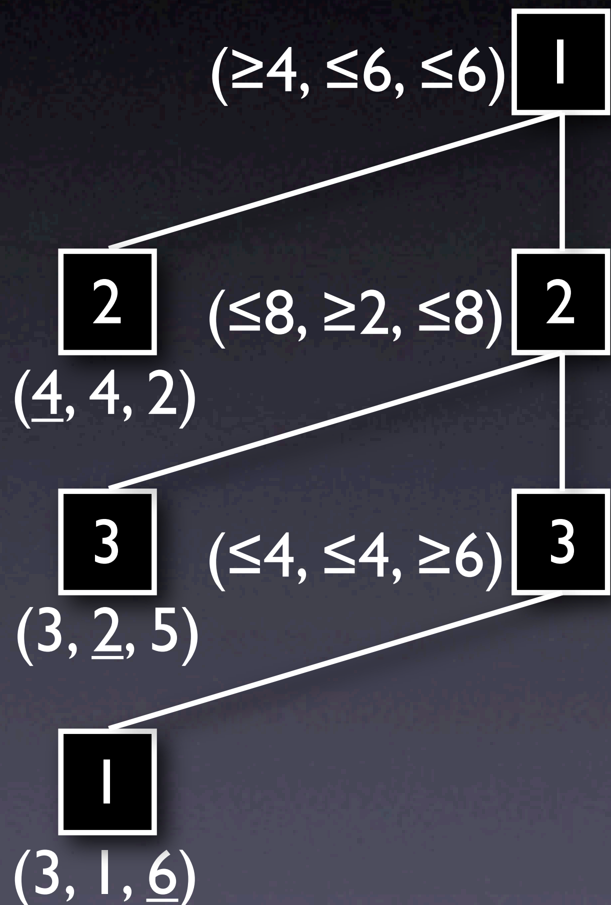

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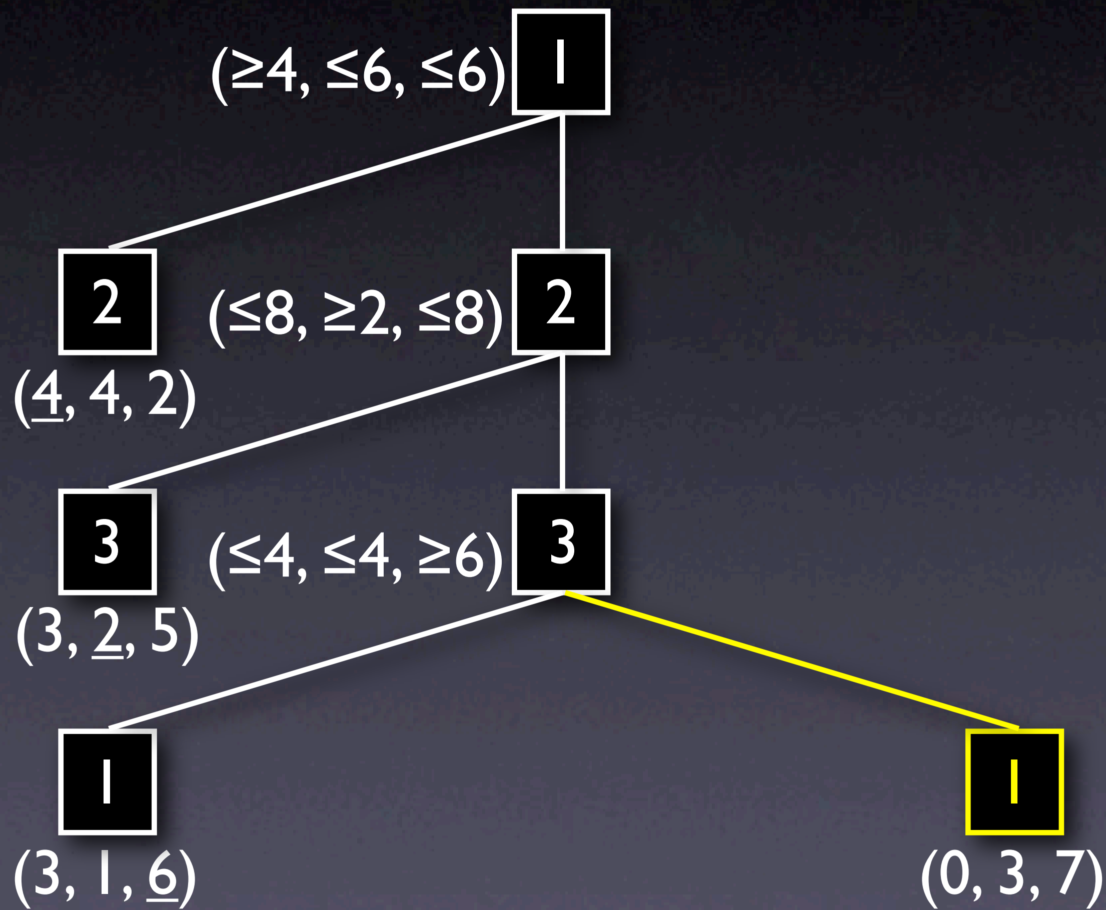

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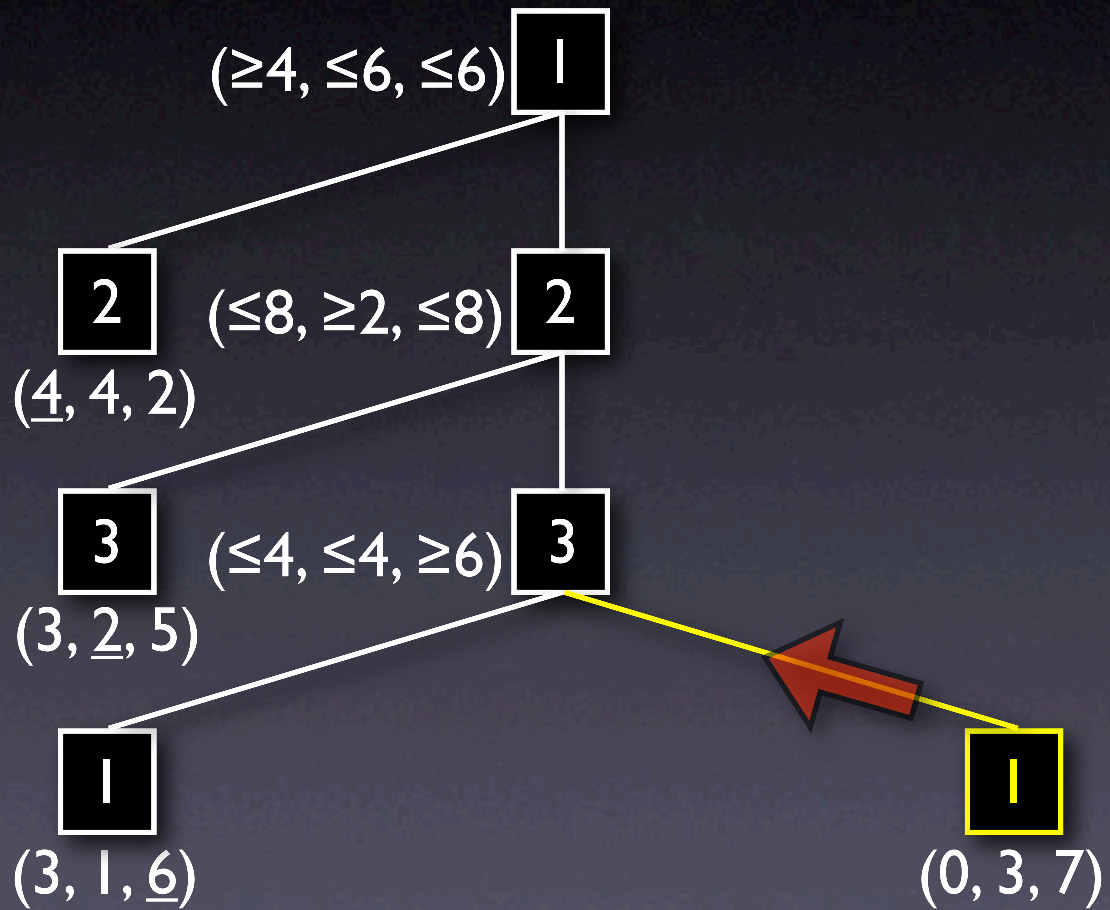

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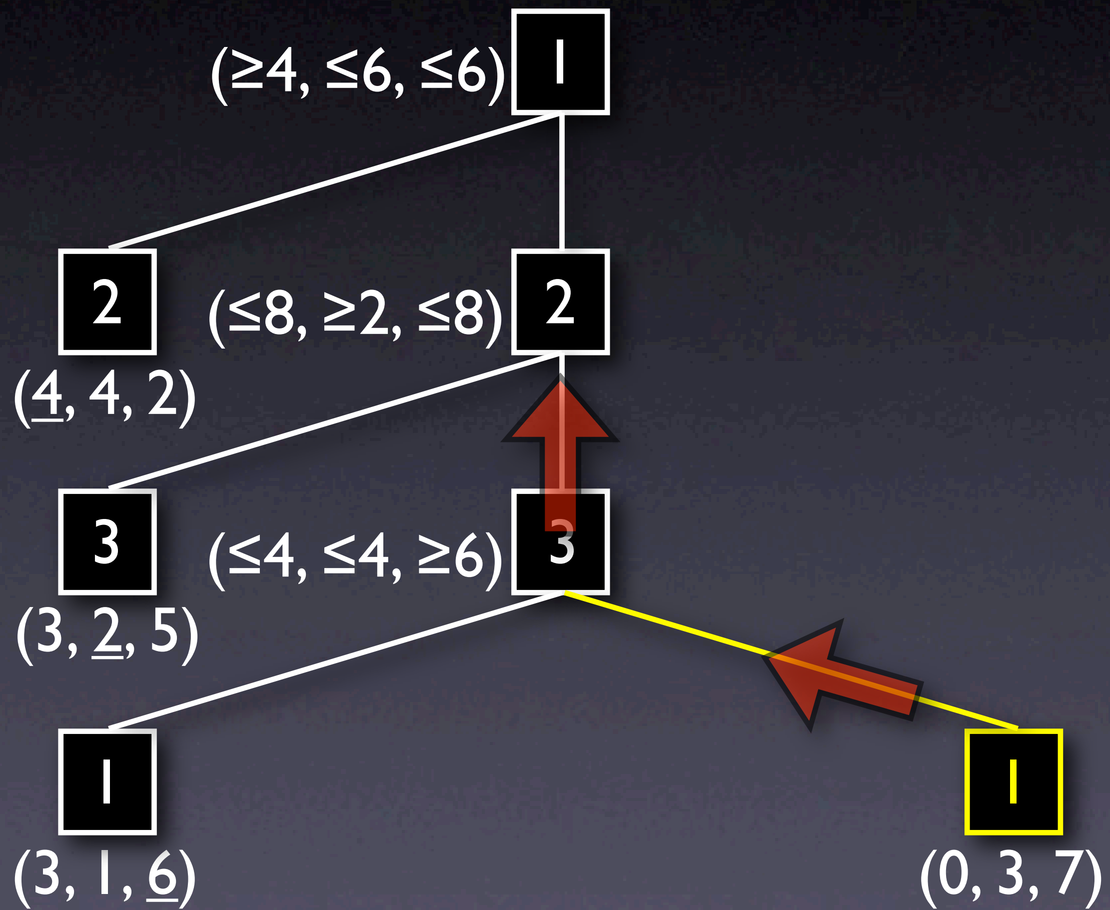

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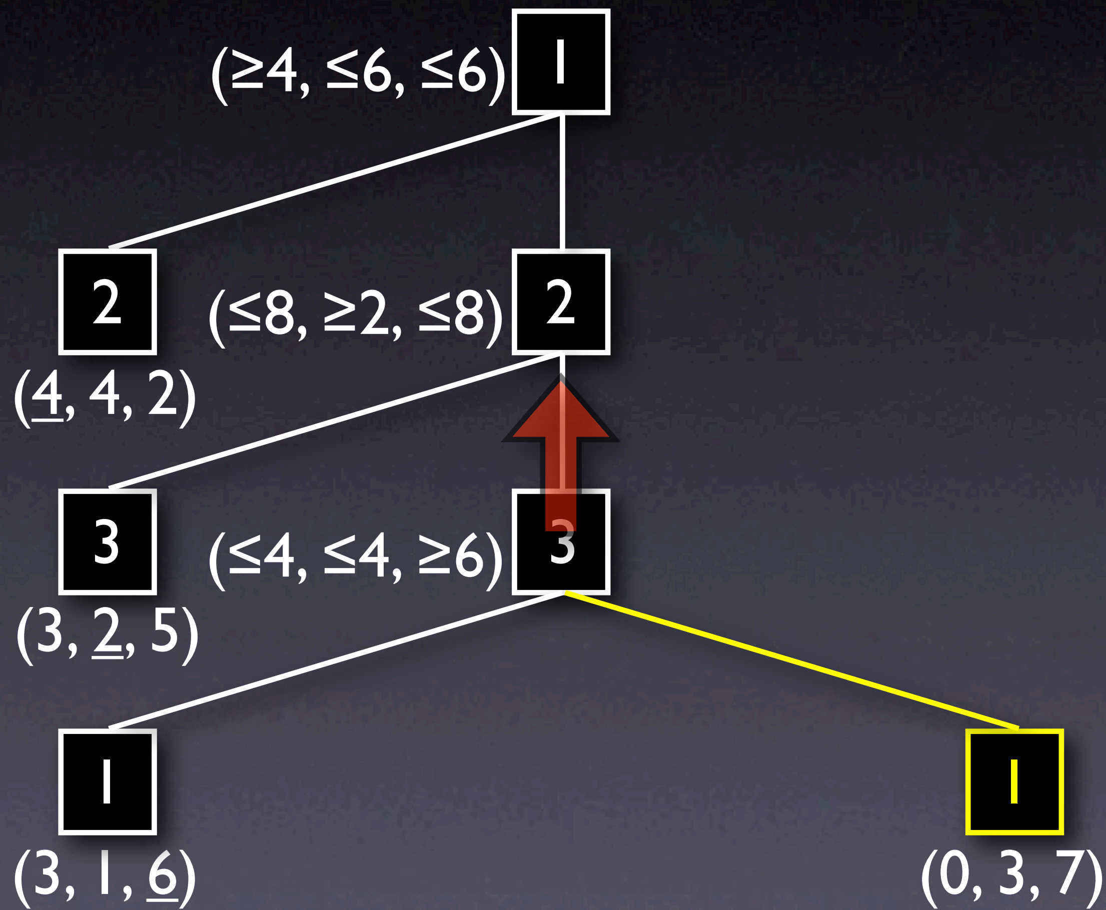

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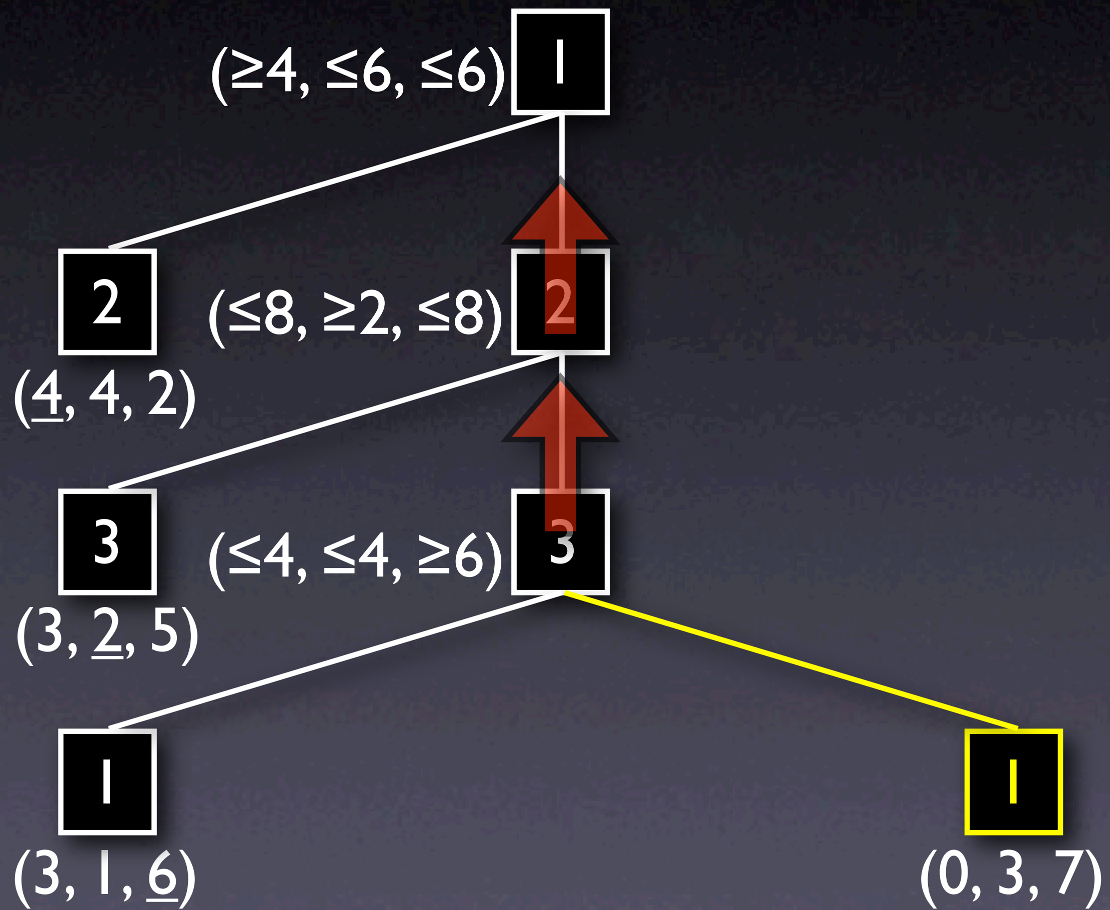

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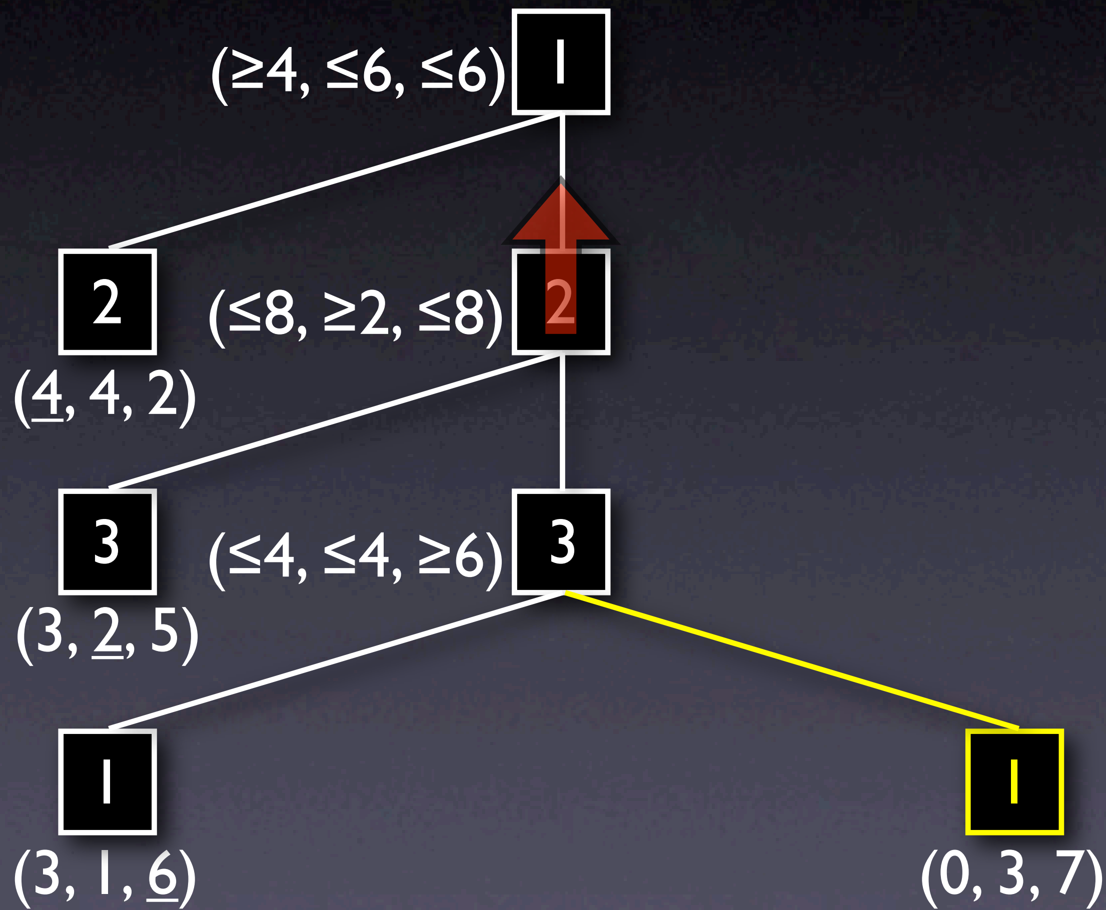

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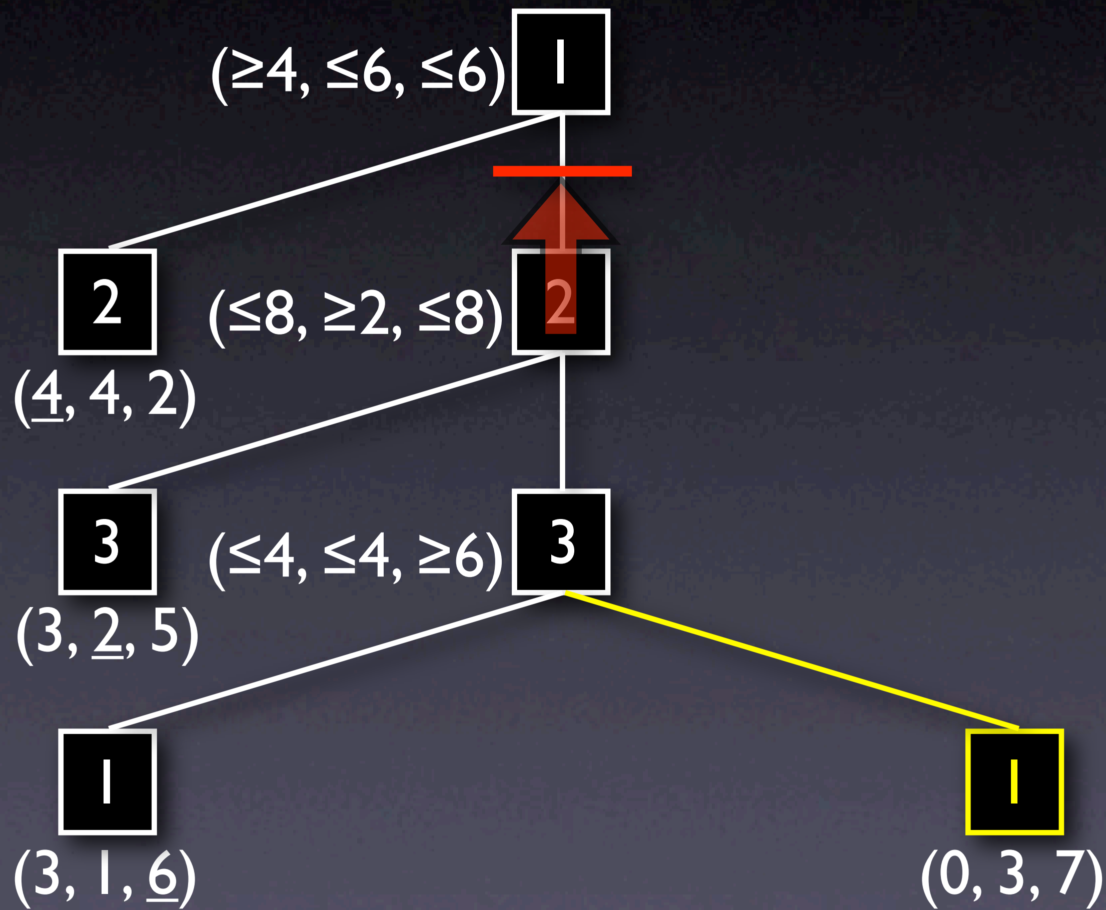

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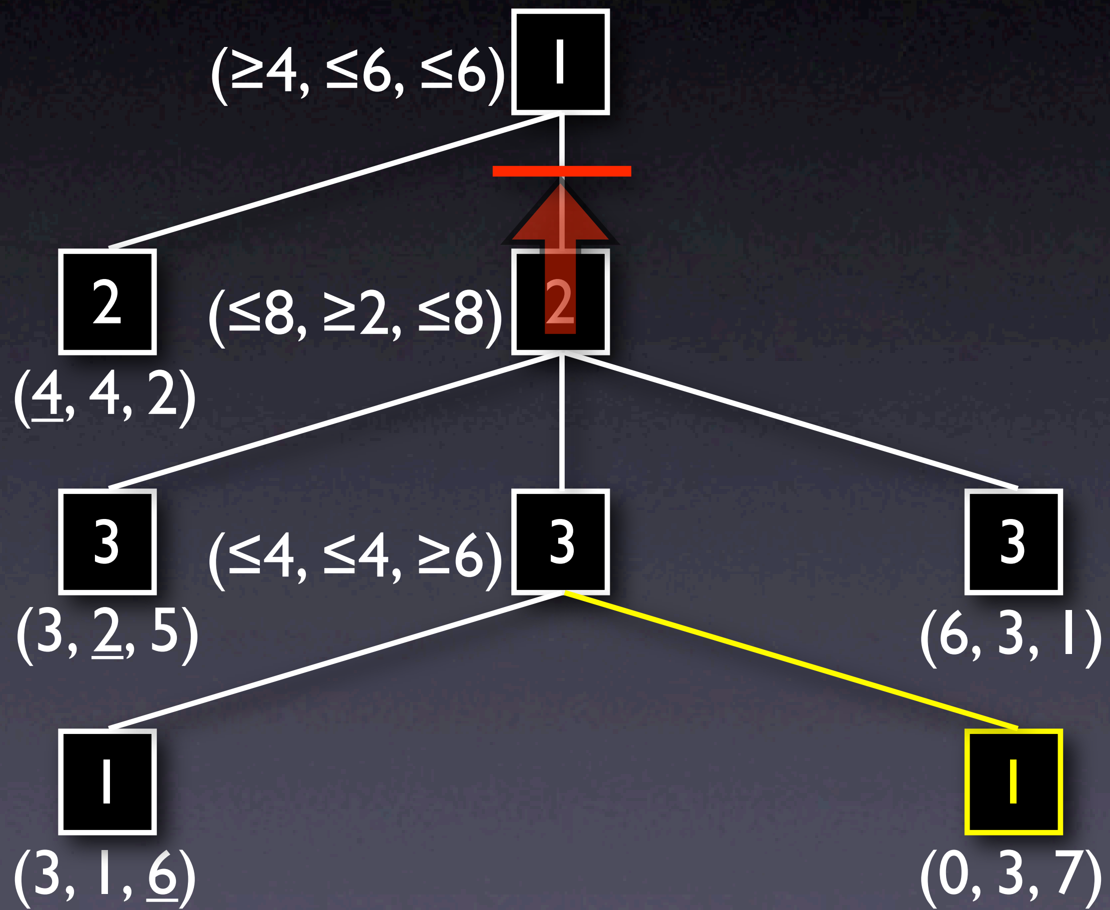

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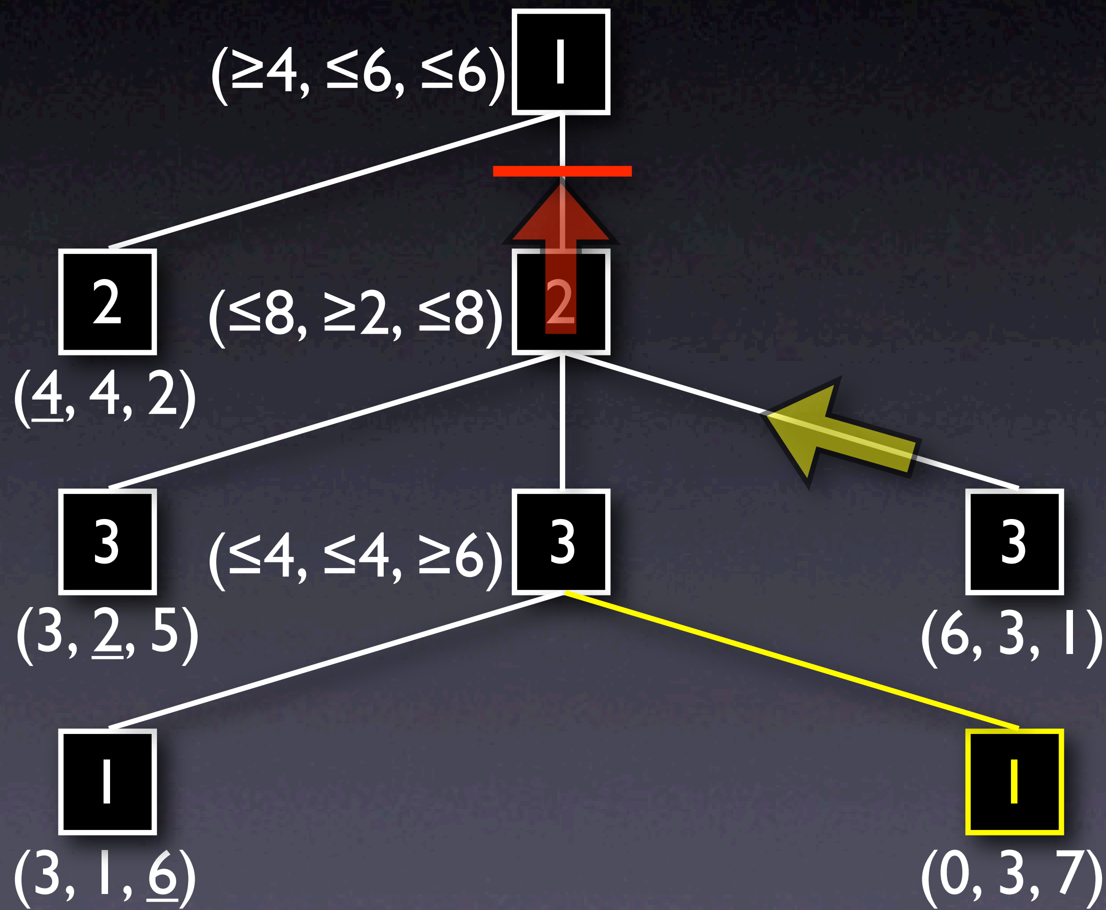

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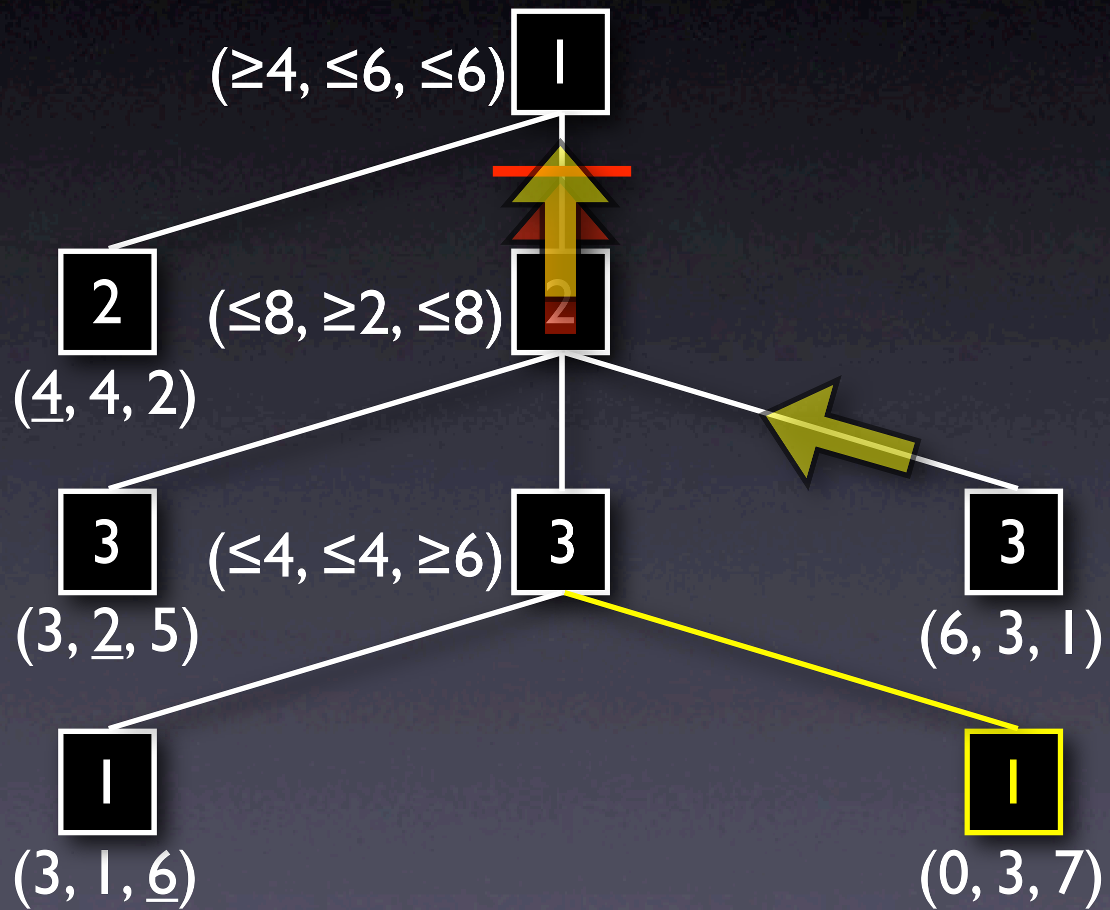

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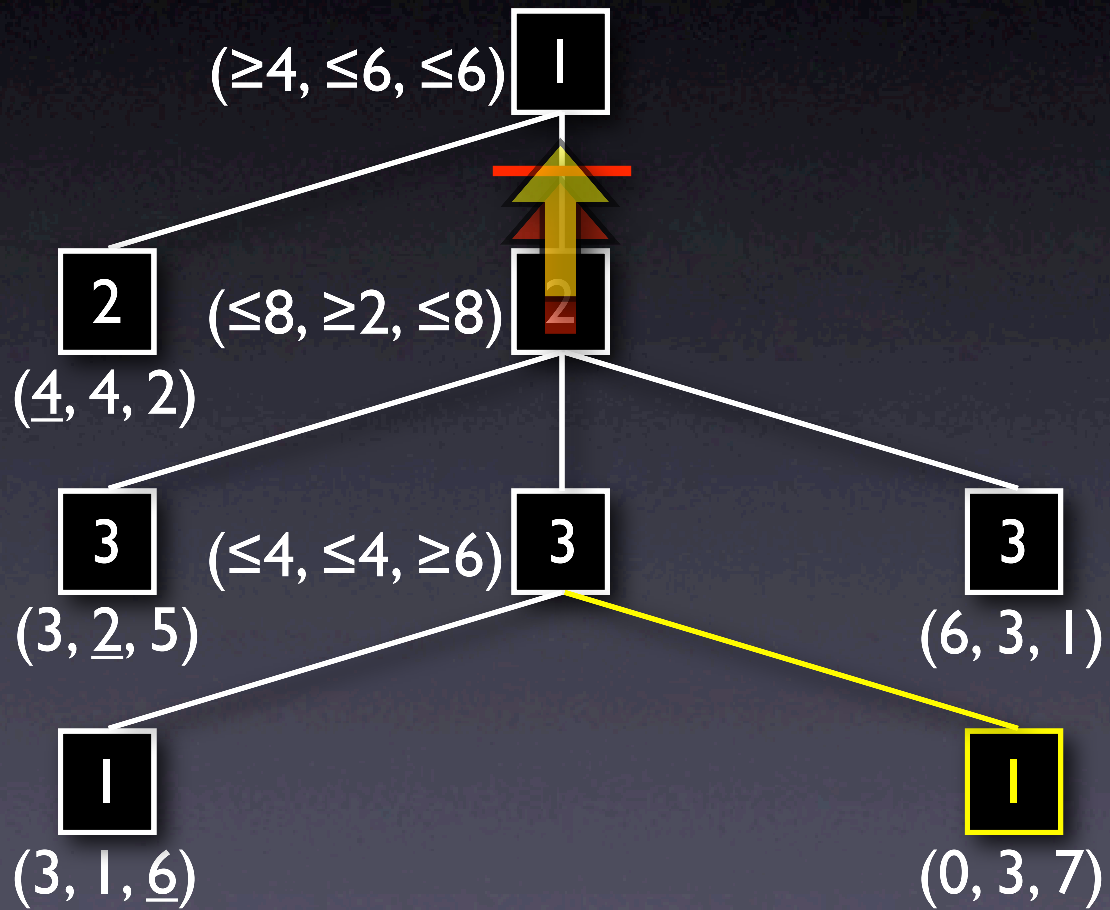

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Deep Pruning

- The max^n value of the last child cannot be the max^n value of the tree
- It can still affect the max^n value of the tree (Korf, 1991)
- Is there a valid way to do this pruning?

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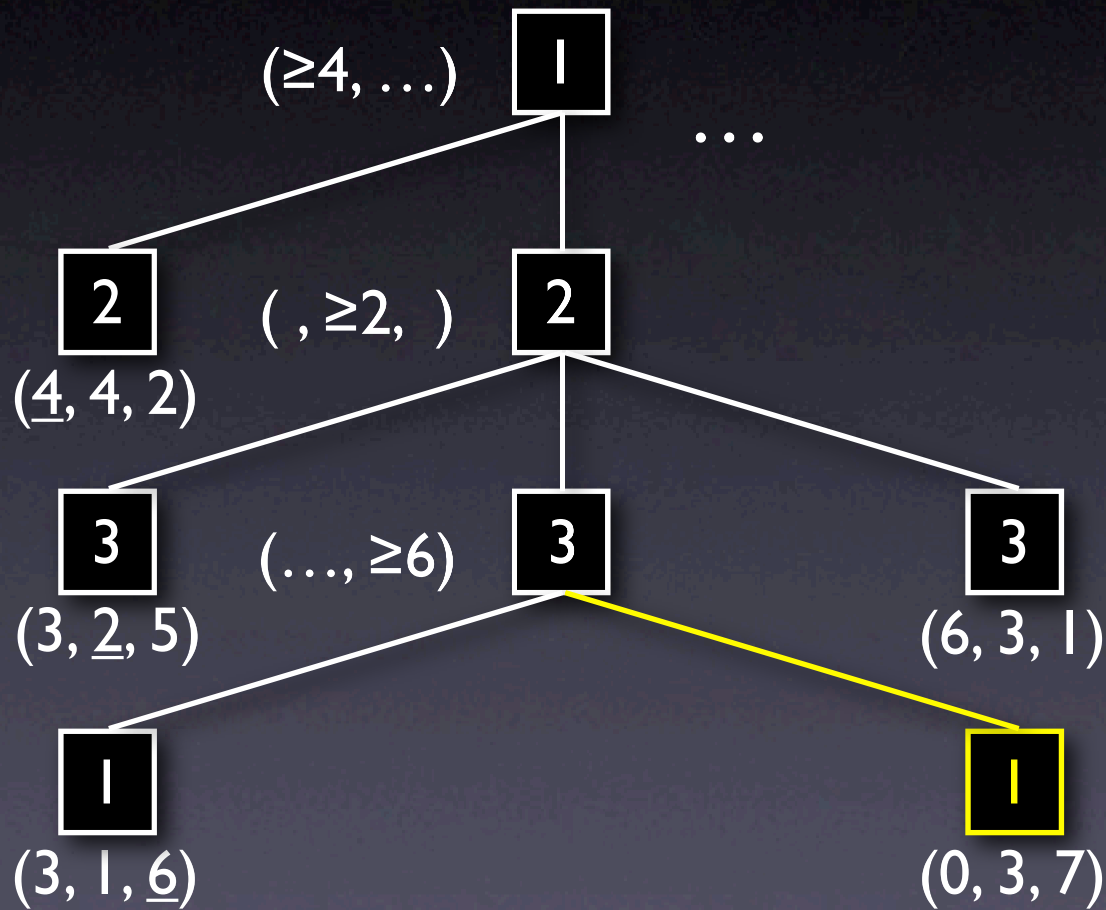
Last-Branch Pruning

(Part 1)

maxsum = 10



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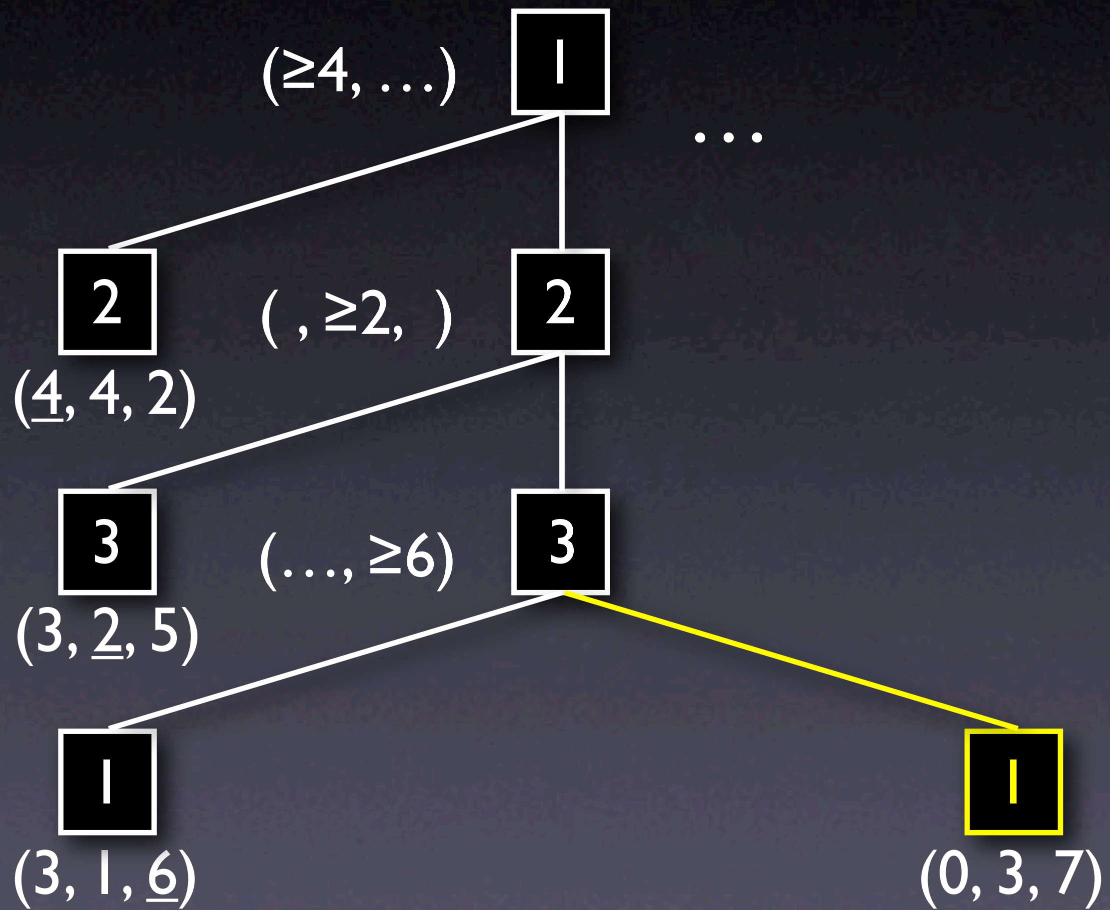

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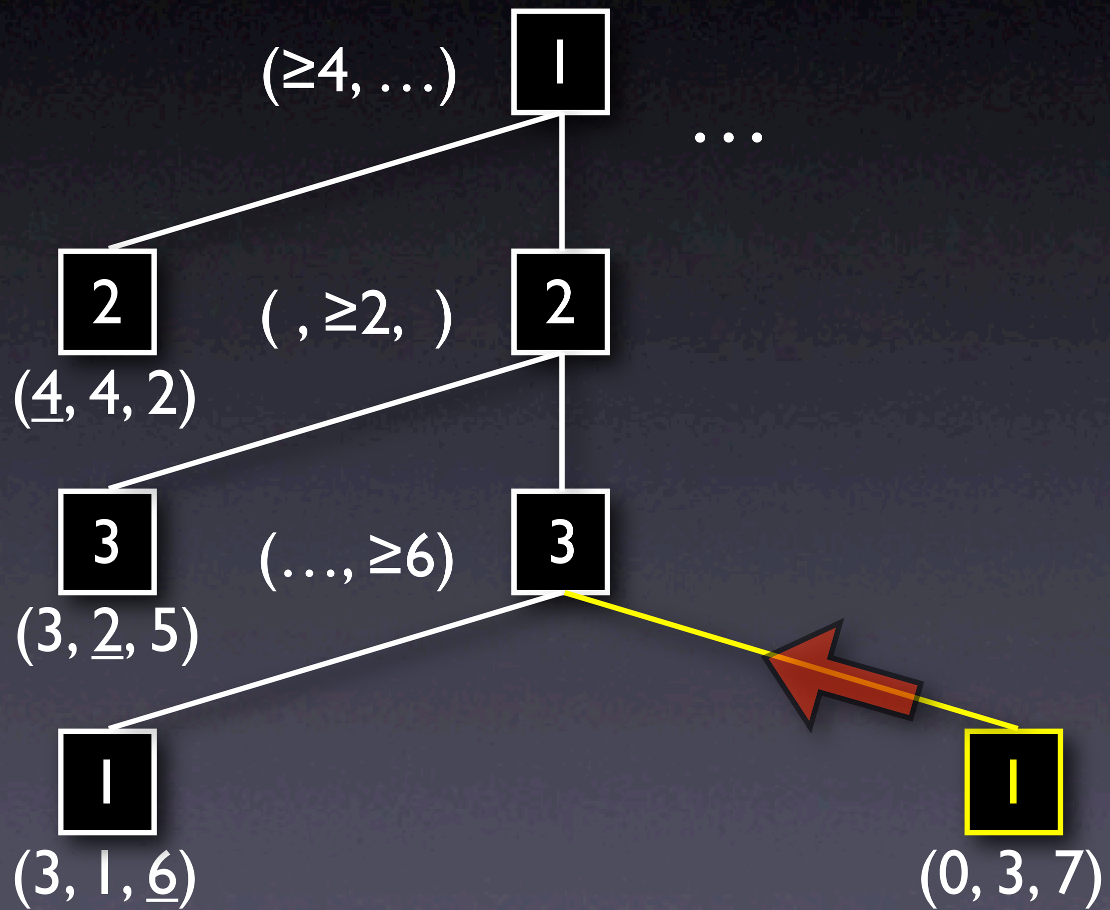

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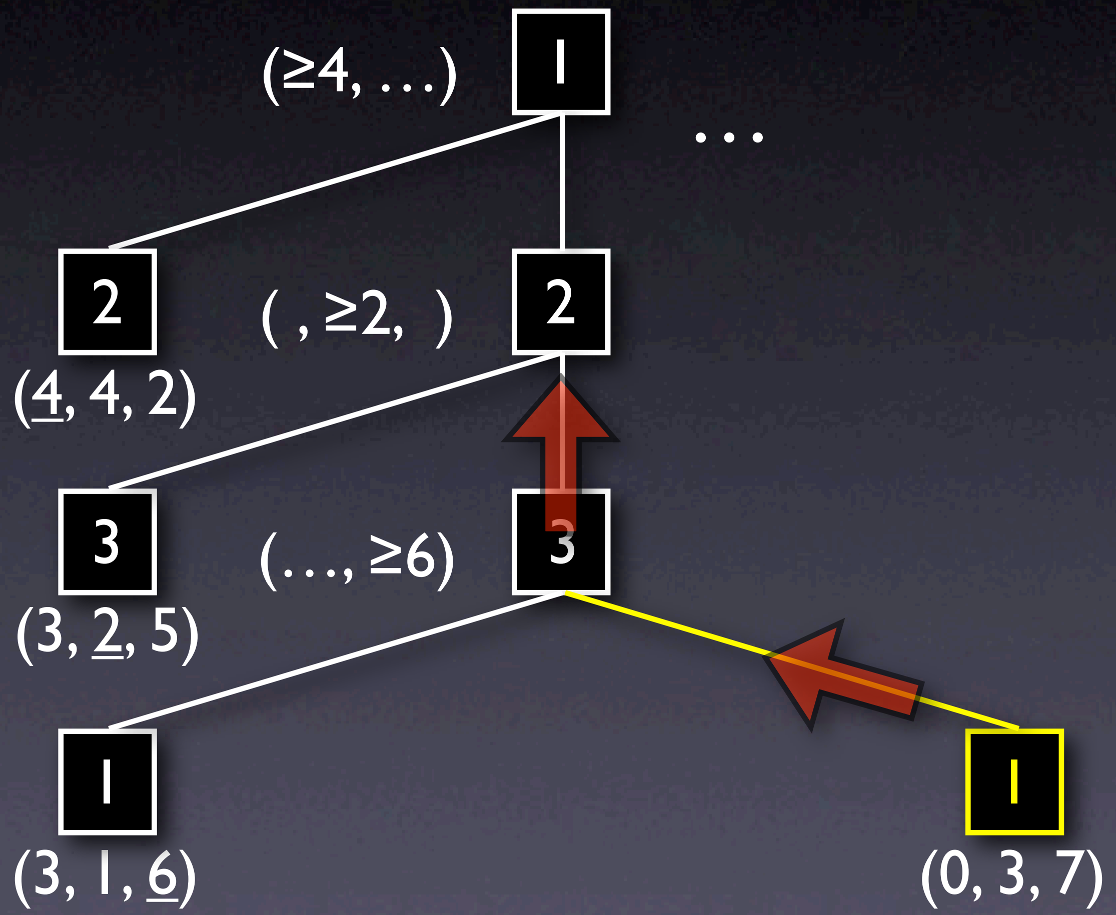

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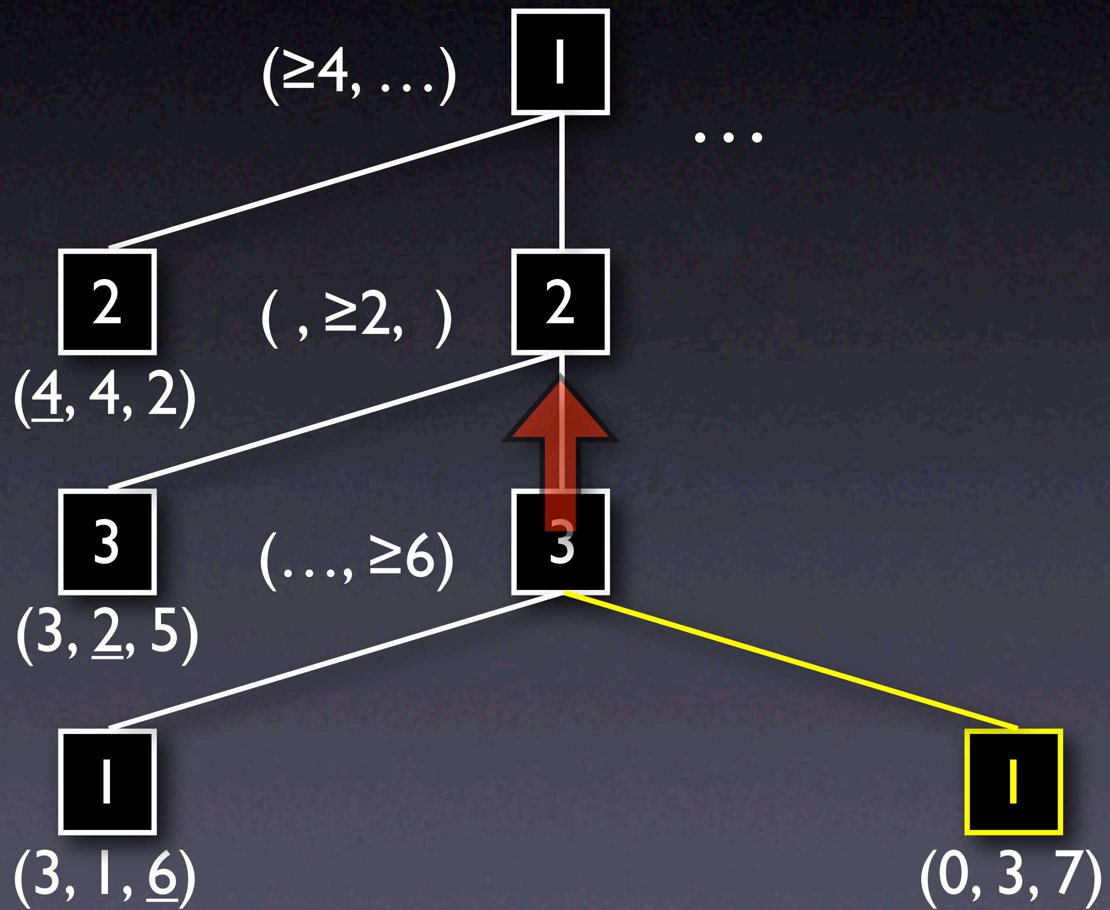

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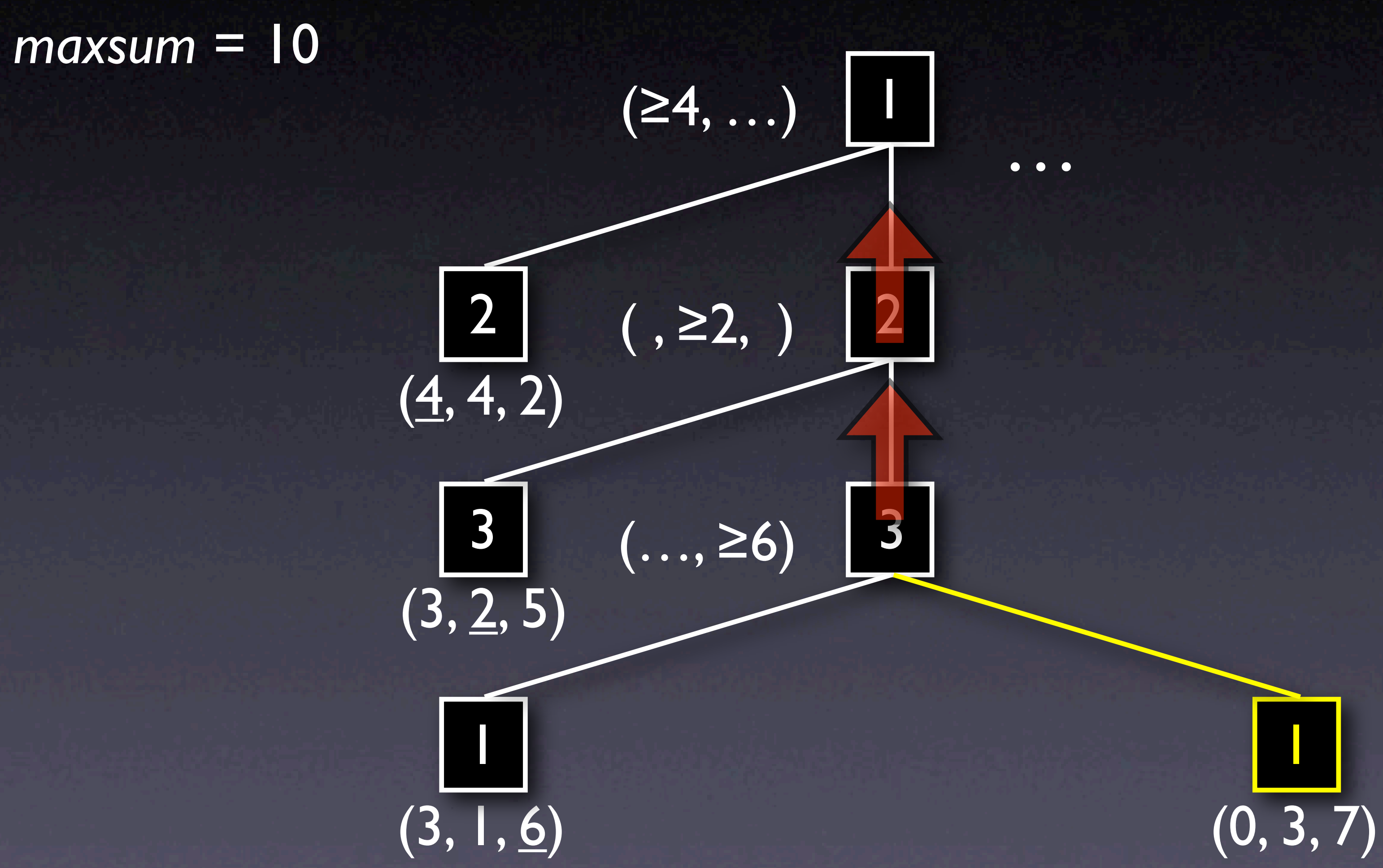

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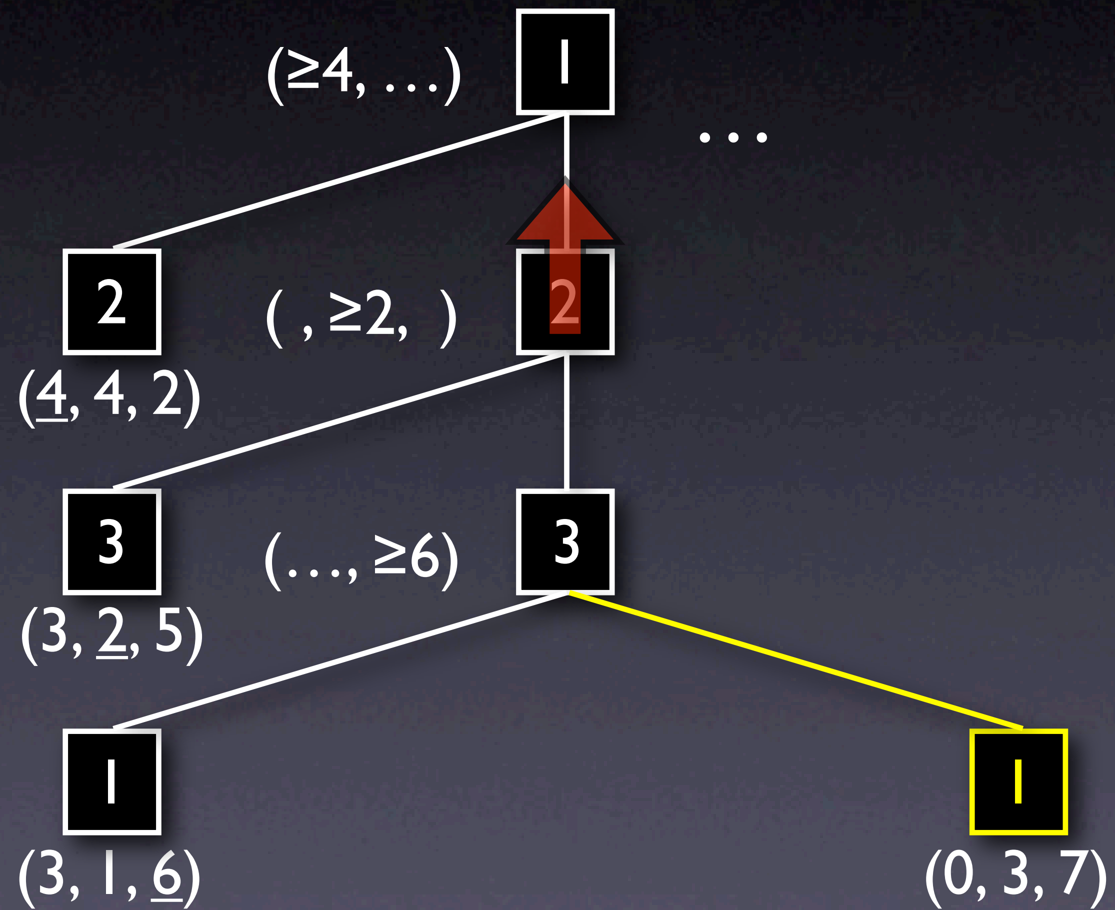

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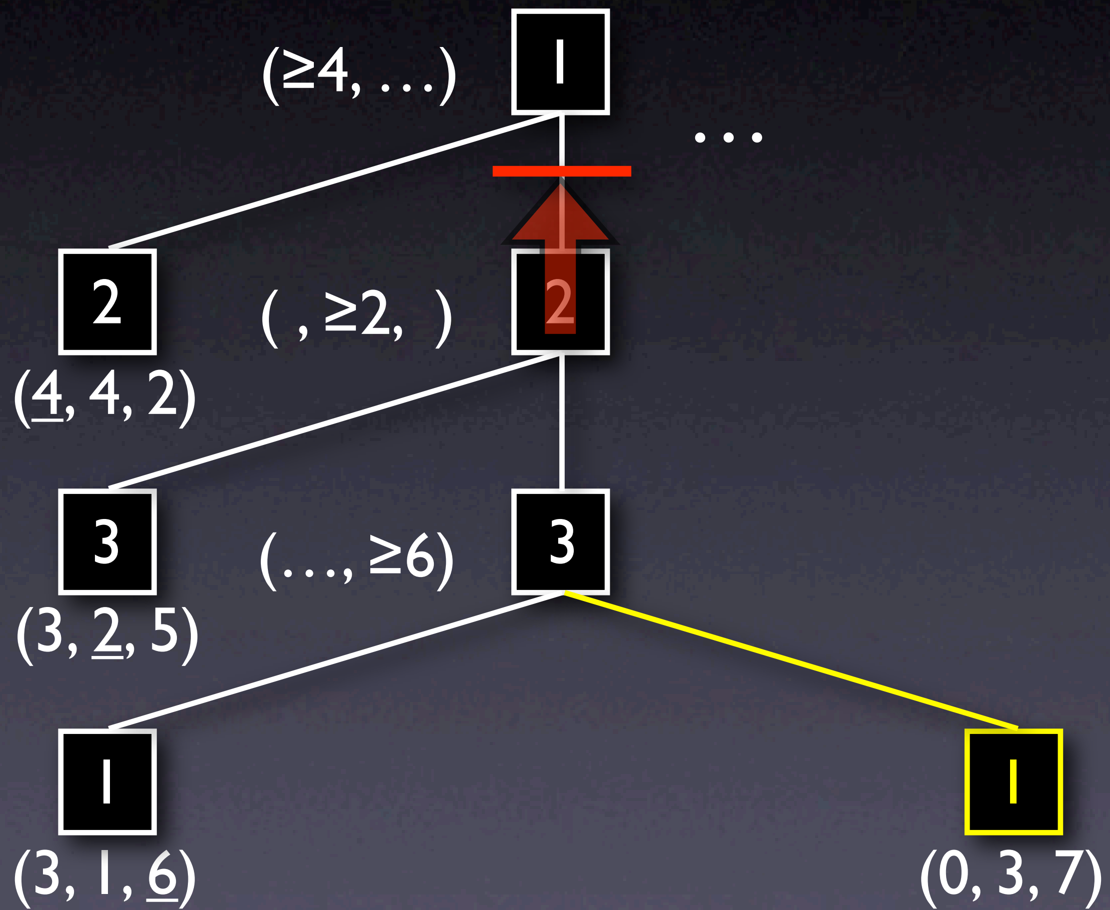

Last-Branch Pruning I

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- We can now deep prune when:

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Last-Branch Pruning I

- We can now deep prune when:
  - Scores sum to *maxsum* or greater

15

Last-Branch Pruning I

- We can now deep prune when:
  - Scores sum to *maxsum* or greater
  - Second player on their last branch

Last-Branch Pruning I

- We can now deep prune when:
  - Scores sum to *maxsum* or greater
  - Second player on their last branch
- Can we still do better?

Last-Branch Pruning
(Part 2)

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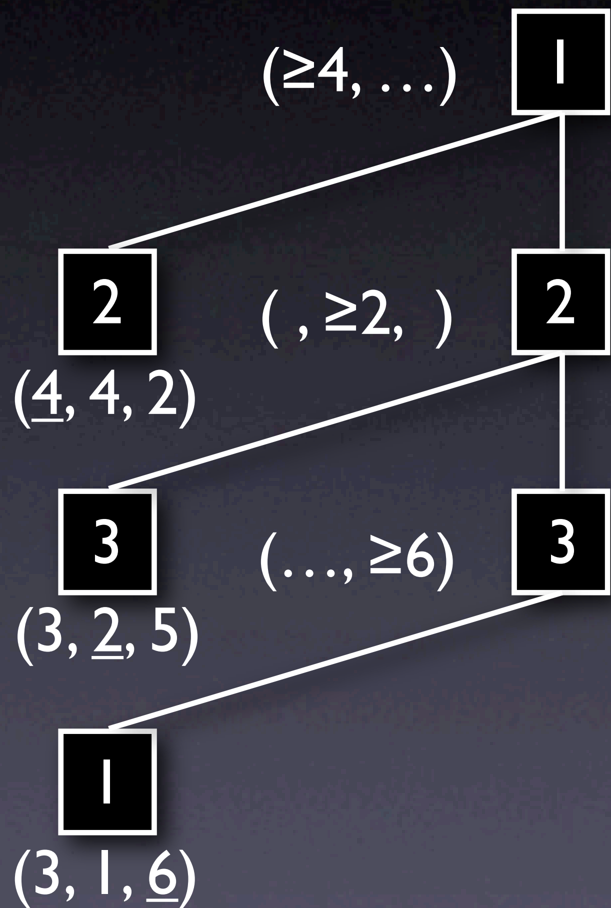

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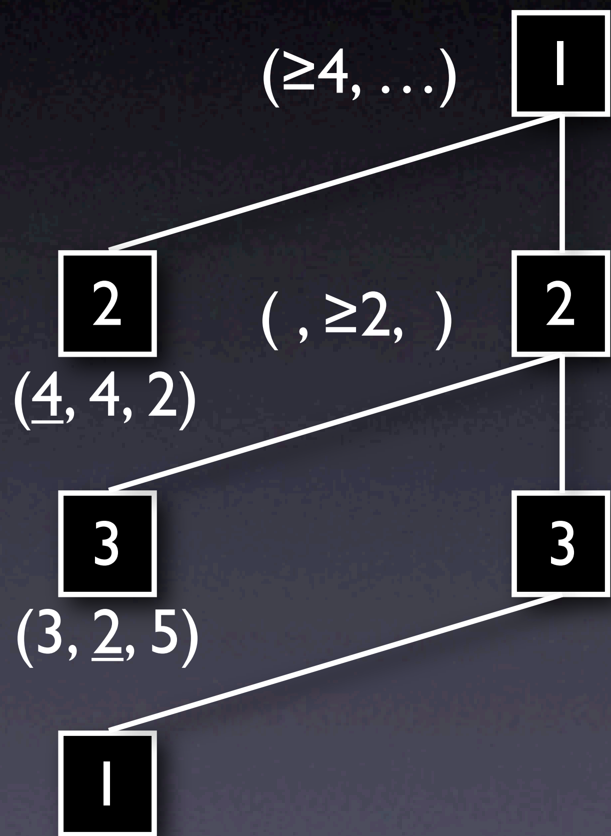

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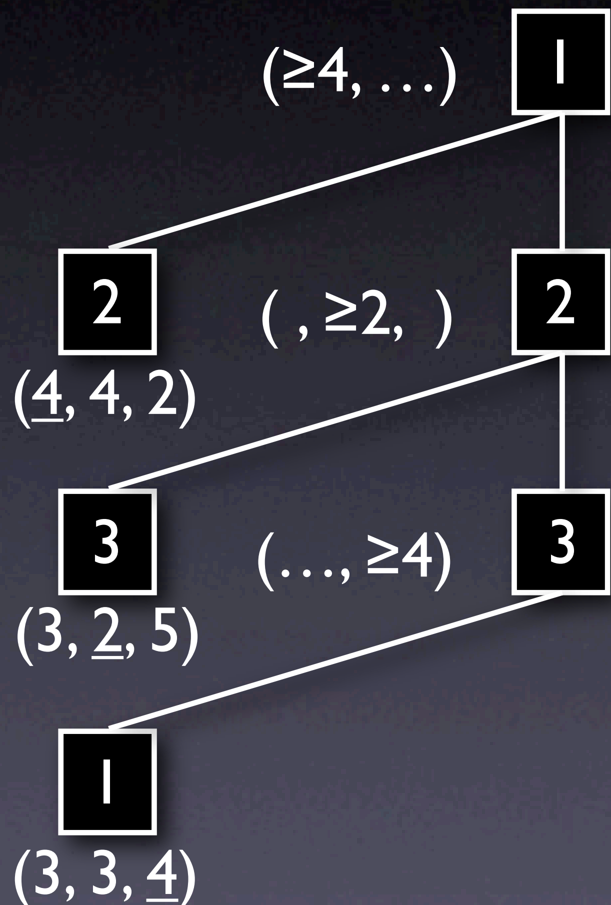

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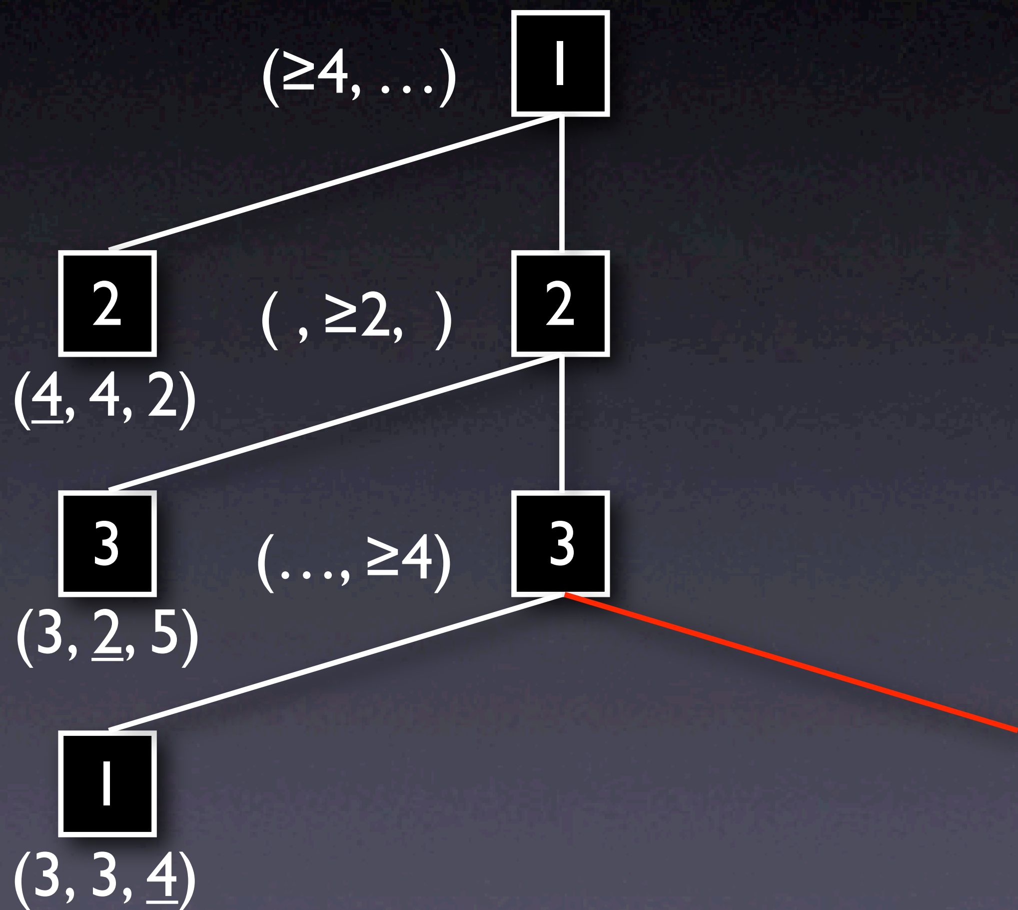

Last-Branch Pruning

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- Directional Algorithm

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- Directional Algorithm
- Prunes when:

Last-Branch Pruning

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- Prunes when:
  - Sum of players' bounds ≥ *maxsum*

17

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- Prunes when:
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  - Second player on their last branch

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Last-Branch Pruning

- Directional Algorithm
- Prunes when:
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  - Second player on their last branch
- Limited to last branch! Can we do better?

Speculative Pruning

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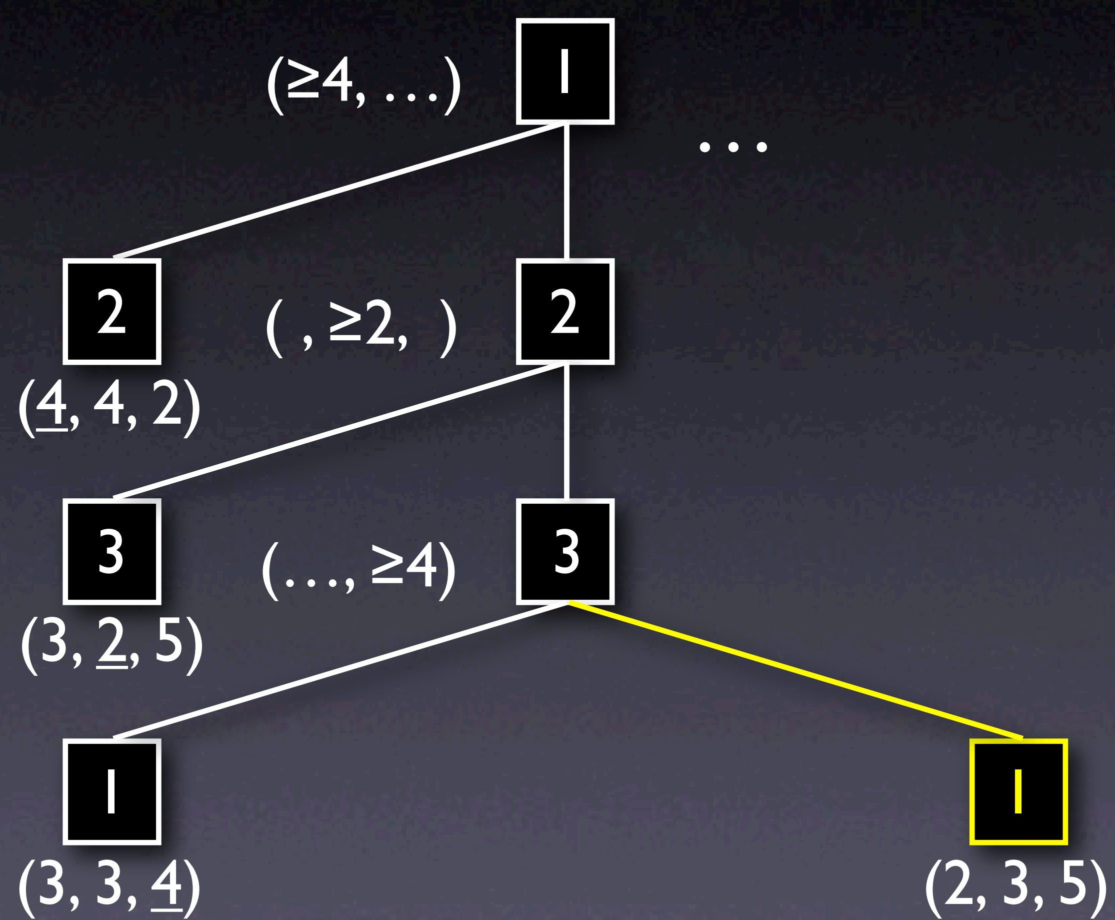

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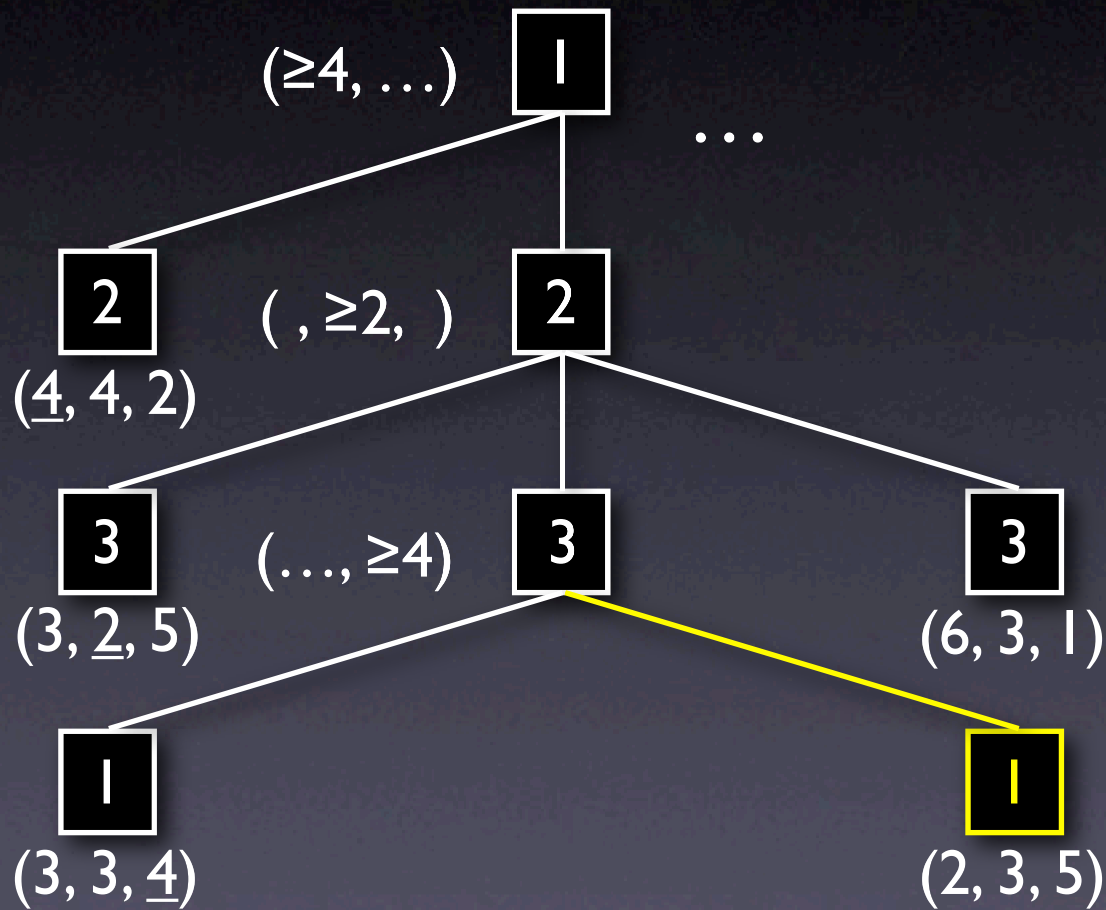

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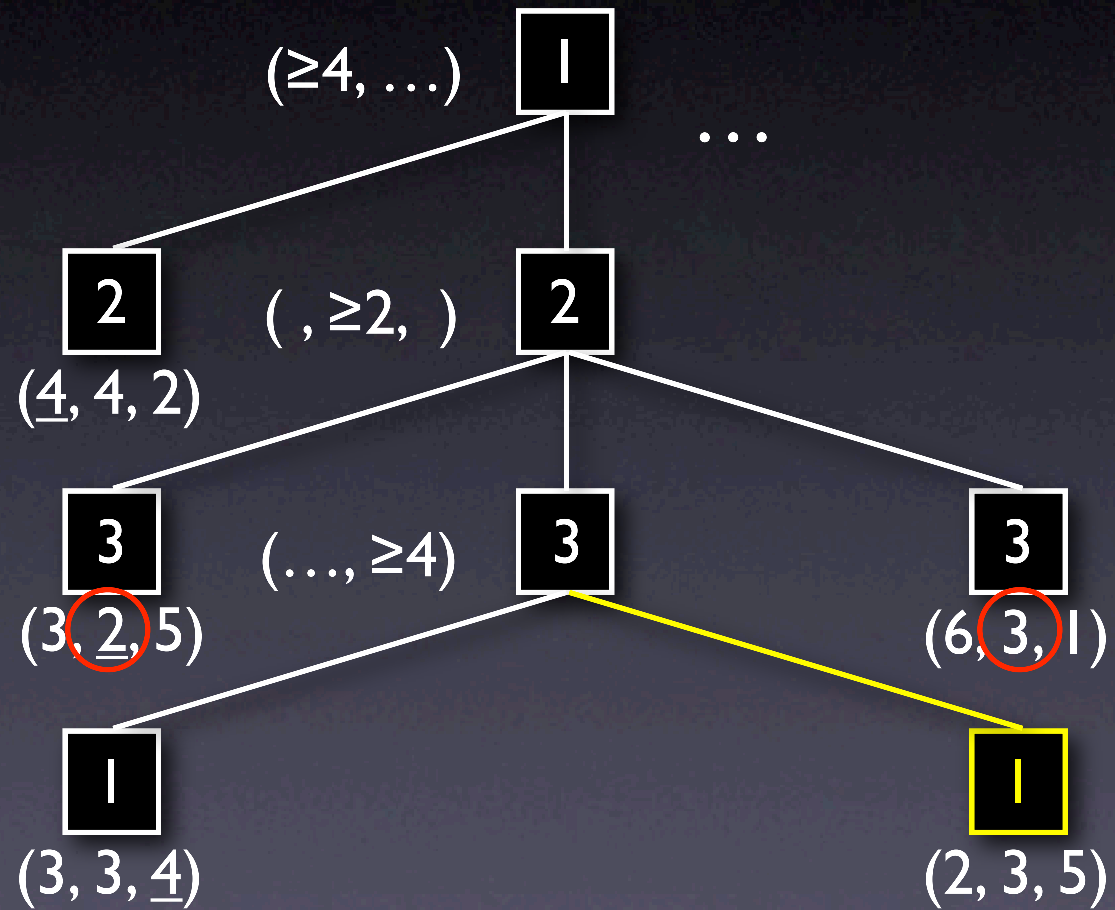

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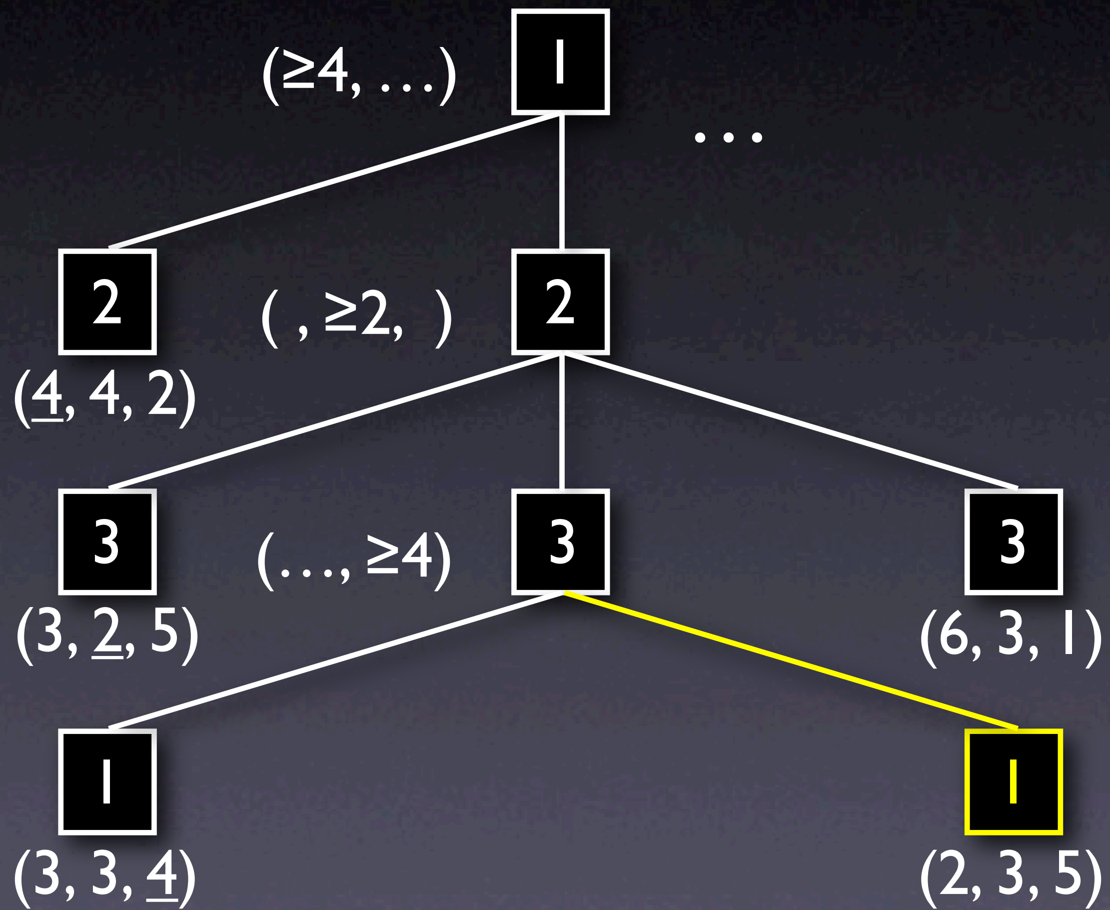

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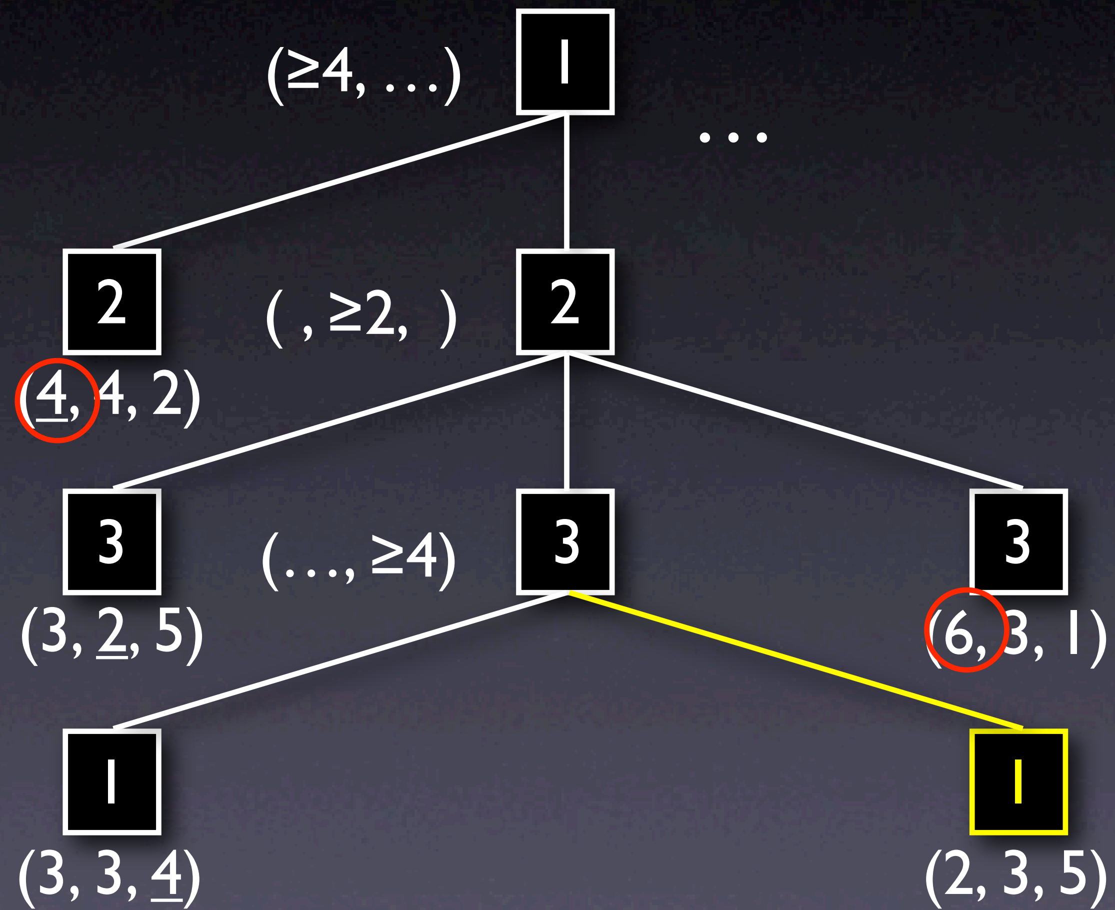

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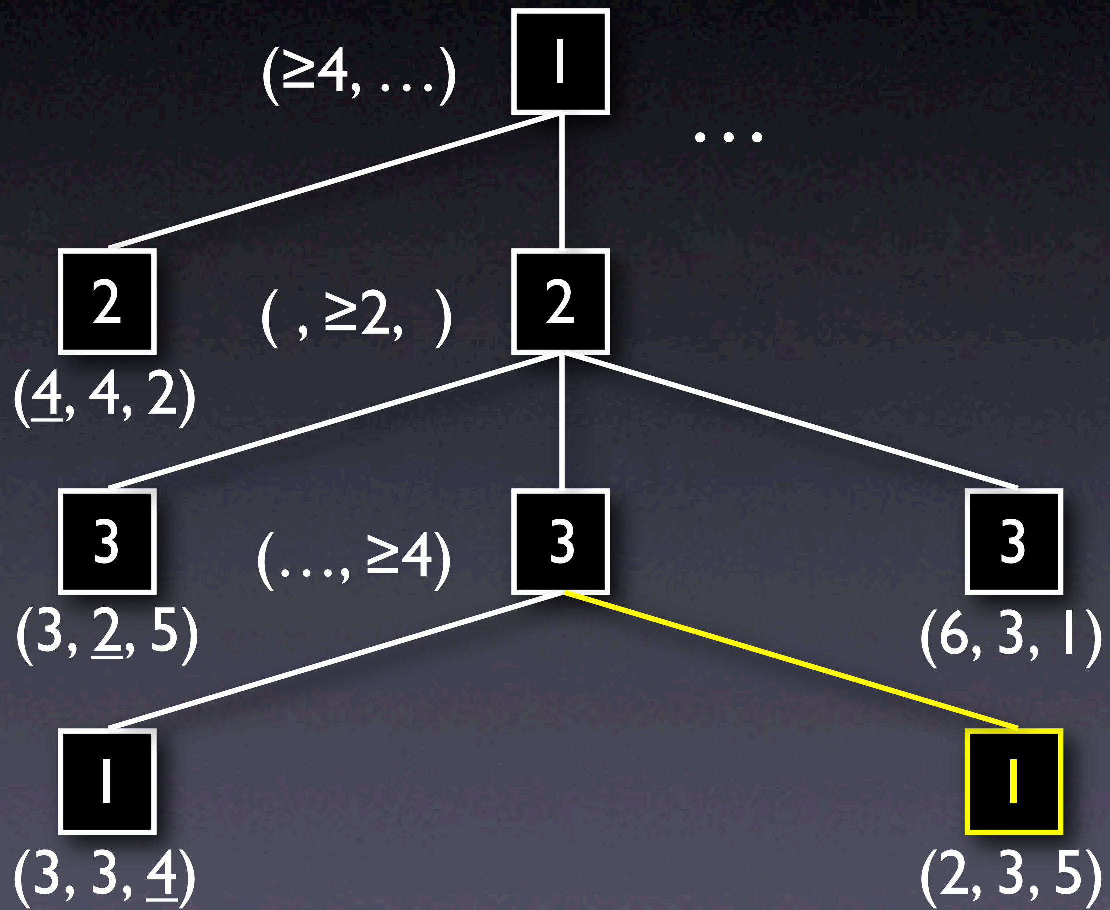

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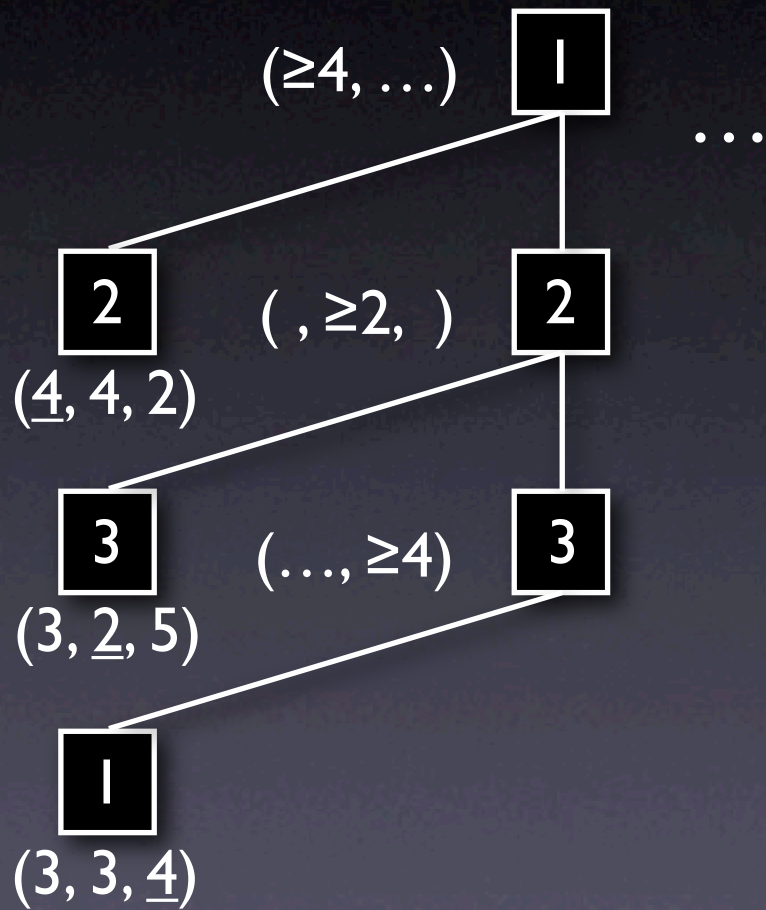

Speculative Pruning

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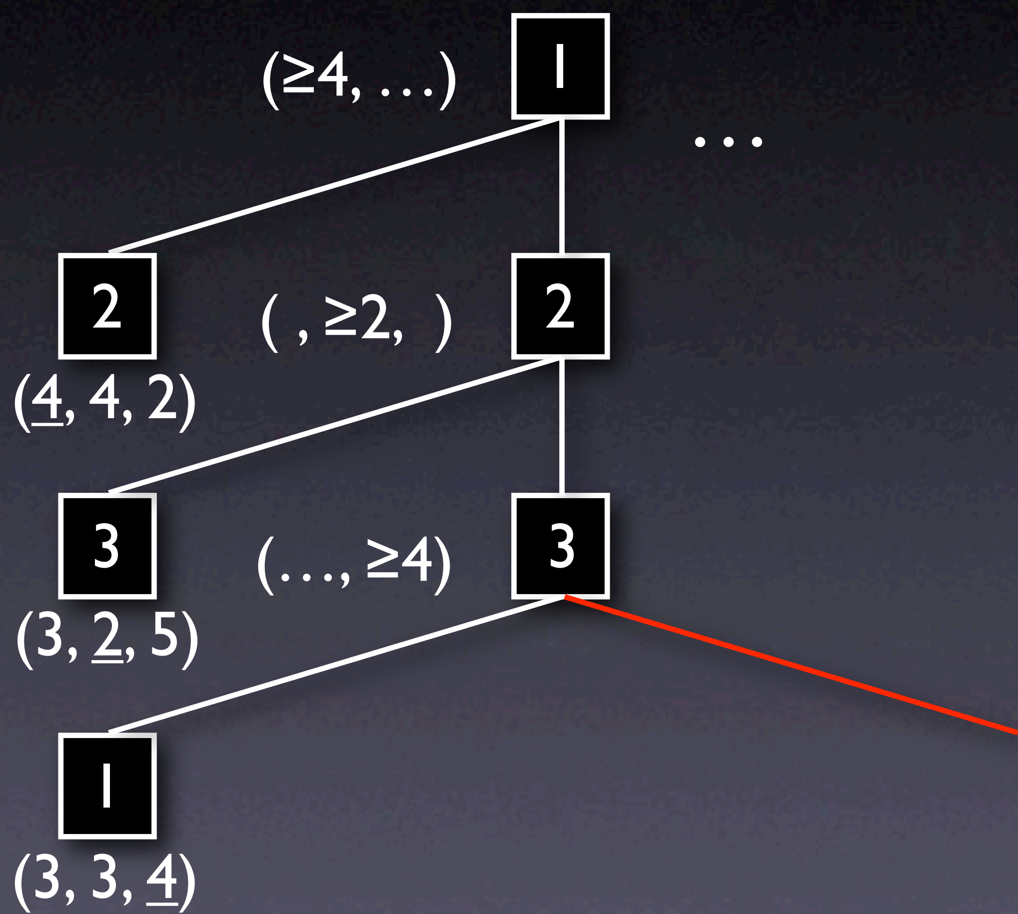

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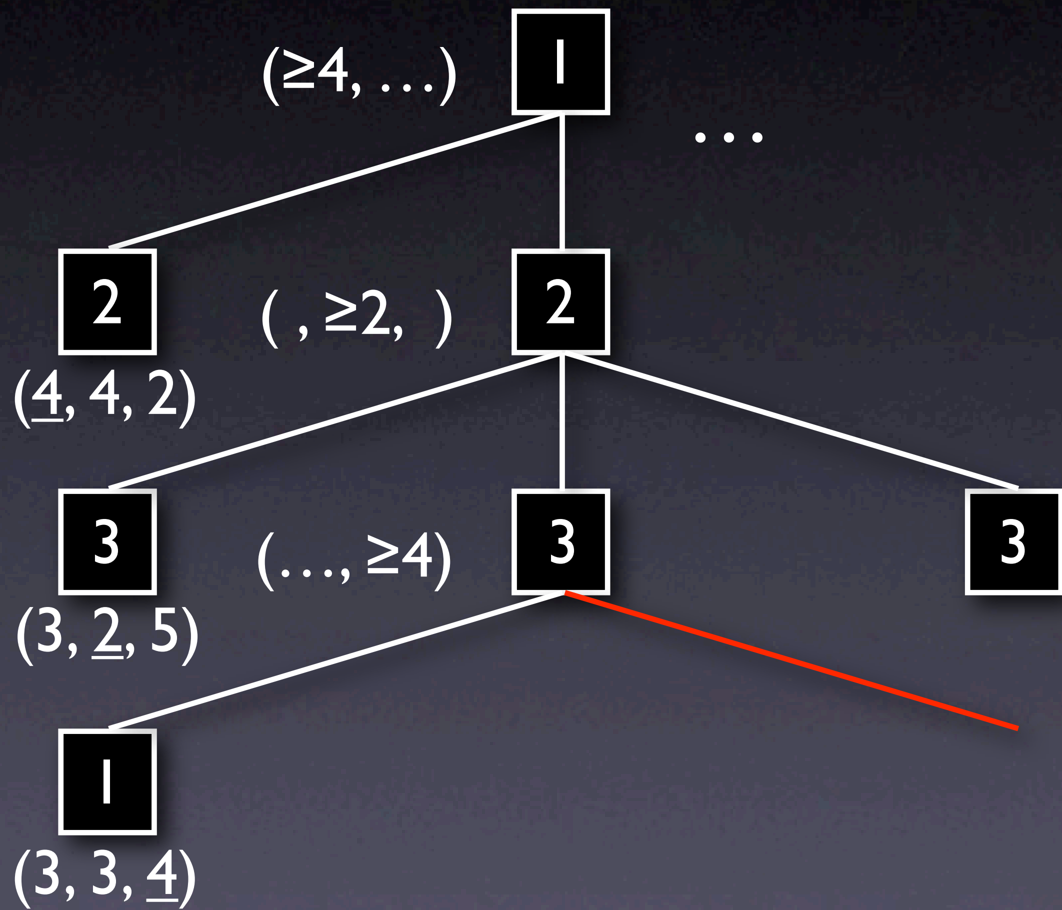

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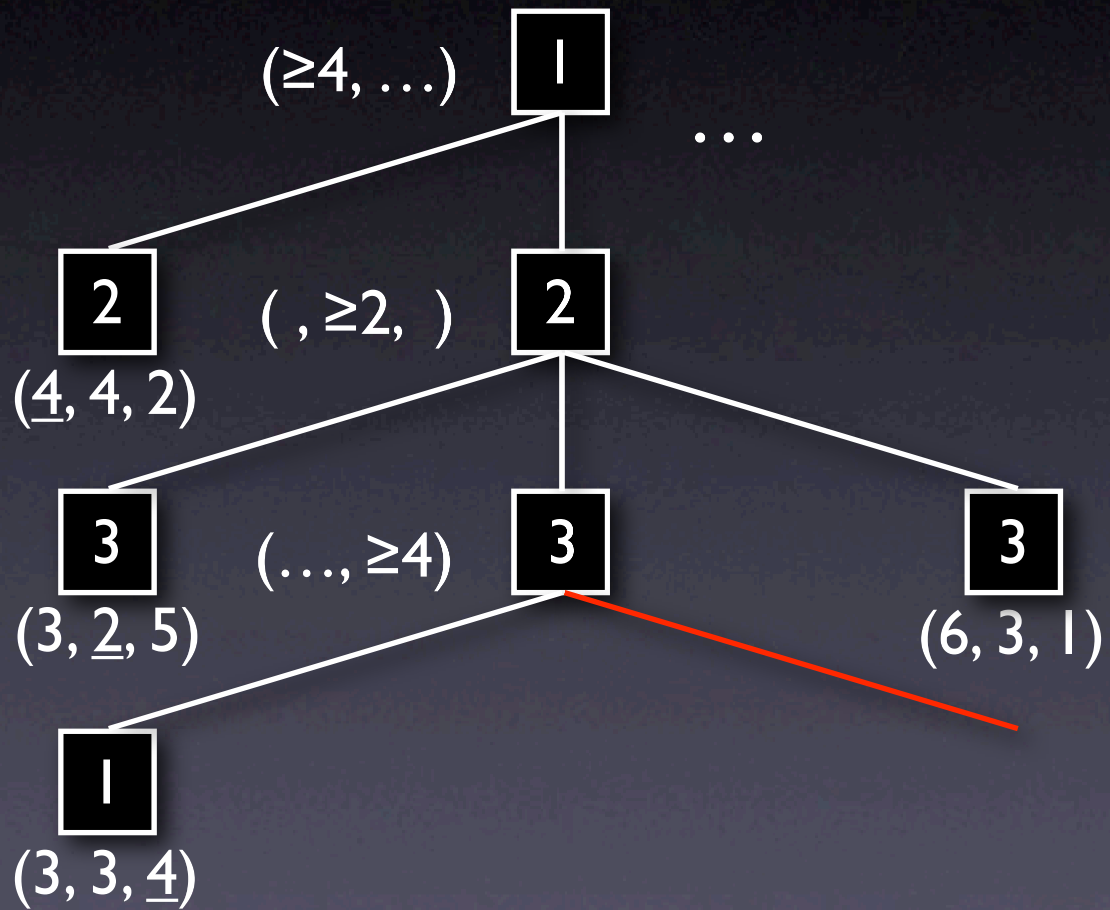

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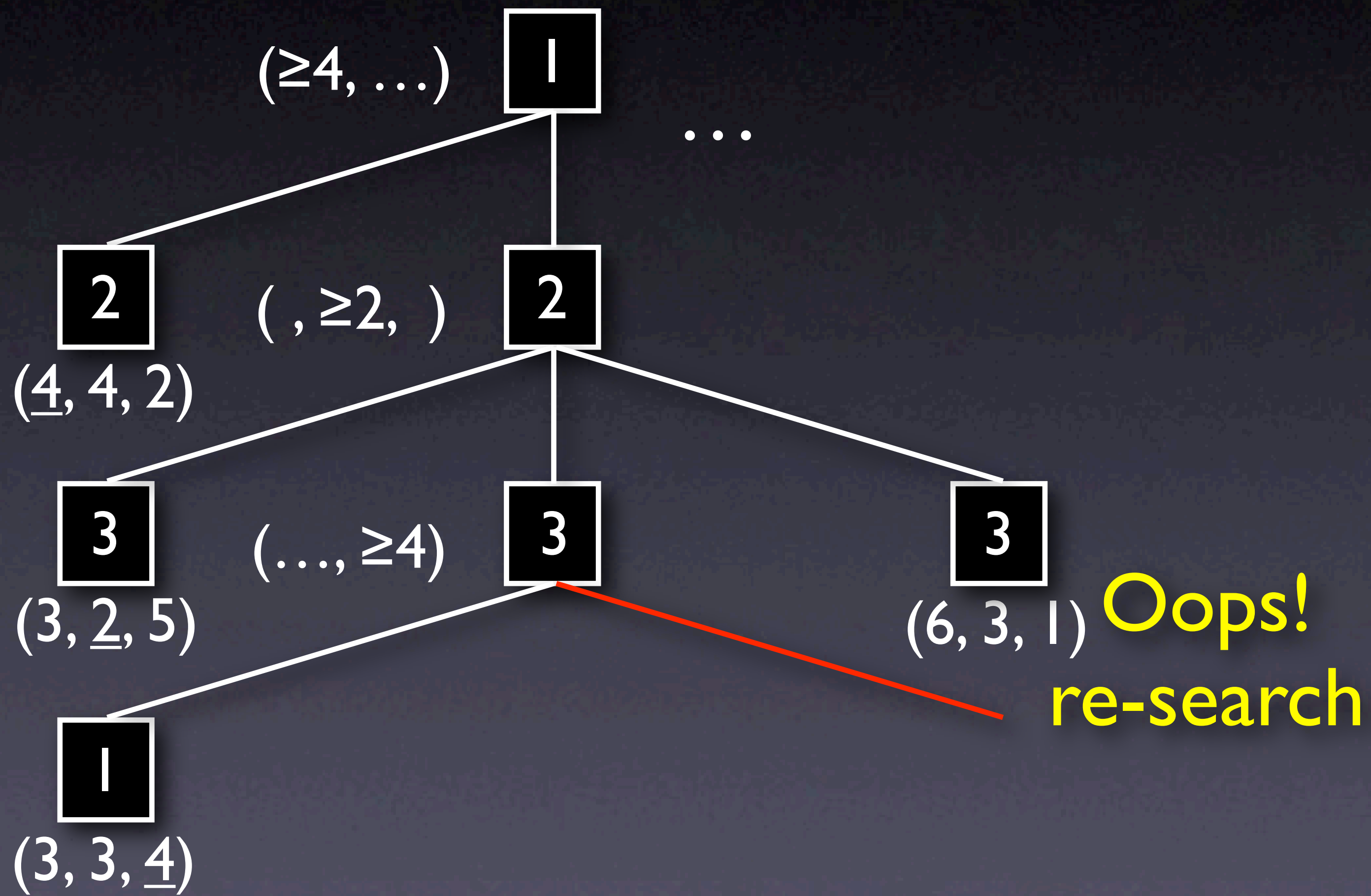

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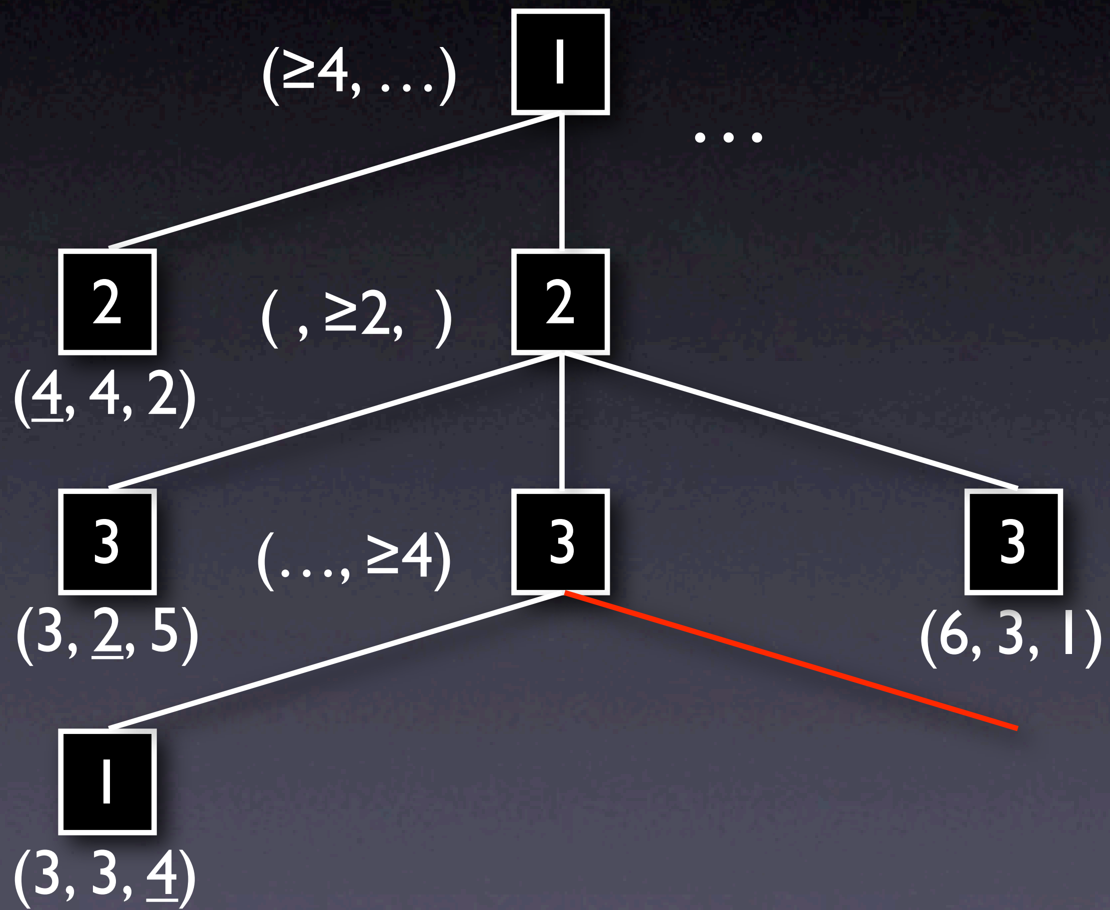

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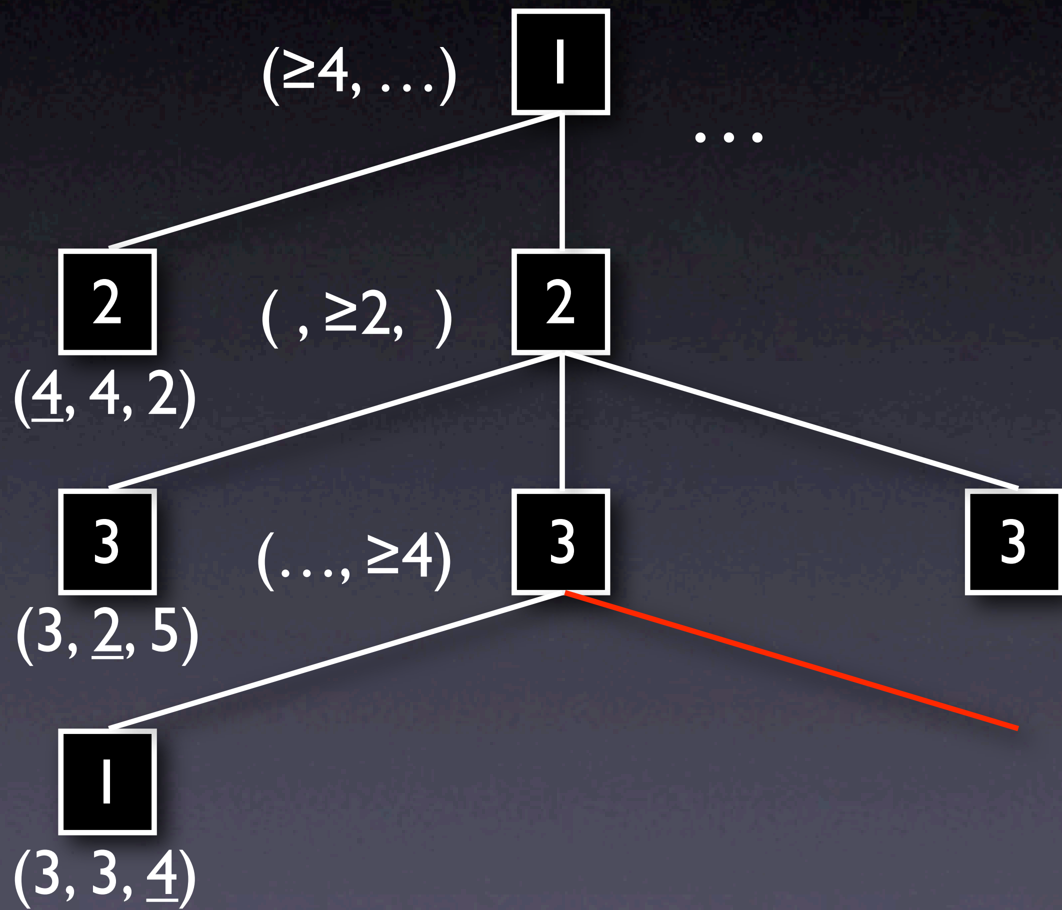

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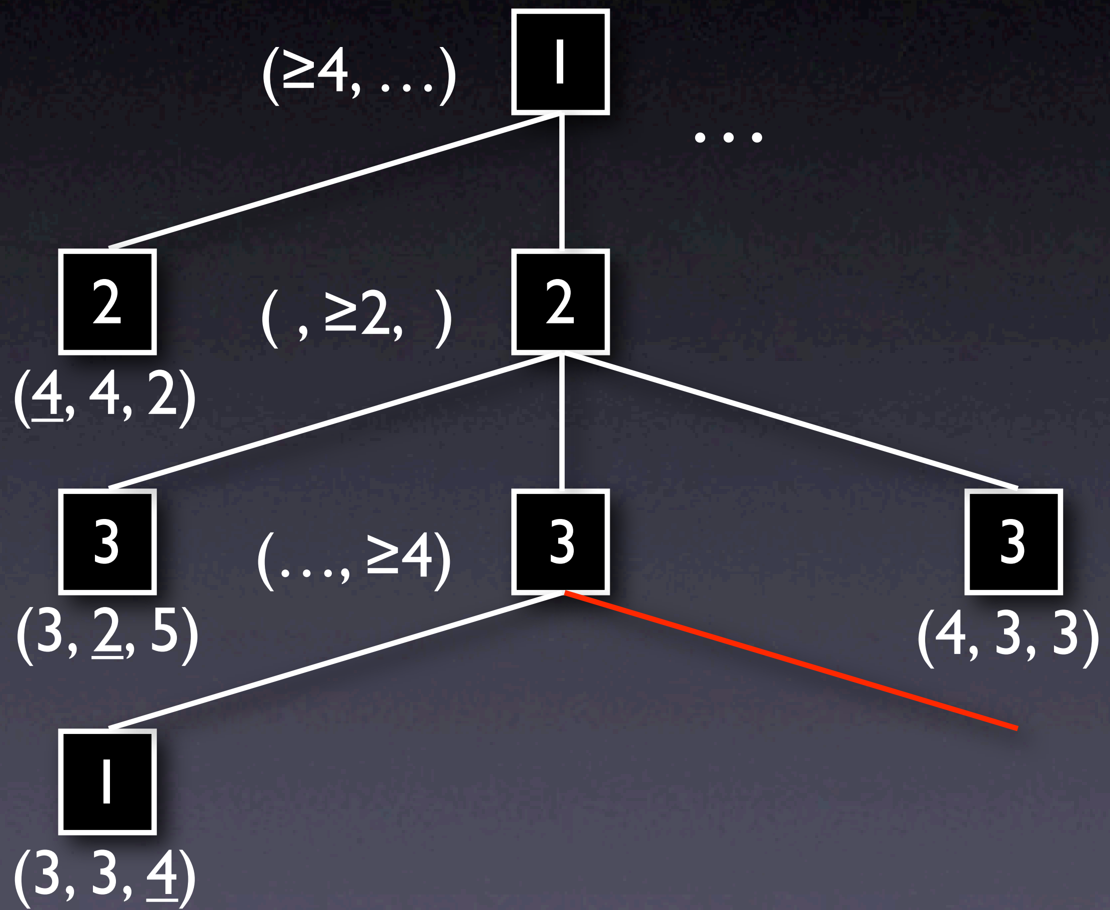

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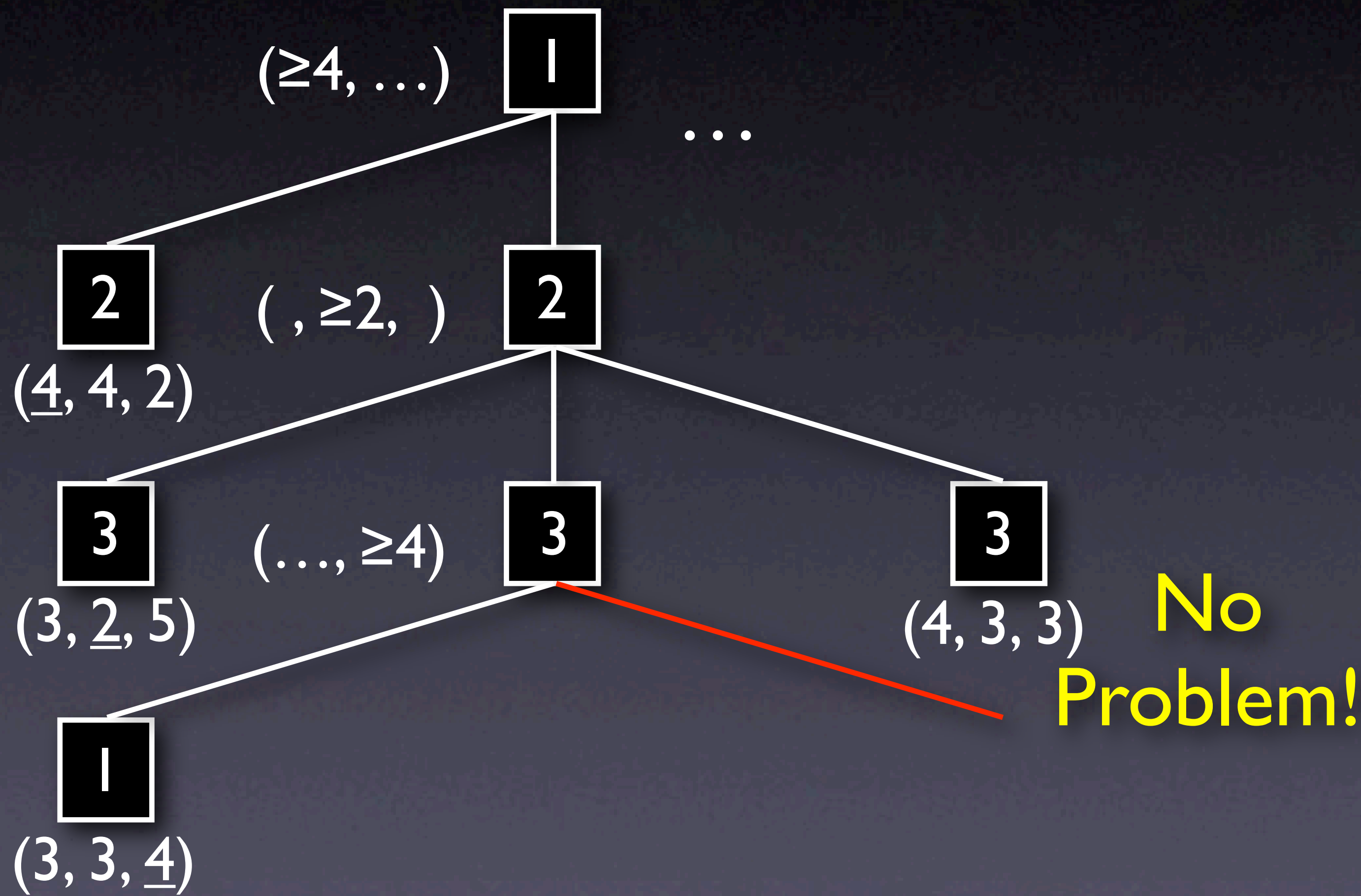

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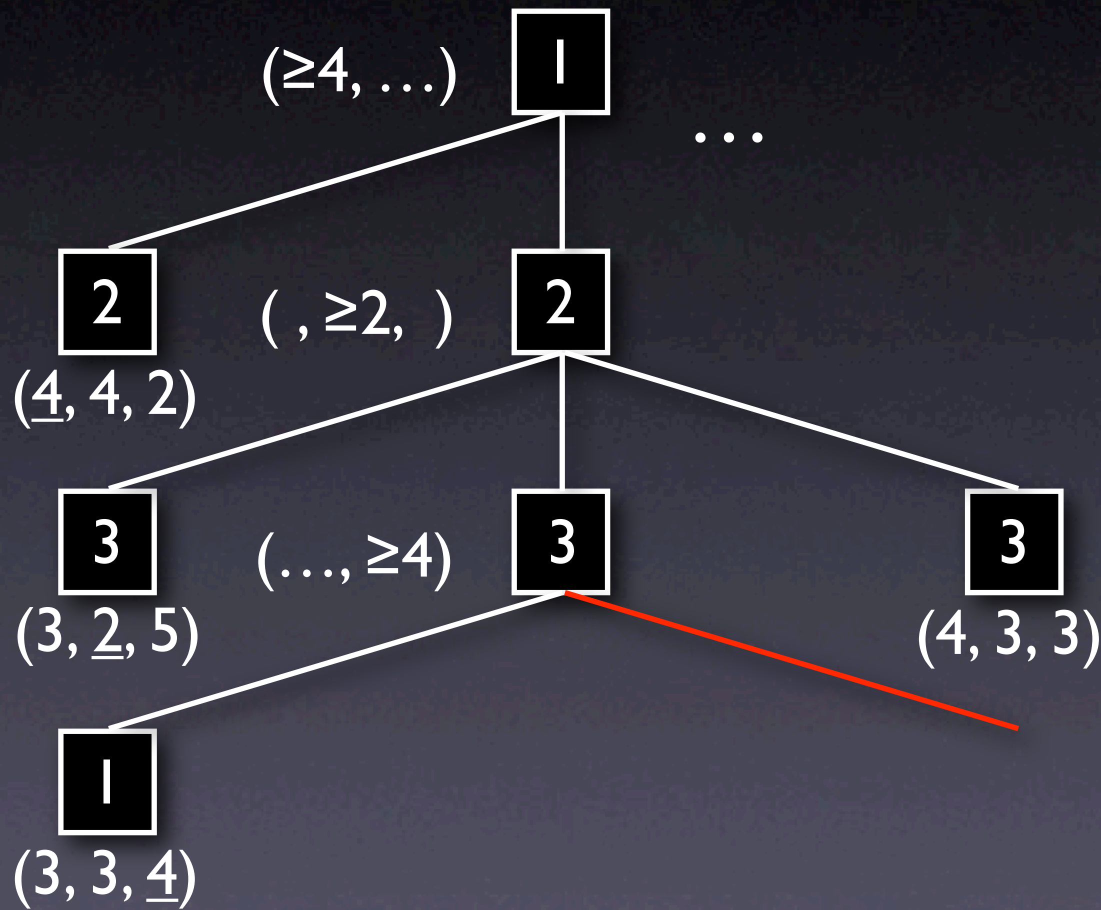

Speculative Pruning

Speculative Pruning

- Non-directional algorithm

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Speculative Pruning

- Non-directional algorithm
  - Prunes are not guaranteed to be correct
  - Can re-search branches if needed
- Works on any constant-sum game tree
  - Effectiveness depends only on node ordering

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Speculative Pruning

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Speculative Pruning

- Reduces branching factor:

Speculative Pruning

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  - In the best case, as *b* gets large

Speculative Pruning

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  - $b^{(n-1)/n}$

Speculative Pruning

- Reduces branching factor:
  - In the best case, as *b* gets large
  - $b^{(n-1)/n}$
- Effective even in the average case

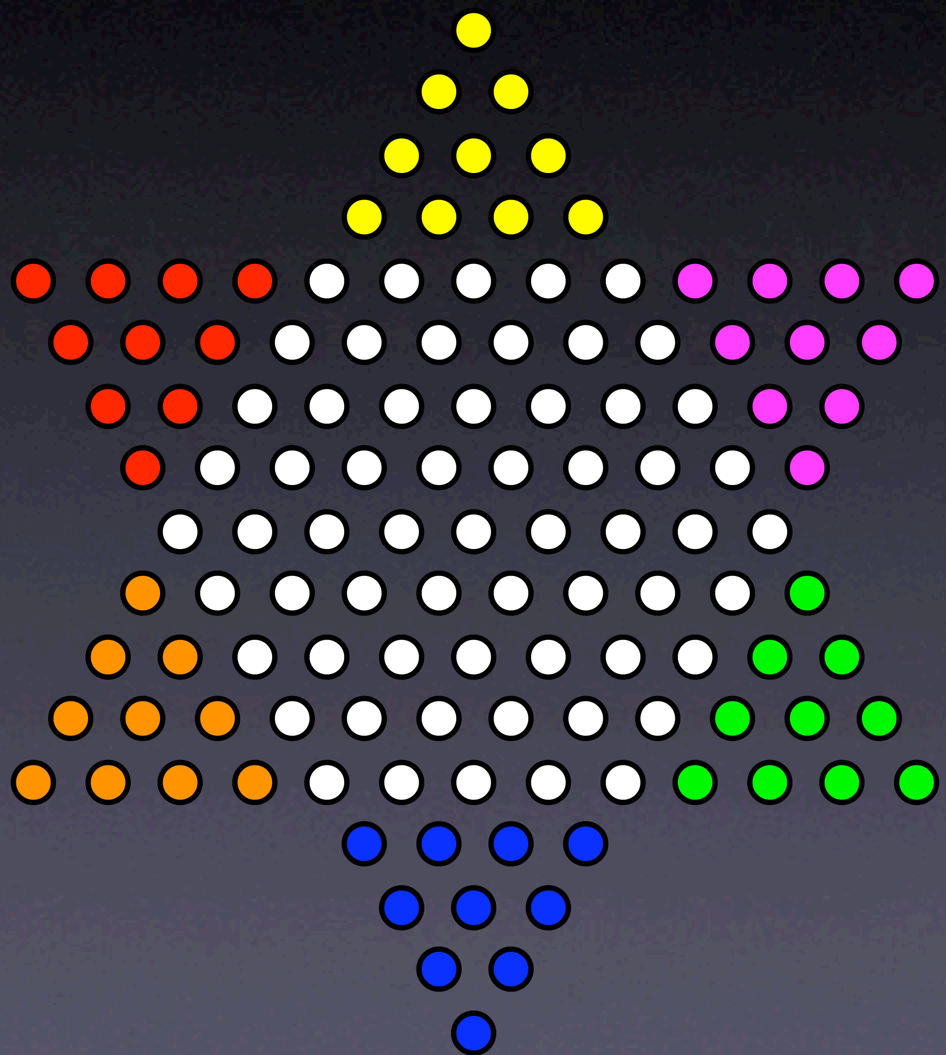
Speculative Pruning
(3-Players)

b	$b^{2/3}$	*actual b*
2	1.59	1.84
3	2.08	2.47
4	2.52	3.00
10	4.64	5.42
1000	100.0	103.6

Outline

- Max^n Decision Rule
- Max^n Pruning Techniques
- Experimental Results
- Conclusions

Chinese Checkers



Chinese Checkers

Chinese Checkers

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Chinese Checkers

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- Played 30 games, ~1500 searches

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Chinese Checkers

- 3 Players
- Played 30 games, ~1500 searches
  - Iterative deepening search
  - Limited branching factor to 10 moves
  - Measured average expansions at depth 6
- Previous algorithms could not prune tree

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Chinese Checkers

Algorithm	Node expansions (depth 6)
Max^n	1.2 million
Spec. Max^n	100k

Hearts and Spades

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- Trick-based card games

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  - Have monotonic heuristics

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- Shallow pruning does not occur in Hearts
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  - Iterative deepening search
  - 500k node search limit

Average Search Depth

	Hearts	Spades
Best Prev. Max^n	20.9	21.7
Spec. Max^n	22.1	24.3

Conclusions

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Conclusions

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Conclusions

- New pruning algorithms for max^n
- Last-Branch Pruning
  - Directional algorithm
- Speculative Pruning
  - Non-directional algorithm
- First algorithms effective in pruning any constant-sum game