Min-Max Multiway Cut

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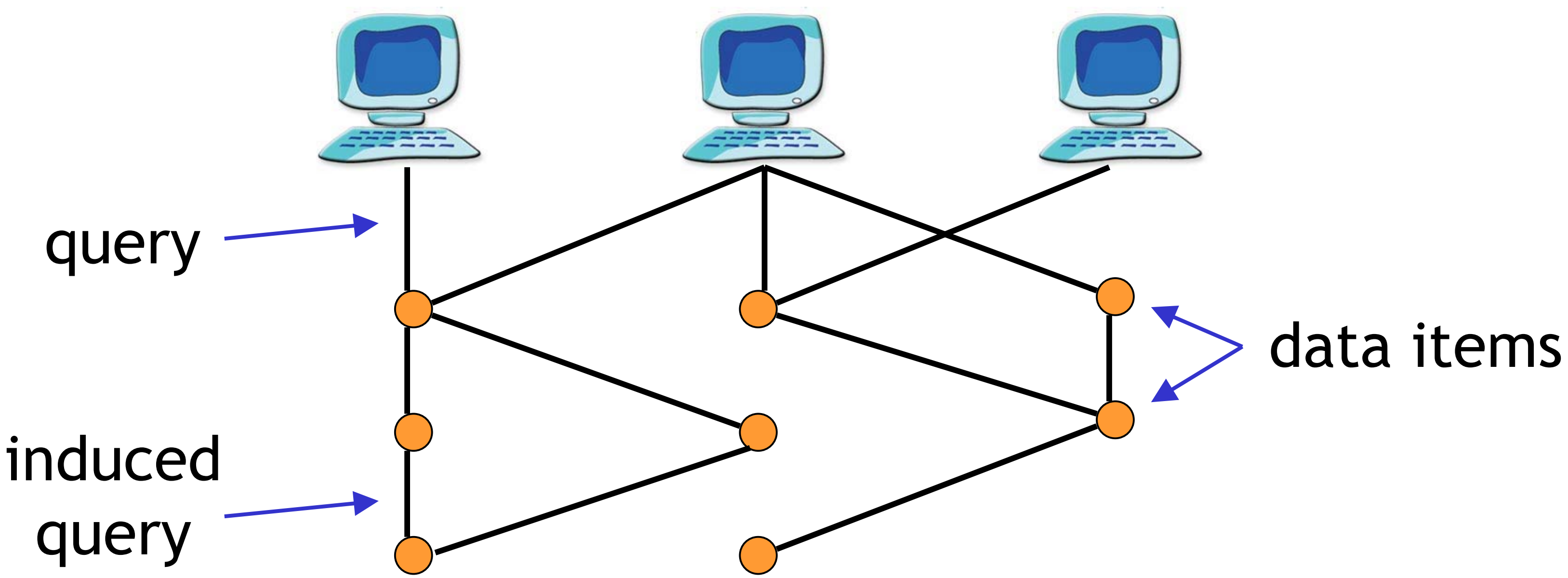
joint work with Éva Tardos

Motivating example

- Partition data among servers
- Minimize communication load on servers



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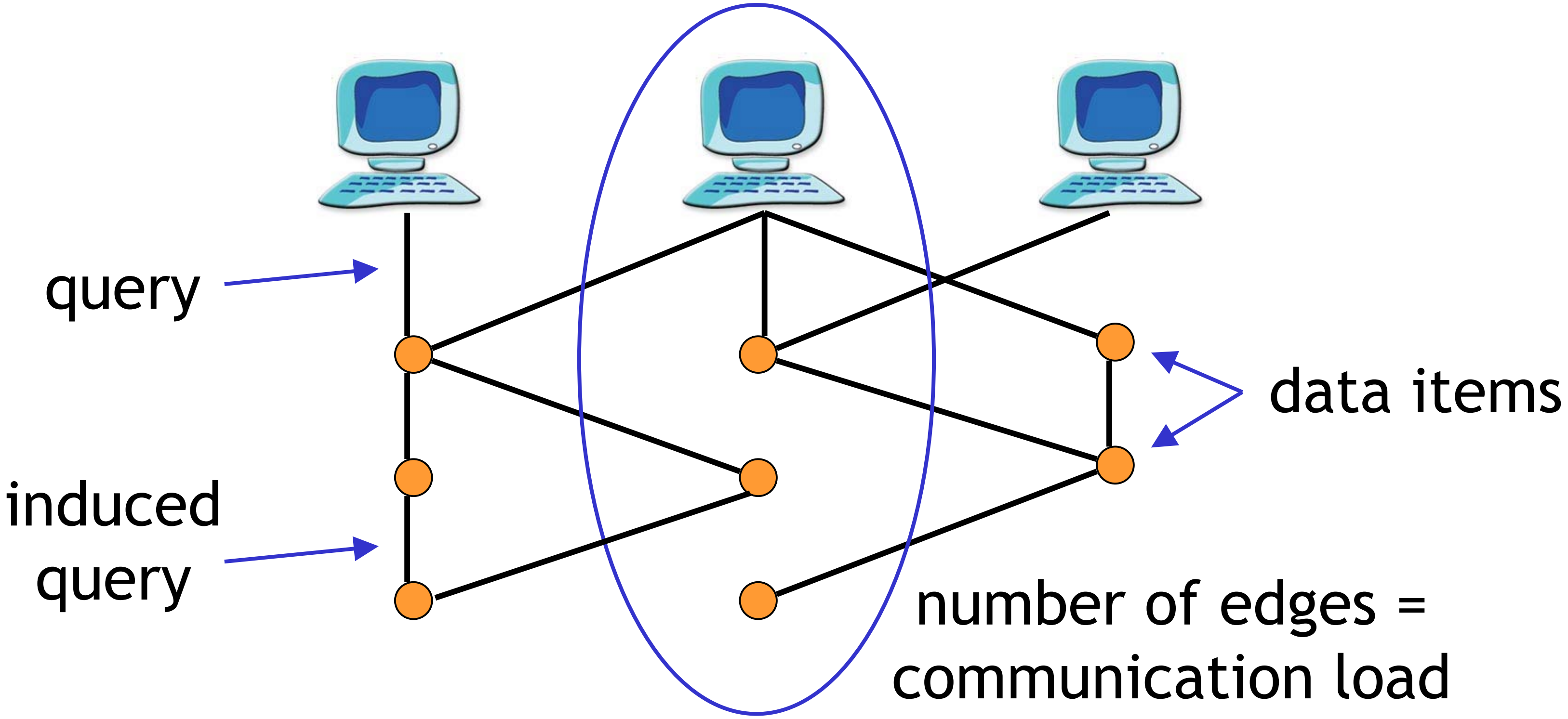
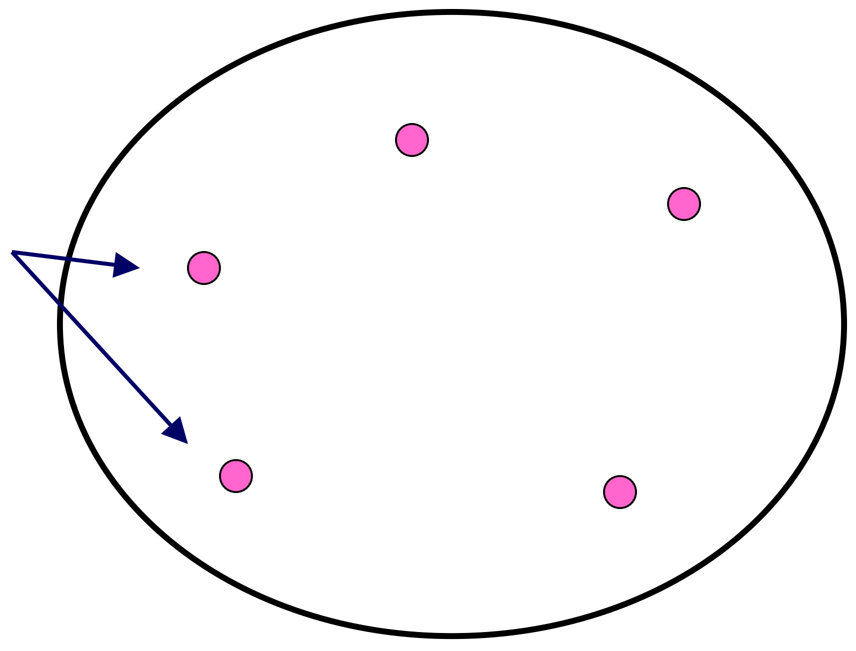
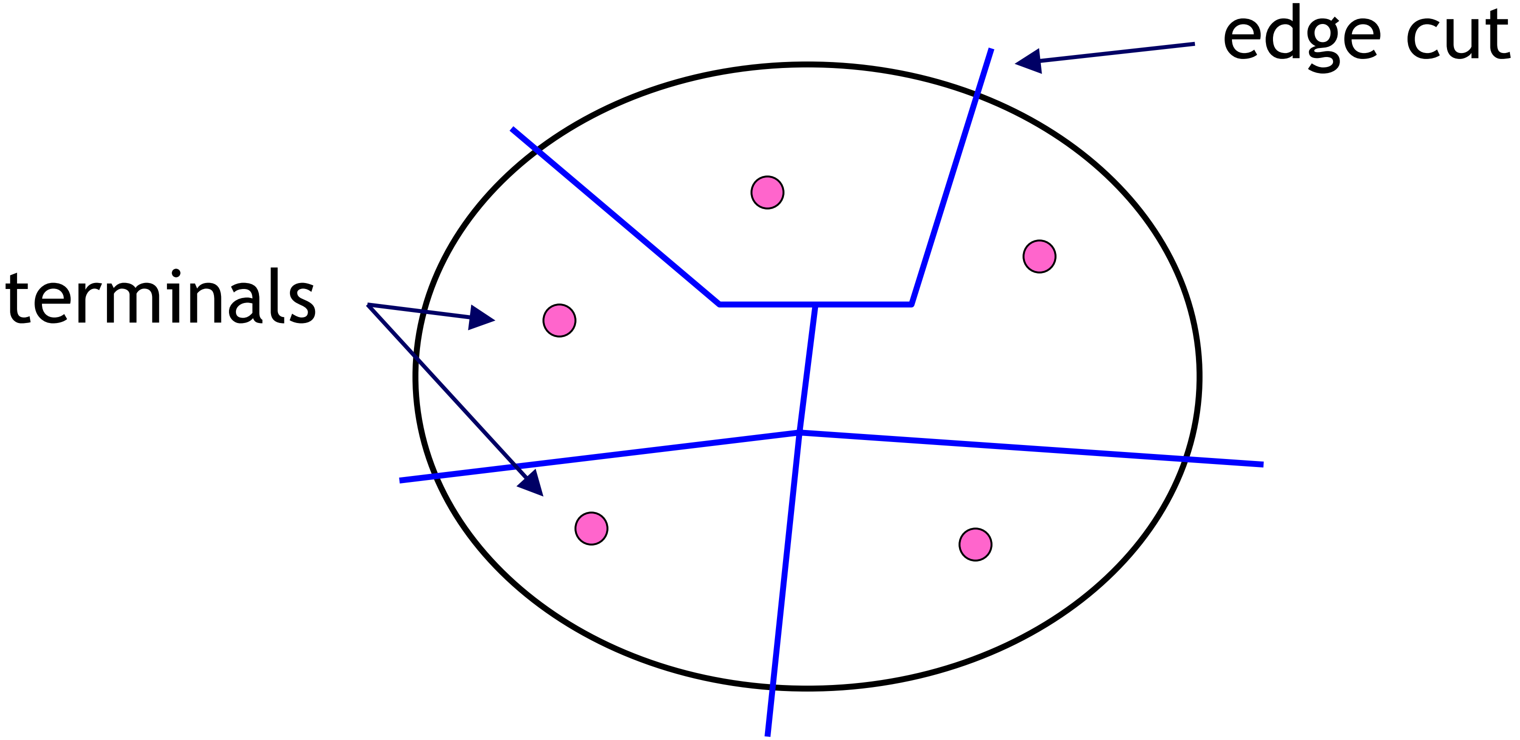


Multiway Cut



Multiway Cut

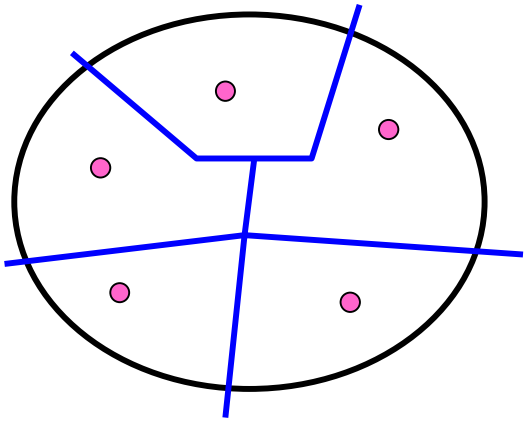


Objective

Traditional:

- Total number of edges cut

[E. Dahlhaus, D.S. Johnson, C.H. Papadimitriou, P.D.Seymour, and M. Yannakakis, 1994]



Min-max:

- Maximum number leaving any component

NP-hardness

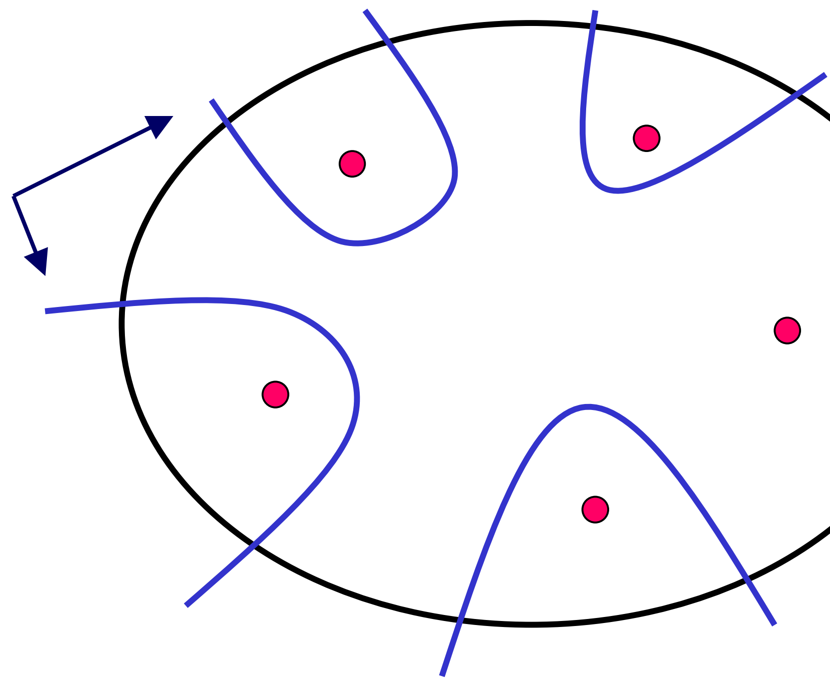
Min-max multiway cut is NP-hard:

- on graphs with only 4 terminals
- on trees with edge capacities

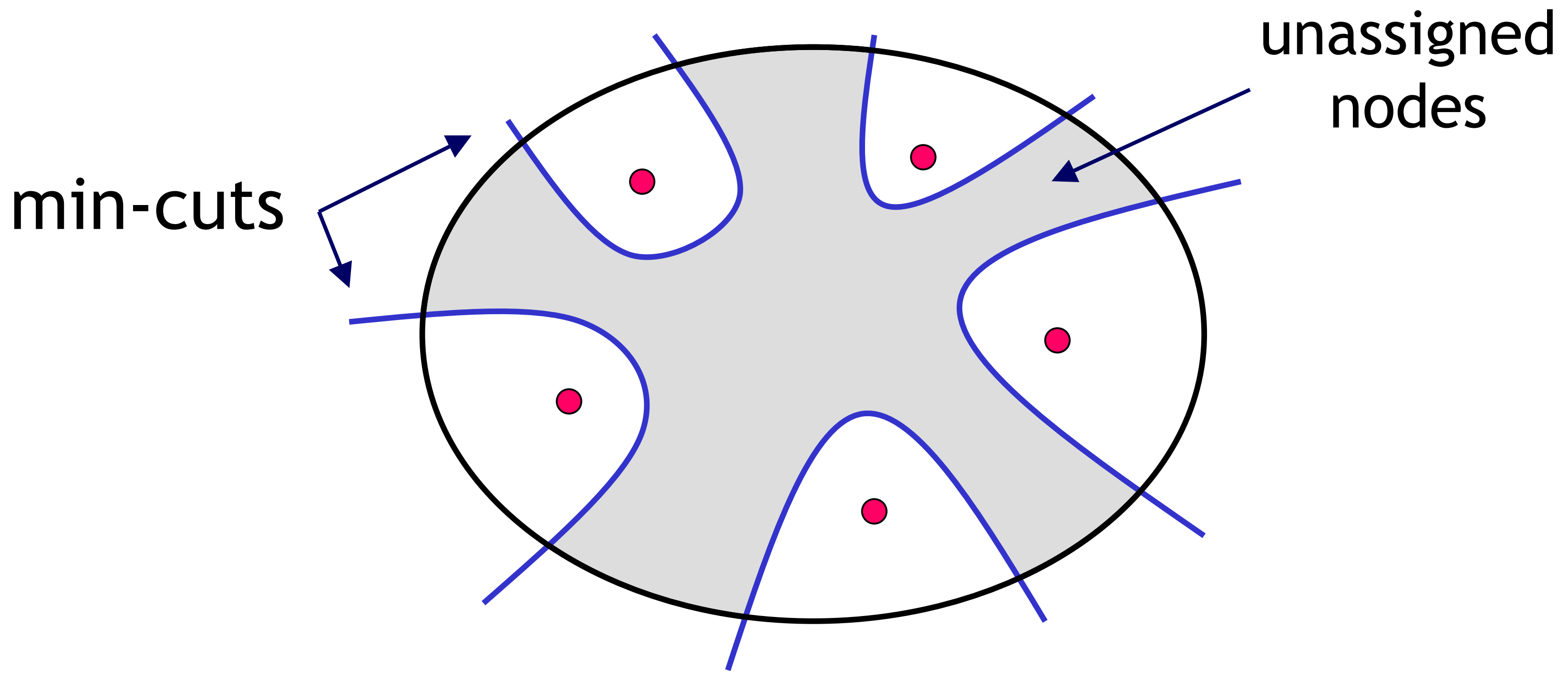
Algorithm for Multiway Cut

[Dahlhaus et al]

- 2-approximation for (sum) multiway cut
- one large cut

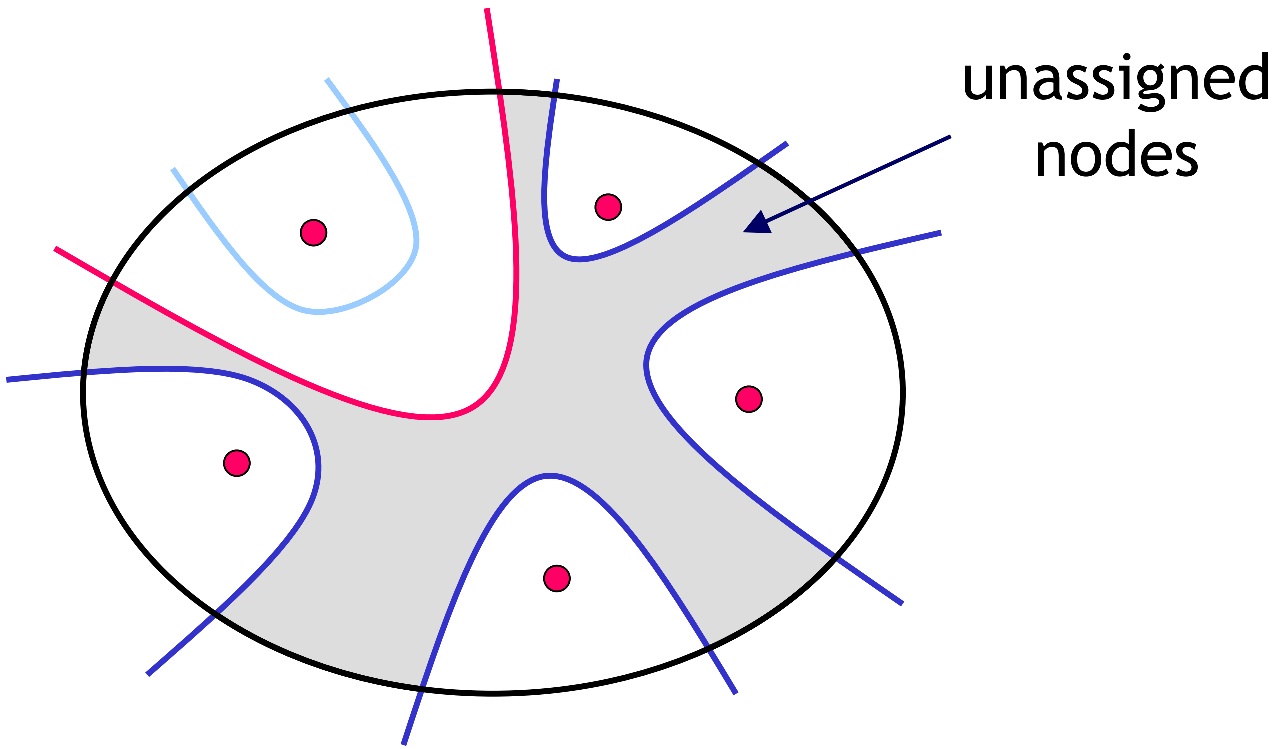


Intuition for our algorithm



Intuition for our algorithm

- Find cuts that take more nodes



Maximum size bounded capacity cut (MaxSBCC)

Given:

Graph G, nodes s,t, bound B

Find:

- s-t cut (S, T)
  - maximize $|S|$
  - capacity $\leq B$

Maximum size bounded capacity cut (MaxSBCC)

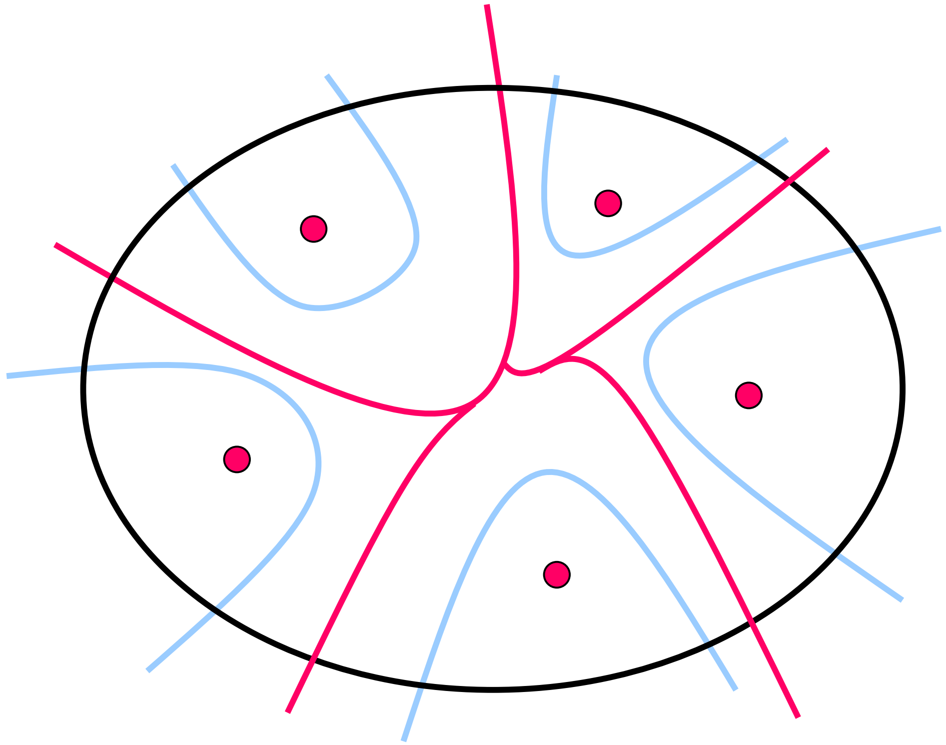
- $O(log^2 n)$ – approximation to find min-capacity cut of given size

[Feige, Krauthgamer, 2002]

$\Rightarrow$ $O(log^2 n)$ bicriteria for MaxSBCC

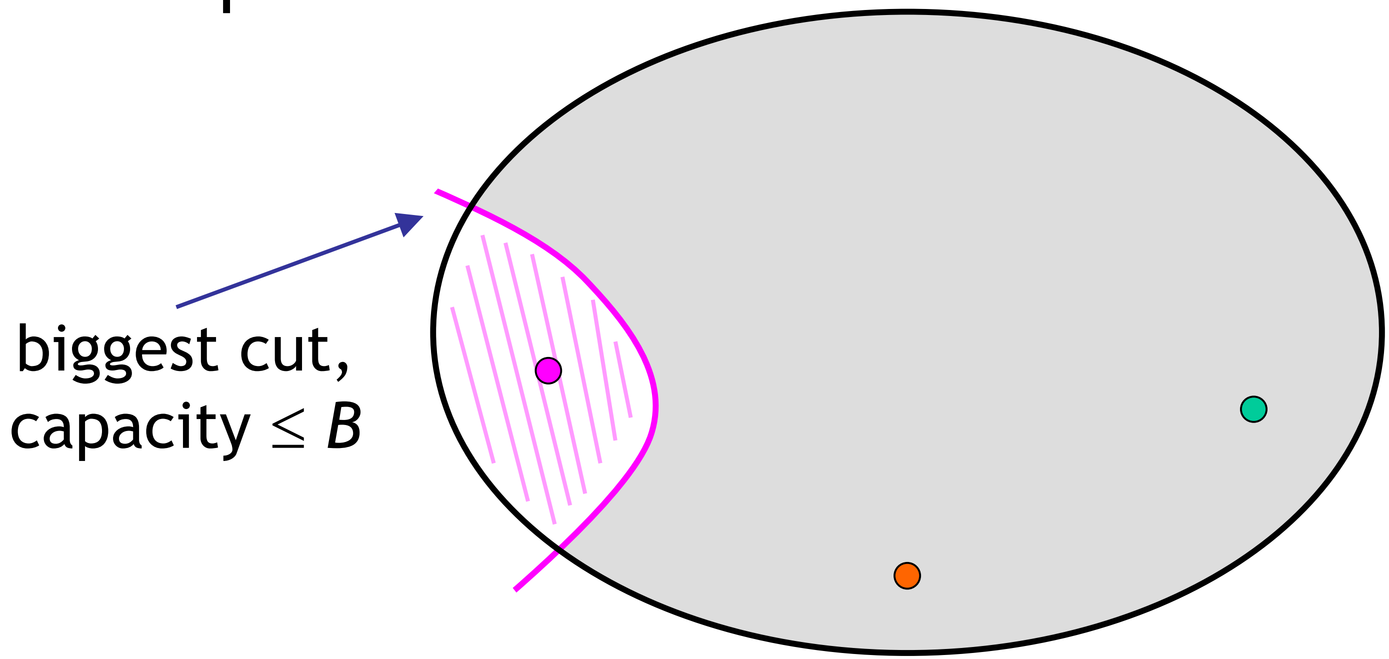
Intuition for our algorithm

- Find maximum-size cuts
- Ideally, disjoint and cover whole graph



The algorithm

B = max capacity of optimal cut



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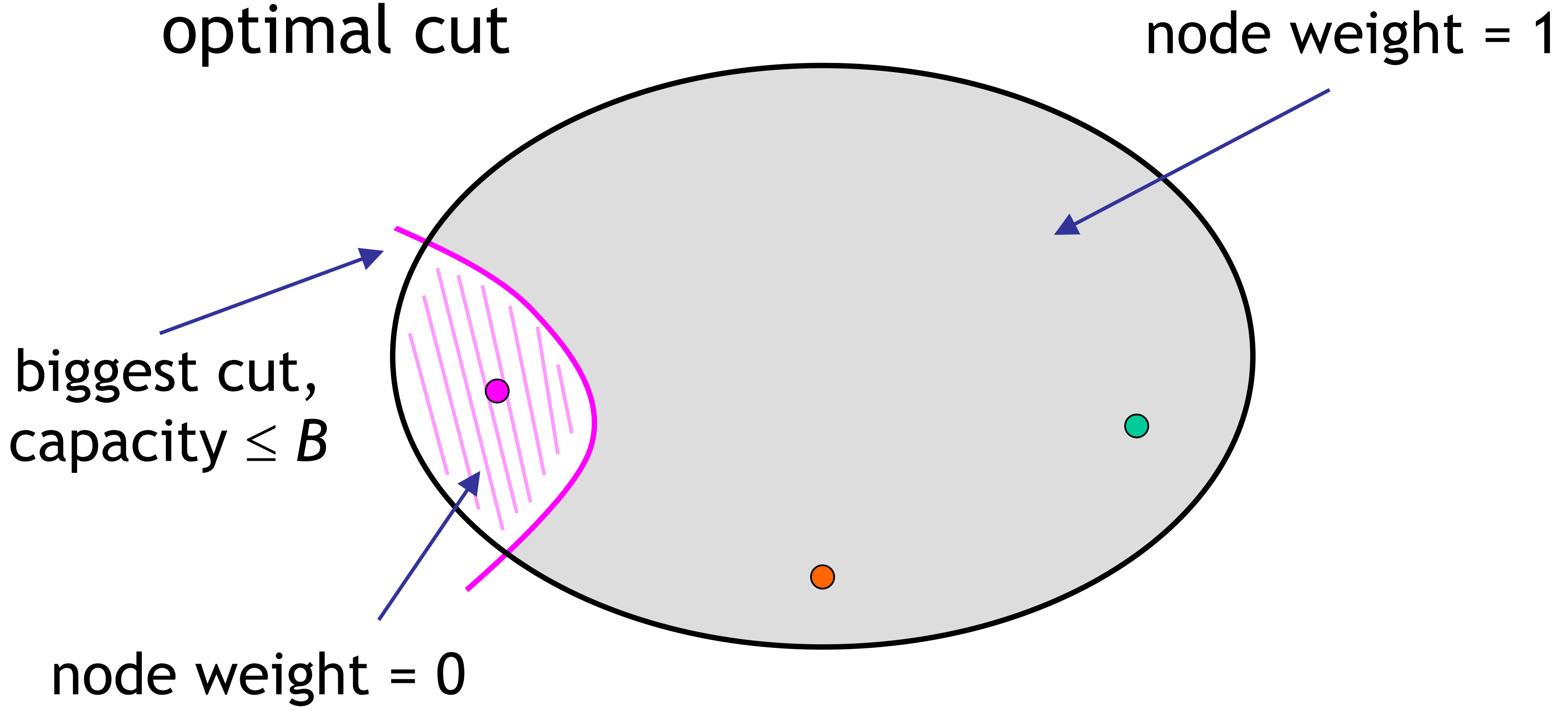

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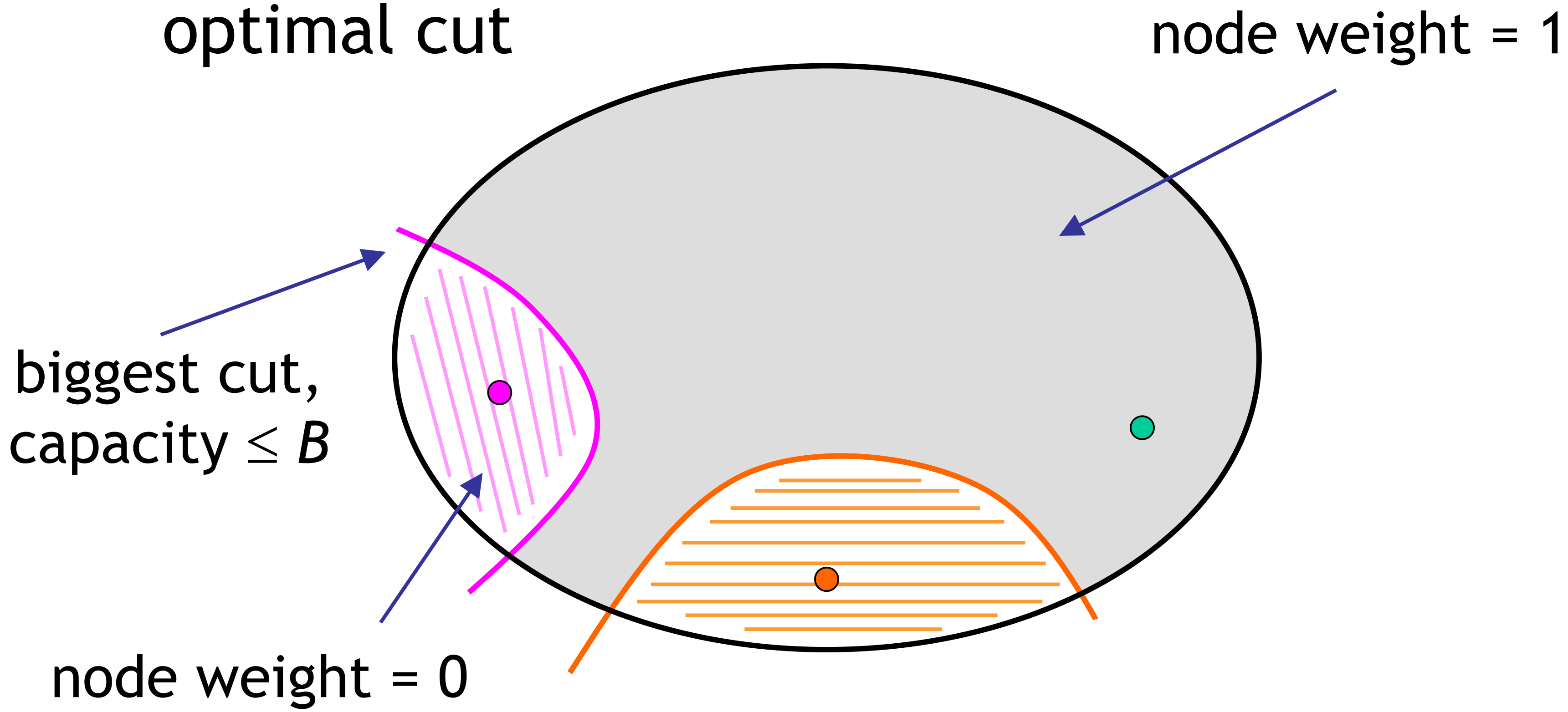

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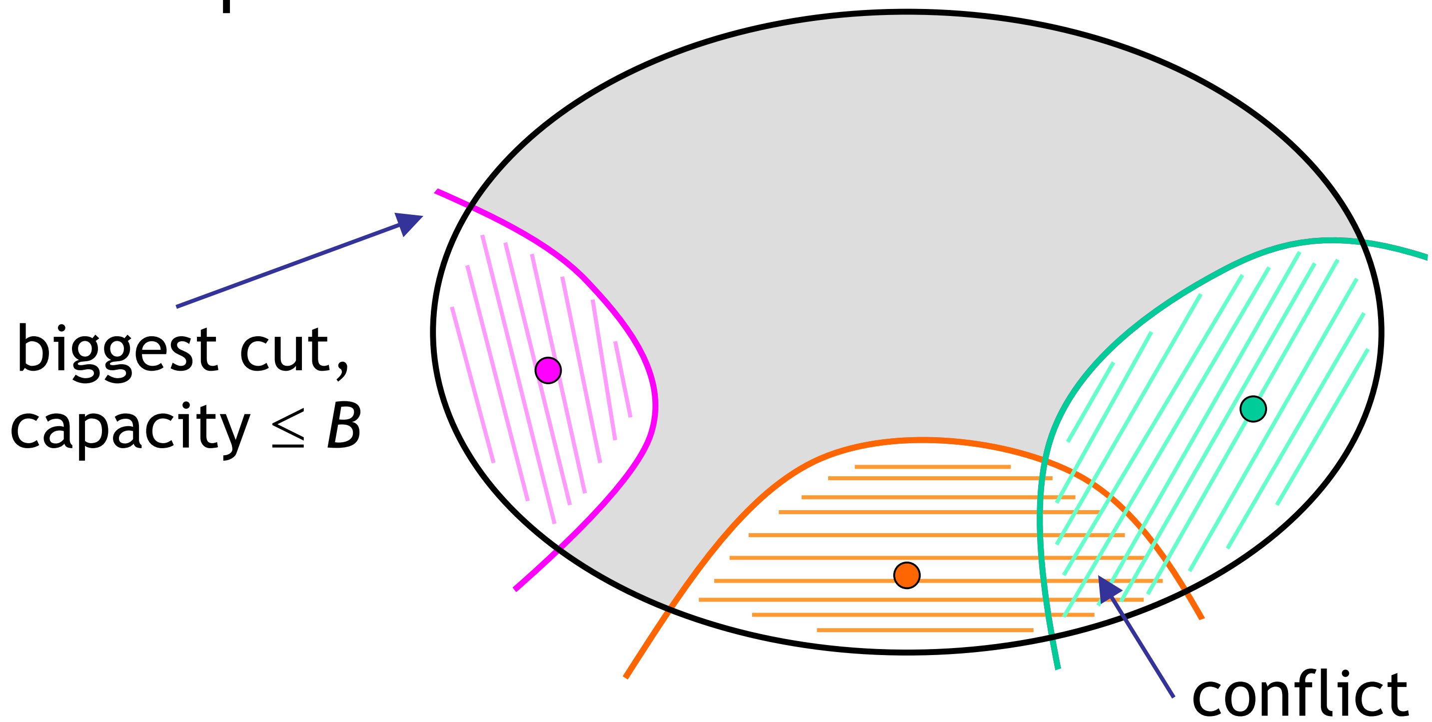

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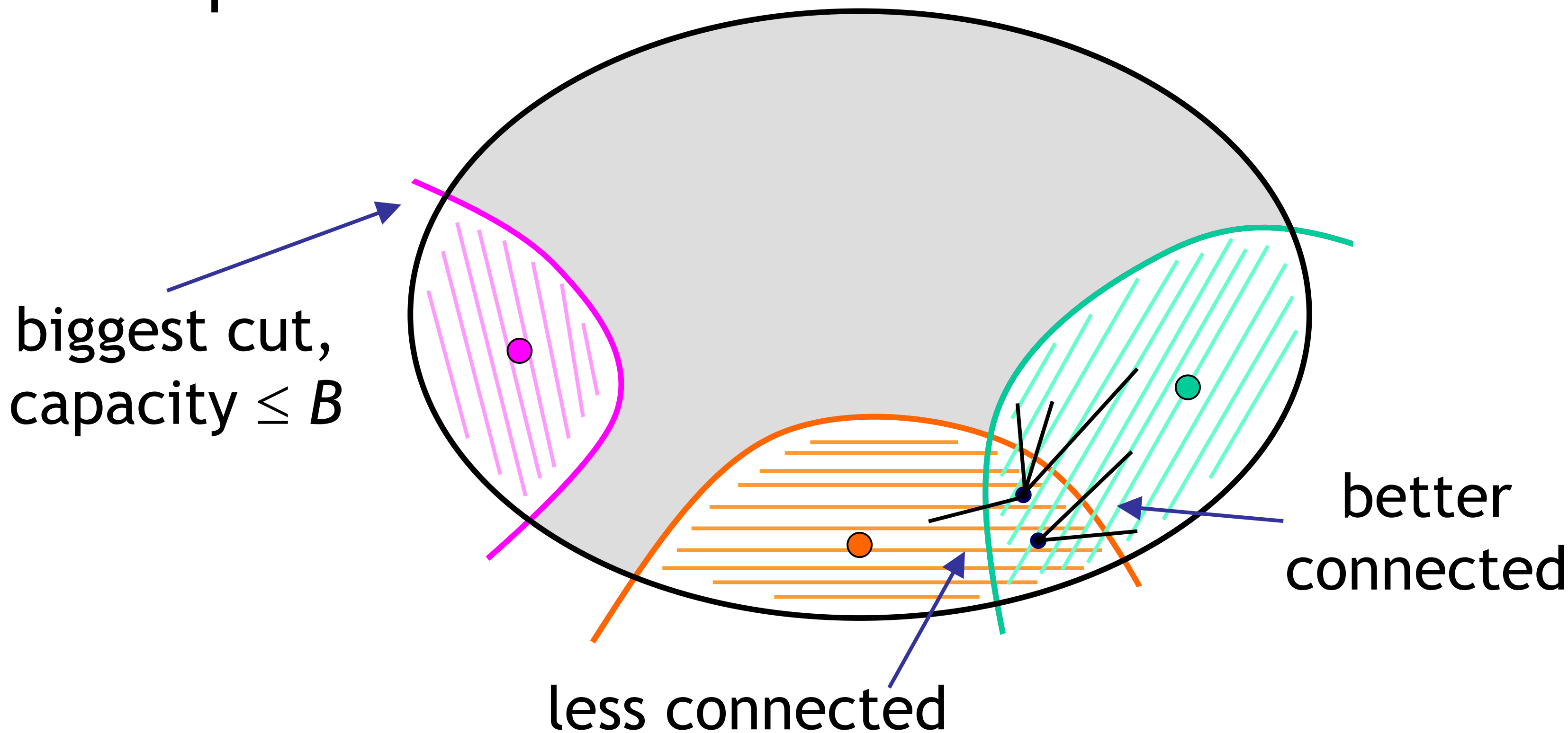

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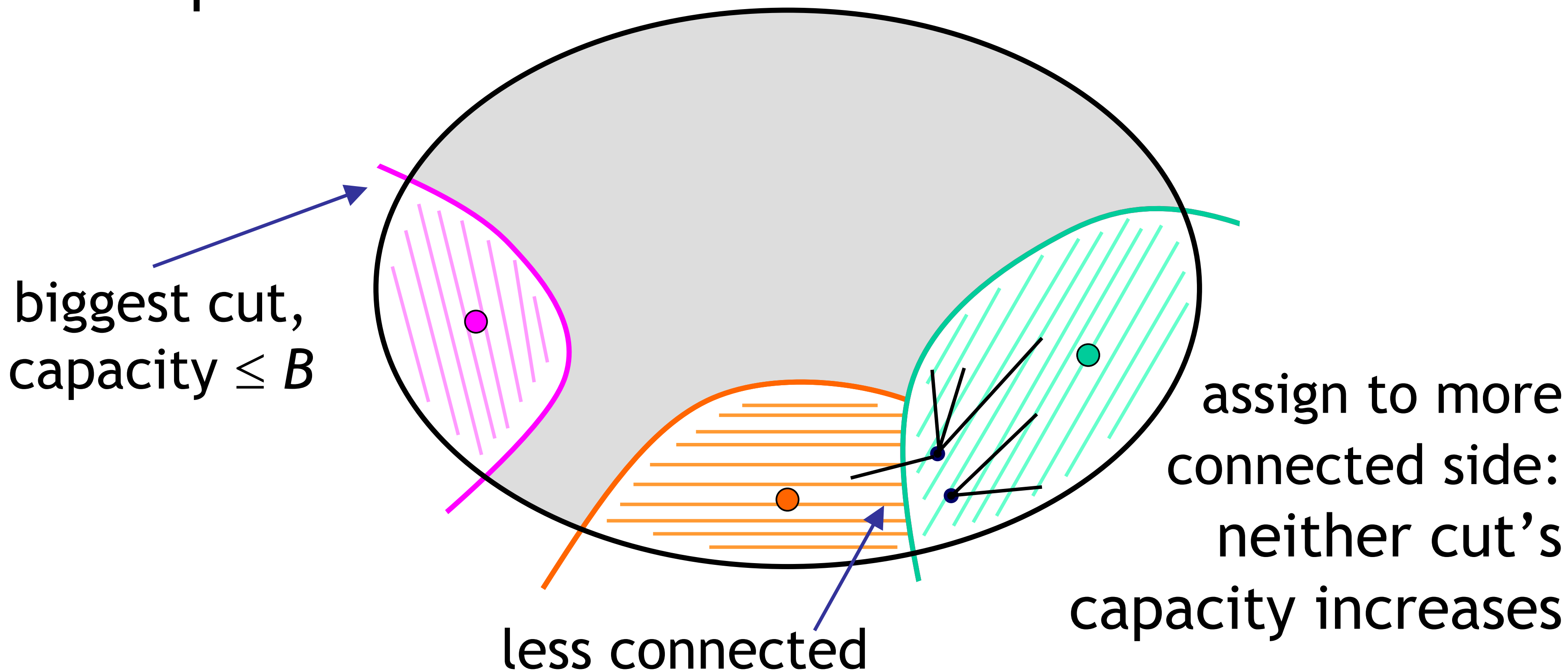
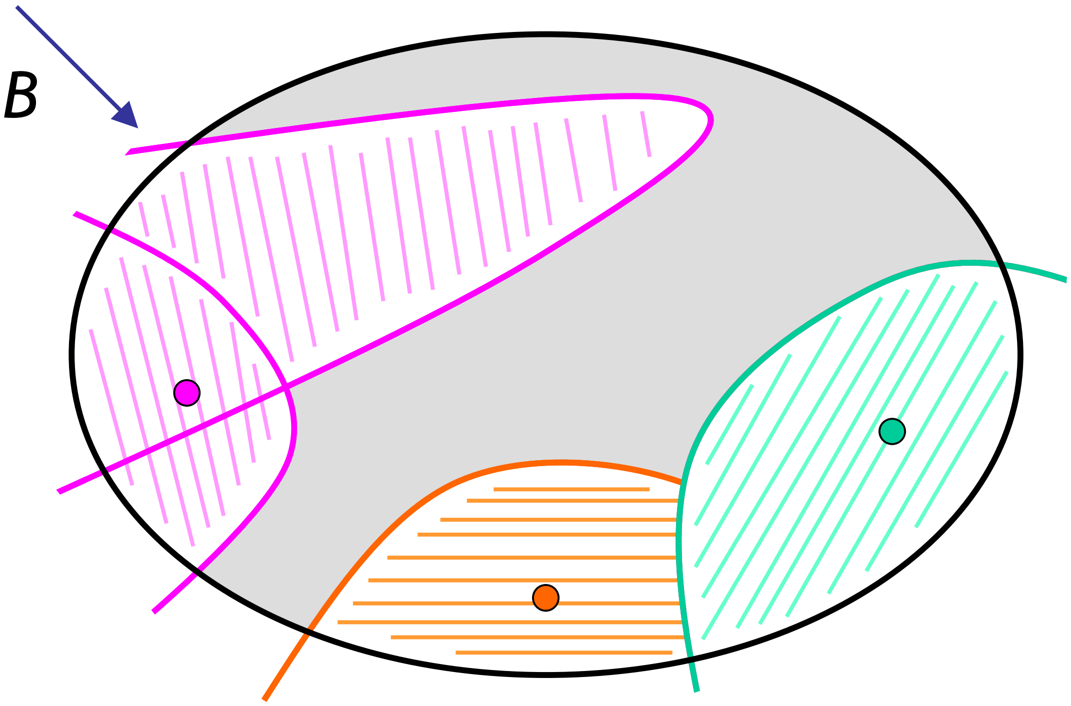


The algorithm

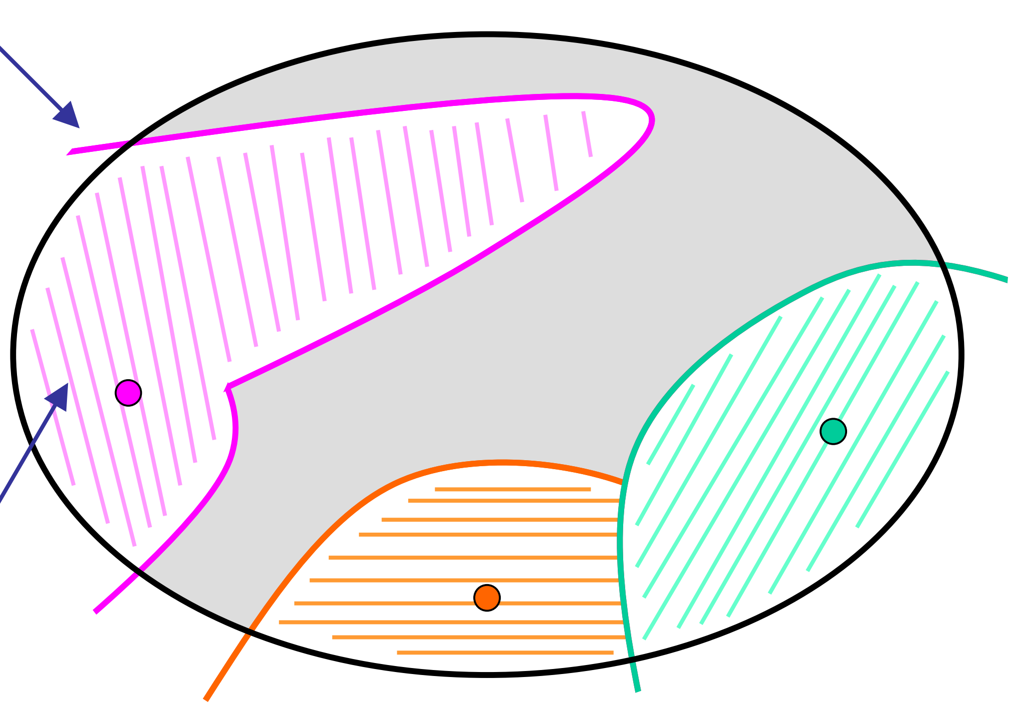
new biggest cut with capacity $\leq B$



The algorithm

new biggest cut with capacity $\leq B$

union over all rounds

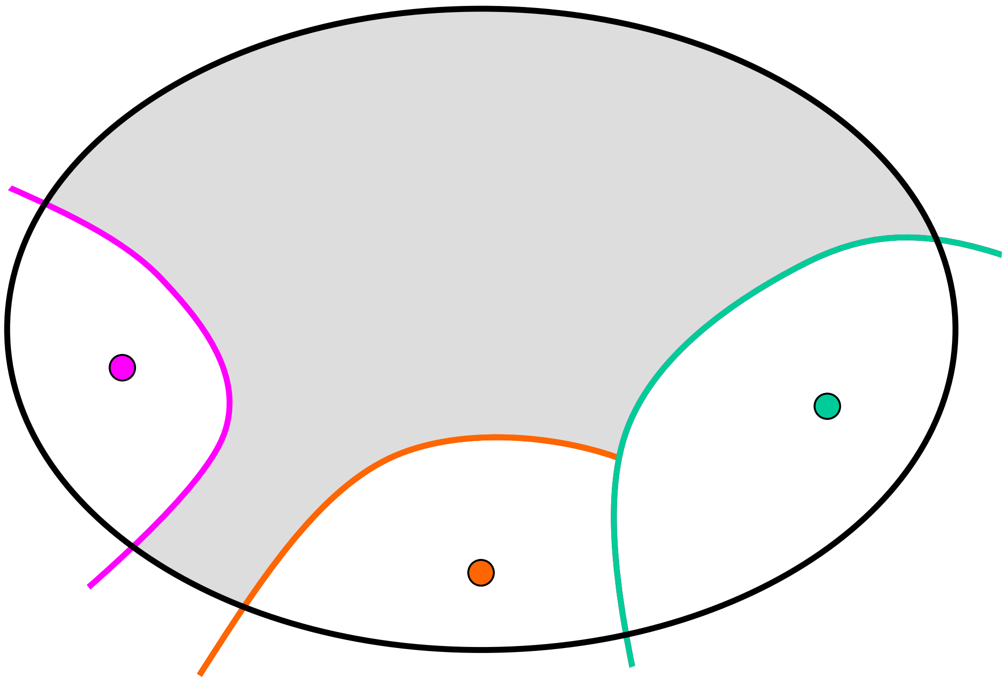


The algorithm

- Guess the optimal value *B*
- Repeat until all nodes are assigned:
  - Find MaxSBCC around each terminal
    - Resolve conflicts
- Output unions of terminals' cuts

Analysis

≥ ½ of remaining nodes are assigned in each round



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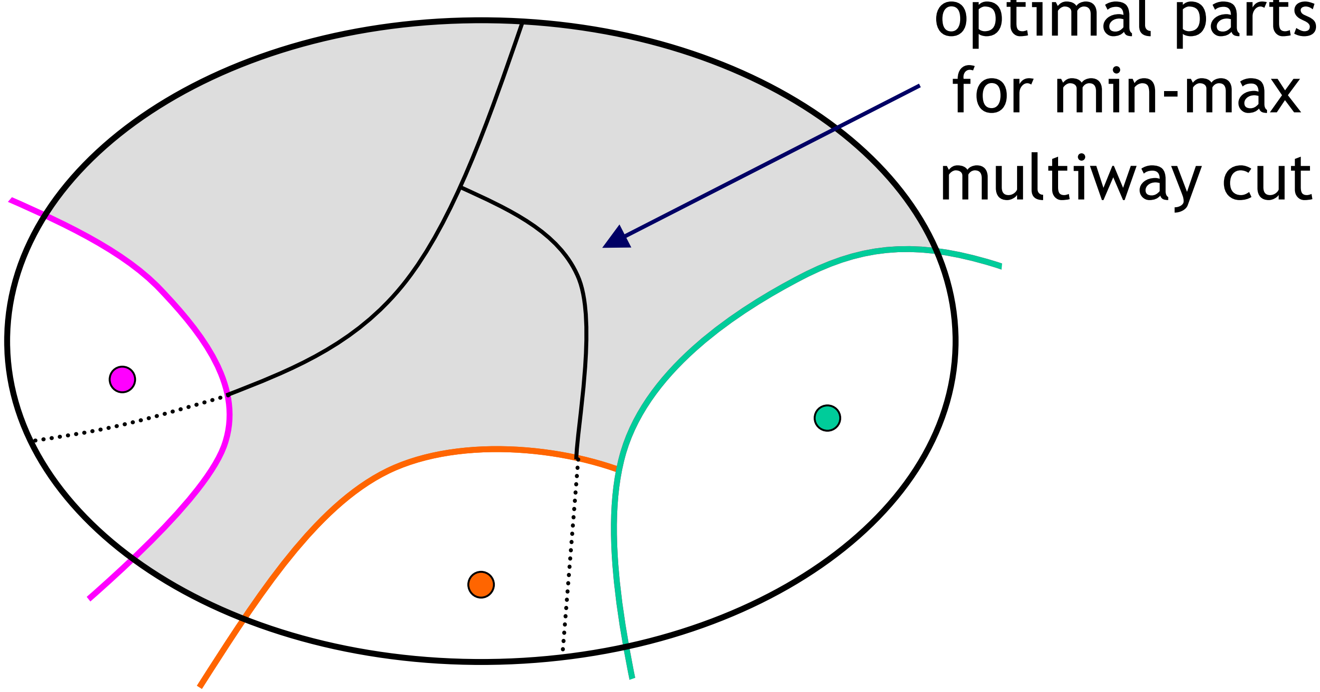

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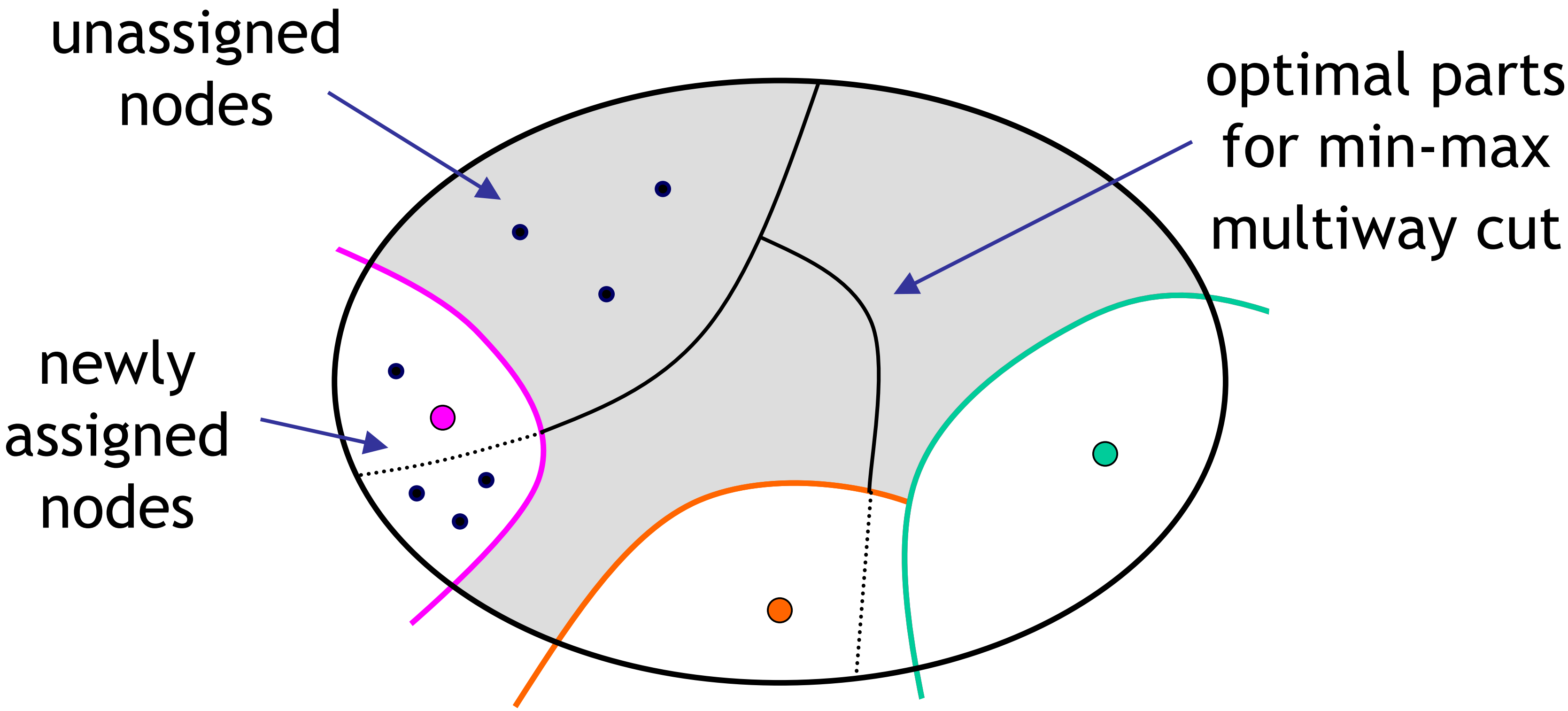
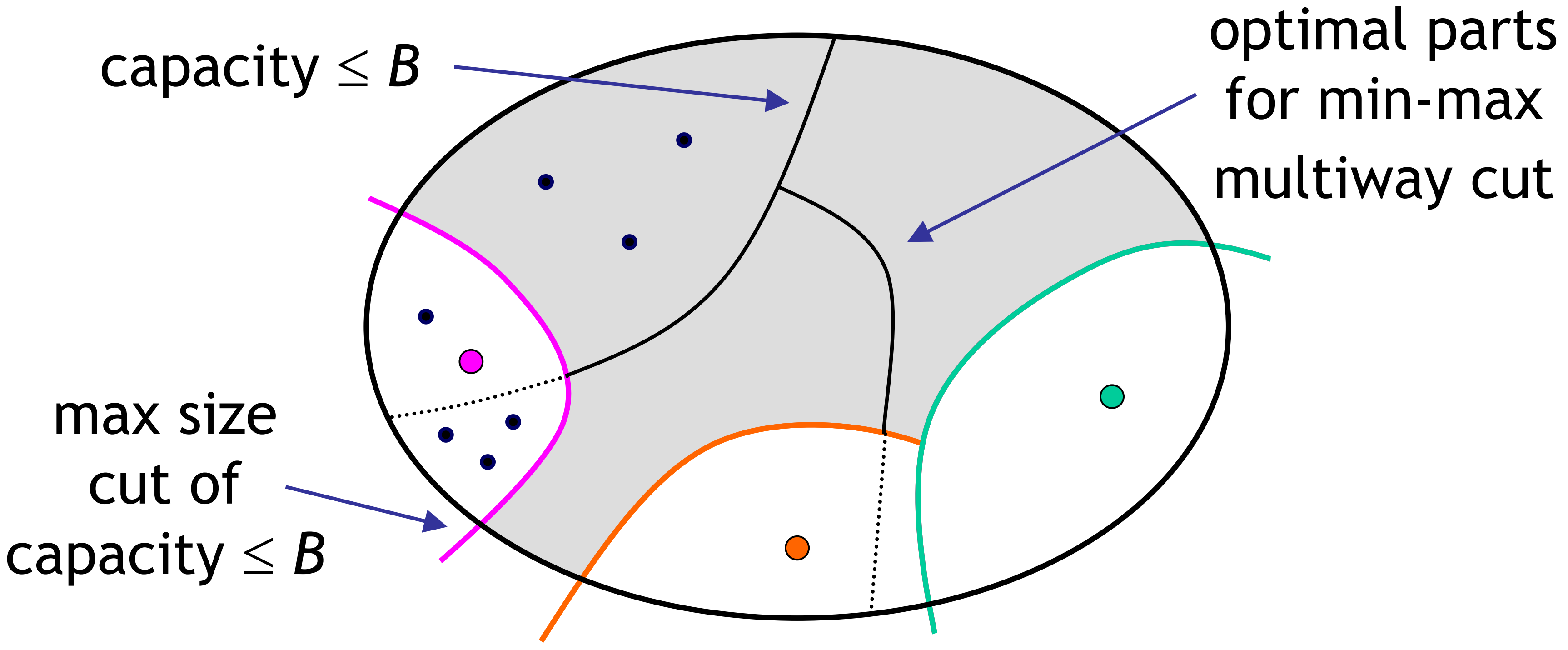

unassigned
nodes
optimal parts
for min-max
multiway cut
newly
assigned
nodes

Analysis

- for each unassigned node, another is assigned



Analysis

- In each round, half of remaining nodes is assigned

$\Rightarrow$ *log n* iterations

$\Rightarrow$ total capacity around each terminal $\leq B \cdot \log n$

Main result

- Combining with $O(log^2 n)$ approximation for MaxSBCC,

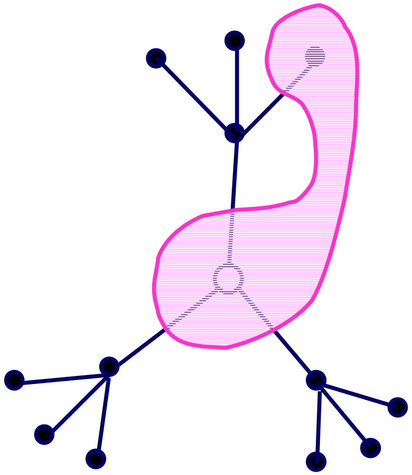
$\Rightarrow$ $O(log^3 n)$ approximation for Min-max multiway cut

Other results

Better approximations for planar graphs

$(2+\varepsilon)$ – approximation for

- trees
- graphs with bounded treewidth



Open problems

- Improved approximations for
  - Max size bounded capacity cut
  - Min-max multiway cut
- Min-max criteria for other graph cut problems