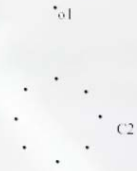


# Enhancing Effectiveness of Outlier Detections for Low Density Patterns

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CI .....



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## Overview

- Previous Outlier Detection Schemes
- The LOF Scheme
- Motivation for an Improved Scheme
- COF Scheme
- Examples
- Summary

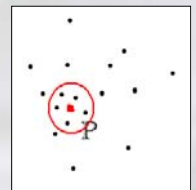
## What is an outlier?

*“An outlier is an observation that deviates so much from other observations as to arouse suspicion that it was generated by a different mechanism.”* Hawkins

- Application of outlier detection would be credit card fraud

## Previous Outlier Detection Schemes

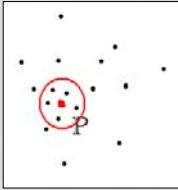
- Clustering
  - Generate outliers as a by-product
  - Outliers are highly dependant on algorithm
- Statistics
  - Examines deviations of individual data objects
  - Assumes prior knowledge of data distribution
- Distance Based Schemes
  - Based on number of other objects in neighborhood
  - More appropriate for detecting outliers w/o previous knowledge
    - DB(n,q)-outlier (Knorr and Ng)
    - (t,k)-nearest neighbor (Ramaswamy et al.)



## LOF Detection Scheme

Density Based Scheme – Local Outlier Factor (LOF)  
(Breunig, et al.)

- Idea of k-distance = set of objects whose distance from point P is not greater than a distance k
- Local reachability density of P = density of P



$$LOF_k(p) = \frac{\sum_{o \in N_{k\text{-distance}(p)}(p)} \frac{Ind_k(o)}{Ind_k(p)}}{|N_{k\text{-distance}(p)}(p)|}$$

- measures how strong an object can be an outlier
- determined by comparing its density with those in its neighborhood

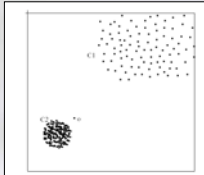


Figure 1: A Data Set showing the strength of LOF

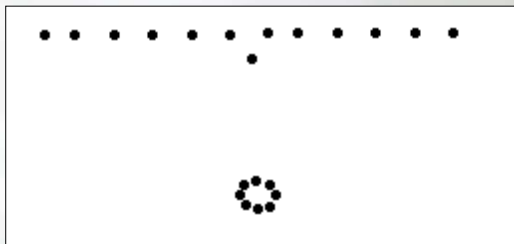
## Weakness Inherent in LOF

- The weakness of LOF is that it may rule out outliers close to some non-outliers pattern that have similar low density



## Connectivity-Based Outlier Factor (COF)

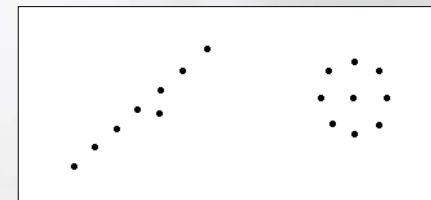
Based on the idea that an outlier does not always need to be of a lower density than a pattern it deviates from.



**The LOF Scheme would not successfully find the outlier!**

## Connectivity-Based Outlier Factor (COF)

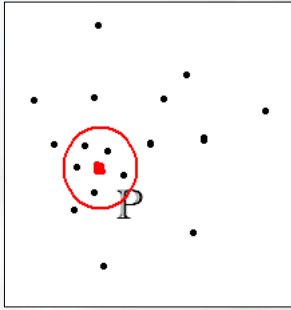
- Differentiates between “low-density” and “isolativity”
  - Low-density = number of objects in a close neighborhood
  - Isolativity = refers to the degree that an object is “connected” to other objects



- Observe that patterns with low density usually exhibit low dimensional structures

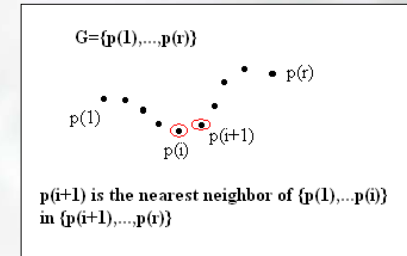
## Connectivity-Based Outlier Factor (COF)

- Definitions...
  - k-nearest neighborhood - radius of a circle encompassing the k nearest objects or points.



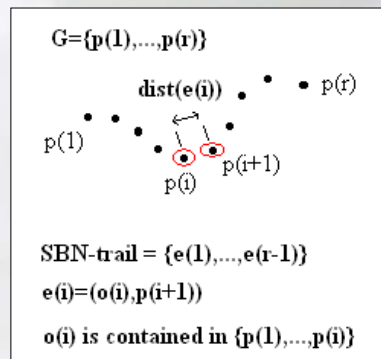
## Connectivity-Based Outlier Factor (COF)

- Definitions...
  - Set based nearest path (SBN-path) – indicates order in which the nearest objects are presented
  - If next item is not unique, impose pre-defined order among its neighbors to break tie



## Connectivity-Based Outlier Factor (COF)

- Definitions...
  - Set based nearest trail (SBN-trail) – a sequence of edges based on the set based nearest path
  - The distances of these edges is called the cost description of the SBN-trail

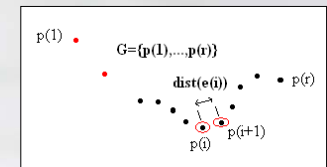


## Connectivity-Based Outlier Factor (COF)

- Definitions...
  - Average Chaining Distance = average of the weighted distances in the cost description of the SBN-trail

$$ac - dist_G(p_1) = \frac{1}{r-1} \cdot \sum_{i=1}^{r-1} \frac{2(r-i)}{r} \cdot dist(e_i)$$

- This means that if edges close to  $p_i$  are larger than those further away, then they contribute more to the average chaining distance

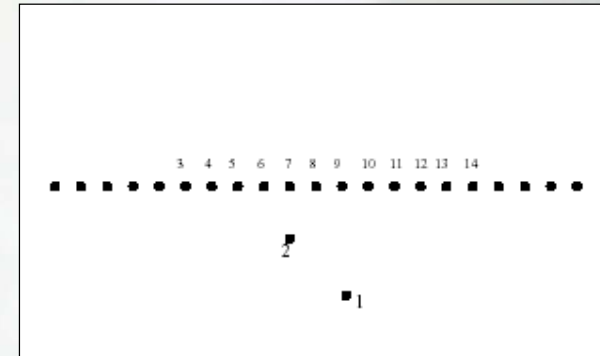


## Connectivity-Based Outlier Factor (COF)

- What we have all been waiting for.... COF!
  - The connectivity-based outlier factor indicates how far away a point shifts from a pattern
  - Compares the point to the points around it to influence the outlier factor

$$COF_k(p) = \frac{|N_k(p)| \cdot ac - dist_{N_k(p)}(p)}{\sum_{o \in N_k(p)} ac - dist_{N_k(o)}(o)}$$

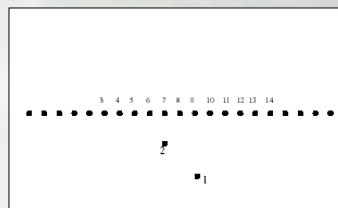
## Example



## Example

- Shows how values shift away from pattern

• K=10



- For Point 1

- $N_k(1) = \{2,9,10,8,11,7,12,6,13,5\}$
- SBN-path =  $\{1,2,7,6,5,8,9,10,11,12,13\}$
- SBN-trail =  $\{(1,2),(2,7),(7,6),(6,5),(7,8),(8,9),(9,10),(10,11),(11,12),(12,13)\}$
- Cost Description =  $\{5,3,1,1,1,1,1,1,1,1\}$
- Average chaining distance = 2.05 (1.46, 0.98)
- $COF(1) = 2.1$  (1.35, 0.96)

## Criticisms

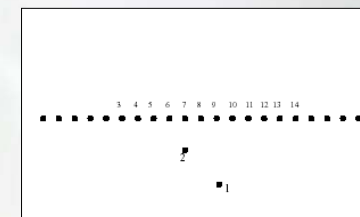
- My calculations did not match calculations in paper...

$$ac - dist_{N_k(1) \cup \{1\}} = 2.05 \quad (2.05)$$

$$ac - dist_{N_k(1) \cup \{1\}} = 1.46 \quad (1.44)$$

$$ac - dist_{N_k(1) \cup \{1\}} = 0.98 \quad (1.04)$$

- Could not complete problem to verify results for COF



## Comparison LOF and COF

- Connectivity based scheme has similar power to the density based scheme in detecting outliers which deviate from high density patterns
- However the connectivity-based scheme can detect outliers in low density patterns

- Introduce idea of ON-COMPATABILITY

$$D = D_o \cup D_n$$

- Not ON-COMPATIBLE IF

$$a \in D_n \text{ and } o \in D_o, \\ \text{such that } f(o, S) \leq f(a, S)$$

## Example 2

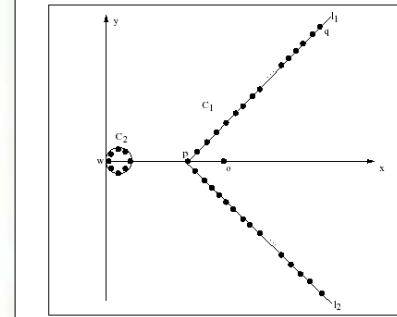


Figure 4: Data Set for Comparison

LOF: (outliers w, o)

for  $k = 1$  to  $7$ :  $\text{LOF}_k(q) > \text{LOF}_k(w)$

for  $k = 8$  to  $99$ :  $\text{LOF}_k(q) > \text{LOF}_k(o)$

for  $k = 99$ :  $\text{LOF}_k(p) > \text{LOF}_k(w)$

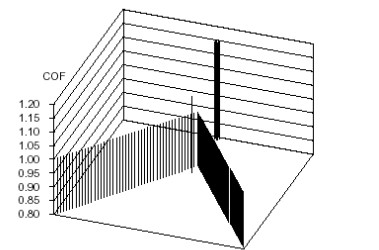


Figure 6: COF Values of All Points When  $k=13$

## Time Complexity

- COF can be split into two sections
  1. Find  $k$ -nearest neighborhoods and SBN-trails
    - $O(n)$  for low dim. data to  $O(n^2)$  for high dim. data
  2. Compute the COF
    - $O(n)$

## More Criticisms...

- Paper does not present examples where the outlier is of similar density as the low density pattern.
- Examples are not fully developed.
- Method is not tested for a wide variety of patterns.
- Paper does not discuss in detail the effect of the choice  $k$

## Conclusions

- By separating the idea of density from isolativity, outliers can be detected in low density patterns thus achieving better results than LOF

Thank you!

Questions?