# Overview of Dual Miner 

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## Problem Statement

"Find all frequent itemsets whose total price is at least $\$ 50$."

- Constraining mining improves its speed and usefulness.
- Many practical constraints can be expressed as a conjunction of monotone and antimonotone predicates.
- Other constraints can be approximated this way.

Dual Miner finds frequent itemsets by leveraging monotone and antimonotone constraints at the same time.

## Related Work

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- Other ways to solve the problem:
- Run existing algorithm twice and intersect.
- Run existing algorithm and post-process.
- Melish's Algorithm

All of these require two distinct phases.

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- Other types of constraints:
- Succinct
- Convertible


## Dual Miner has to offer...

- First algorithm to leverage $P()$ and $Q()$ simultaneously
- Extreme flexibility
- Non-trivial optimizations
- New issues
- Nice summary of analytical properties

Definition

- $P()$ is a conjunction of antimonotone predicates.
- $Q()$ is a conjunction of monotone predicates.


## Key Ideas

- Items have attributes and values
- Monotone and antimonotone predicates e.g. sum $($ price $(X))>30$ or $10<\operatorname{support}(X)<100$
- Join predicates of same type
- Approximate other types of constraints
- Different predicates have different cost e.g. support vs. sum


## More Key Ideas

- Trimming values near top or bottom removes many nodes
- Duality

Find all frequent itemsets


D is OUT because it is not frequent.

## More Key Ideas

- Trimming values near top or bottom removes many nodes
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Find all infrequent itemsets

$D$ is $I N$ because $\sim D(A B C)$ is frequent.

## Subalgebras

- don't care about support - look for MFI
- ... but all subsets of MFI may not satisfy $Q()$

How can we represent portions of the result space?

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- don't care about support - look for MFI
- ... but all subsets of MFI may not satisfy Q()

How can we represent portions of the result space?
Any set of itemsets closed under $\cup$ and $\cap$ can be expressed as a subalgebra.

- A subalgebra consists of a bottom set and a top set.
- All itemsets in a subalgebra contain all of the items in the bottom set,
- and only items from the top set.
- All members of a good subalgebra satisfy P() and Q() .


## Subalgebras Con't

For example, the subalgebra ( $\{A\},\{A B C\}$ ) contains the elements:

- A
- $A B$
- AC
- ABC


## $\max (X$. price $)<4 \wedge \min (X$. price $)<2$

| Item | Cost |
| :--- | :--- |
| A | 1 |
| B | 4 |
| $C$ | 3 |
| $D$ | 2 |

- $\mathrm{P}(\mathrm{X})=\max (X$. price $)<4$
- $Q(X)=\min (X$.price $)<2$
- $P(B)$ is false, therefore $B$ is OUT
- $Q(\sim A)=Q(B C D)$ is false, therefore $A$ is $I N$


## $\max ($ X. price $)<4 \bigwedge \min (X$. price $)<2$

- $P(B)$ is false, therefore $B$ is OUT
- $Q(\sim A)=Q(B C D)$ is false, therefore $A$ is $I N$

This leads to the subalgebra ( $\{\mathrm{A}\},\{\mathrm{ACD}\}$ ), which satisfies P() and $Q()$.


## Towards the Basic Algorithm 1/2

As usual, we will traverse nodes in a tree.

- Each node contains IN, OUT, and CHILD sets, which correspond to the subalgebra ( IN, ~OUT ).
- This is a good subalgebra if $P(\sim O U T) \wedge Q(I N)$.


## Towards the Basic Algorithm 2/2

## Dual Miner

- Start with the root node; it is undetermined.
- Repeatedly pick an undetermined node:
- Optionally, move children to OUT, if $\mathrm{P}(\mathrm{IN} \cup$ child) fails or to IN , if $\mathrm{Q}(\sim($ OUT $\cup$ child $))$ fails
- Optionally, check if node is a good subalgebra.
- Pick one child element to split on, and create two child nodes.
- The node is now determined.


## support $(X) \geq 1 \bigwedge$ total_price $(X)>50$

| Item | Cost |  |
| :--- | :--- | :--- |
| A | 26 |  |
| B | 26 | Transactions |
| C | 1 | ABCD |
| D | 1 | E |
| E | 100 |  |

Root Node: ( $\},\{\mathrm{ABCDE}\},\{ \})$

- $P(X)$ true for all elements $X$
- $\mathrm{Q}(\sim \mathrm{X})$ true for all elements X
- ( $\},\{$ ABCDE $\})$ is not a good subalgebra
- ... therefore pick a child (say E) to split on


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Undetermined Nodes:

- $\beta:(\{E\},\{A B C D\},\{ \})$
- $\gamma:(\{ \},\{A B C D\},\{E\})$


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Current Node: $\beta$ : ( $\{\mathrm{E}\},\{\mathrm{ABCD}\},\{ \}$ )

- $\mathrm{P}(\mathrm{EA})$ is false, so A is OUT
- $\beta$ becomes ( $\{\mathrm{E}\},\{\mathrm{BCD}\},\{\mathrm{A}\}$ )


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Current Node: $\beta$ : ( $\{\mathrm{E}\},\{\mathrm{ABCD}\},\{ \}$ )

- $P(E A)$ is false, so $A$ is OUT
- $\beta$ becomes ( $\{E\},\{B C D\},\{A\})$
- $P(E B), P(E C), P(E D)$ are all false too
- $\beta$ becomes ( $\{\mathrm{E}\},\{ \},\{\mathrm{ABCD}\}$ )


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- $P(E B), P(E C), P(E D)$ are all false too
- $\beta$ becomes ( $\{E\},\{ \},\{A B C D\})$
- ( $\{E\},\{E\})$ is a good subalgebra


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Current Node: $\gamma$ : ( $\},\{\mathrm{ABCD}\},\{\mathrm{E}\}$ )

- $\mathrm{Q}(\sim(\mathrm{EA}))$ is false, so A is IN
- $\gamma$ becomes ( $\{\mathrm{A}\},\{B C D\},\{E\}$ )


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| :--- | :--- | :--- |
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Current Node: $\gamma:(\{ \},\{\operatorname{ABCD}\},\{\mathrm{E}\})$

- $Q(\sim(E A))$ is false, so $A$ is $I N$
- $\gamma$ becomes ( $\{A\},\{B C D\},\{E\}$ )
- $Q(\sim(E B))$ is false, so $B$ is $I N$
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- $\gamma$ becomes ( $\{A B\},\{C D\},\{E\}$ )
- ( $\{A B\},\{A B C D\})$ is a good subalgebra


## support $(X) \geq 1 \wedge$ total_price $(X)>50$

In Summary, the good subalgebras were:

- ( $\{\mathrm{E}\},\{\mathrm{E}\}$ )
- ( $\{\mathrm{AB}\},\{\mathrm{ABCD}\})$



## Finishing Touches

- Don't store IN, OUT, and CHILD; just store new_in and new_out for each node.
- Interleave pruning with P() and Q() ; each creates opportunities for the other.


## Heuristics

Dual Miner is flexible; it can be tuned for the problem at hand:

- traversal strategy
- pruning order heuristics
- stop heuristics
- choice order heuristics
- control heuristics


## Heuristics

Dual Miner is flexible; it can be tuned for the problem at hand:

- traversal strategy
- pruning order heuristics
- stop heuristics
- choice order heuristics
- control heuristics
- don't prune with P() unless IN has changed
- don't prune with Q() unless OUT has changed


## Leveraging the Underlying Algorithm

## Goal: Minimize the cost of executing $P()$ and $Q()$

- $Q()$ is often much cheaper
- existing algorithms minimize calls to P() in clever ways reuse them!
- dual situation may occur as well


## Extend MAFIA HUTMFI...

- partial list of maximum itemsets which satisfy P()
- partial list of minimum itemsets which satisfy $Q()$


## Dual HUT

Testing $\mathrm{P}(\sim \mathrm{OUT})$ may be expensive.

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Alternative:

- Examine a sequence of nodes where new children are speculatively added to IN, until you reach a leaf.
- The oldest ancestor in the chain that satisfies $Q()$ is a good subalgebra.
- Don't evaluate any of its children.


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- The oldest ancestor in the chain that satisfies $Q()$ is a good subalgebra.
- Don't evaluate any of its children.

Also:

- We don't need to evaluate $P()$ for any nodes below the top of the chain, even if that node doesn't satisfy $Q()$.
- This applies in the dual case also.

Seek out complete left and right chains.

## Subalgebra Fragmentation

## Problem

If Dual Miner splits on the wrong child nodes, it can split good subalgebras.

Mitigation

- Heuristics can help.
- Keep track of good subalgebras and merge them on the fly.


## Approximations

## Problem

Dual Miner only handles monotone and antimonotone constraints.
Mitigation
A strategy for approximating mean-like functions is proposed.

- average $(X)<$ constant


## Theoretical Evaluation

Summary:

- Same (weak) upper bound on Dual Miner / Apriori and Dual Miner / MAFIA
- P() more selective than Q()$\rightarrow$ CONVERTIBLE wins


## COVERTIBLE

Run Apriori and post-process, but don't test $Q()$ on a superset of something that already passed.

## Emprical Evaluation

- Assume P() costs $100 \times \mathrm{Q}()$
- Test vs. synthetic data
- Observation: Other algorithms operate in two phases
- Competitors
- CONVERTIBLE
- MAFIA + free second phase
- Champions
- Dual Miner
- Dual Miner with good choice order heuristic

Dual Miner wins when $Q()$ is sufficiently selective.

