



# PREDICTING THE EFFECTIVENESS OF BIDIRECTIONAL HEURISTIC SEARCH

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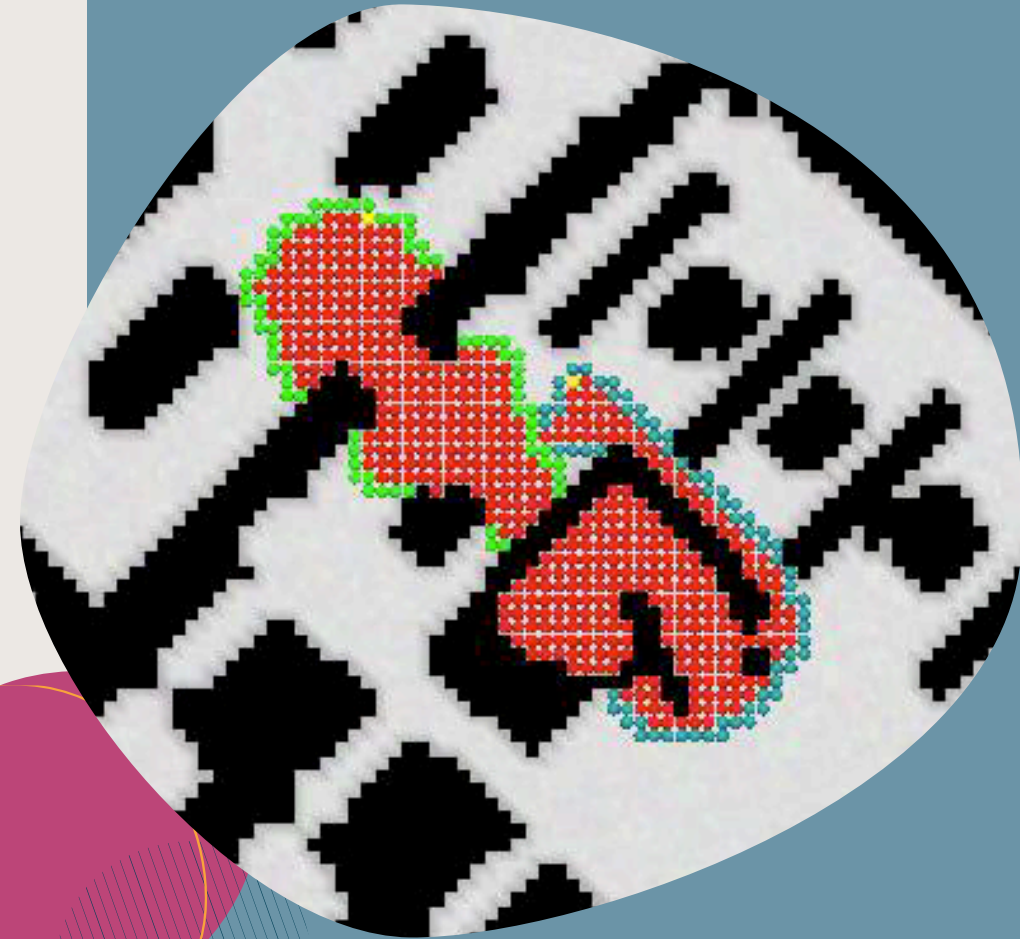
Jingwei Chen, University of Alberta



# BIDIRECTIONAL SEARCH

**Q: When does bidirectional (heuristic) search perform well?**

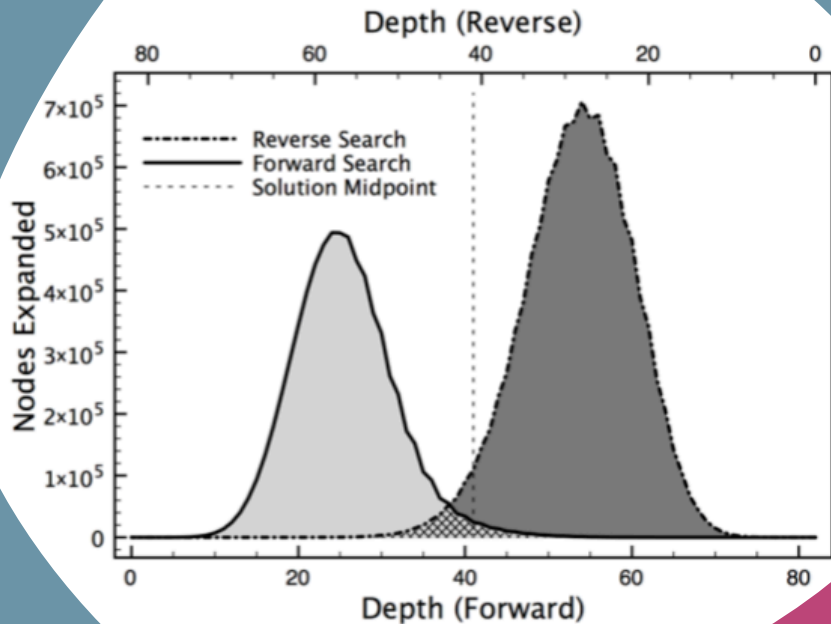
**A: Performance of bidirectional search is positively correlated with the number of states that have heuristics that are both low and inaccurate.**



# BARKER AND KORF (2015)

A strong heuristic expands a majority of the states in the first half of the search.

**BK2: Unidirectional search outperforms bidirectional search with a strong heuristic.**



**Start**

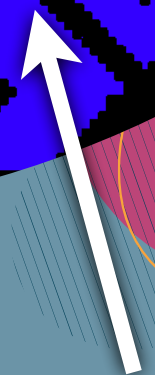


## HOLTE ET AL (2017)

**MM is guaranteed to meet in the middle.**

**If  $|FF| > |RN|$ ,  $A^*$  will expand more states than MM if the heuristic is weak, fewer if the heuristic is accurate.**

**Goal**



**Start**

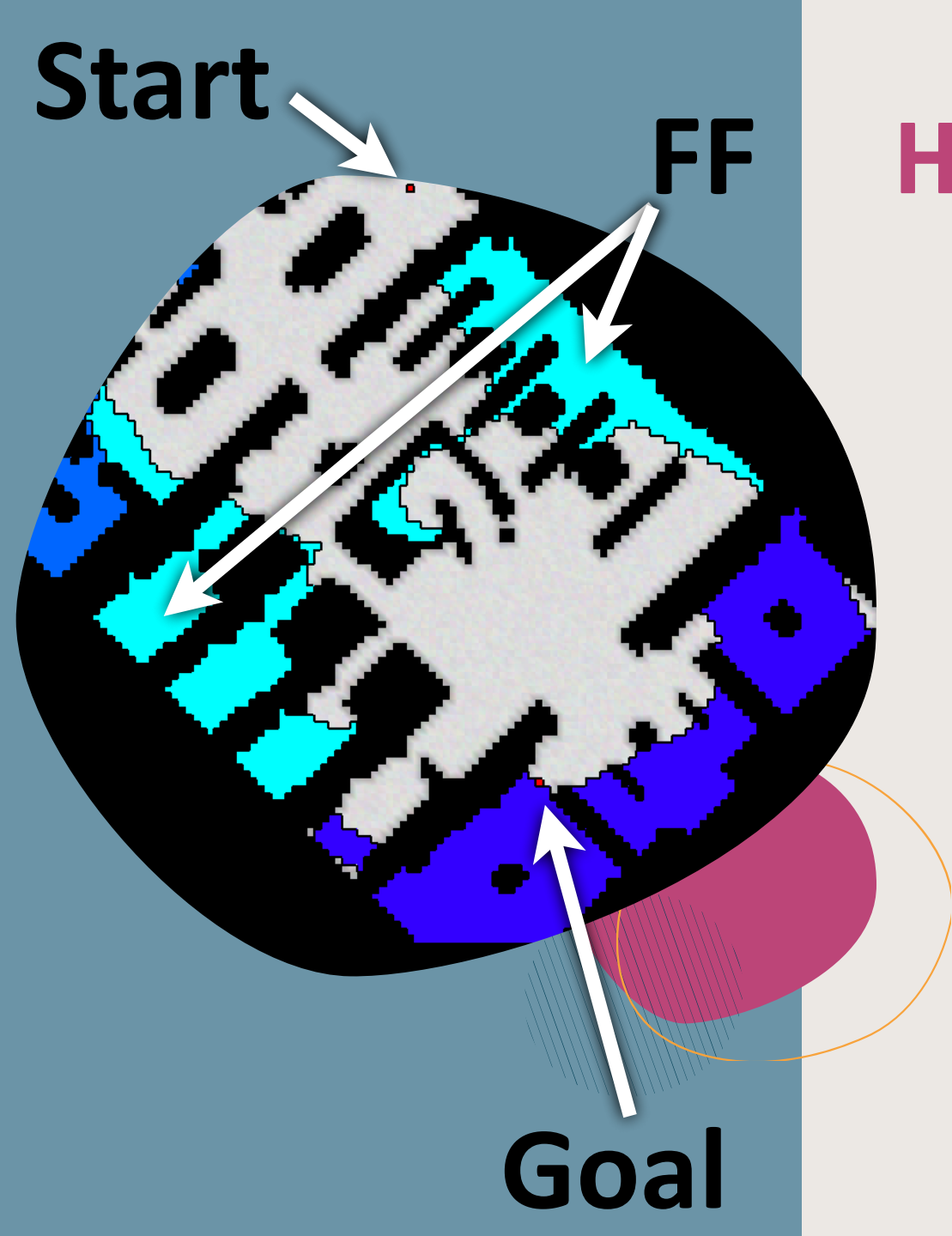
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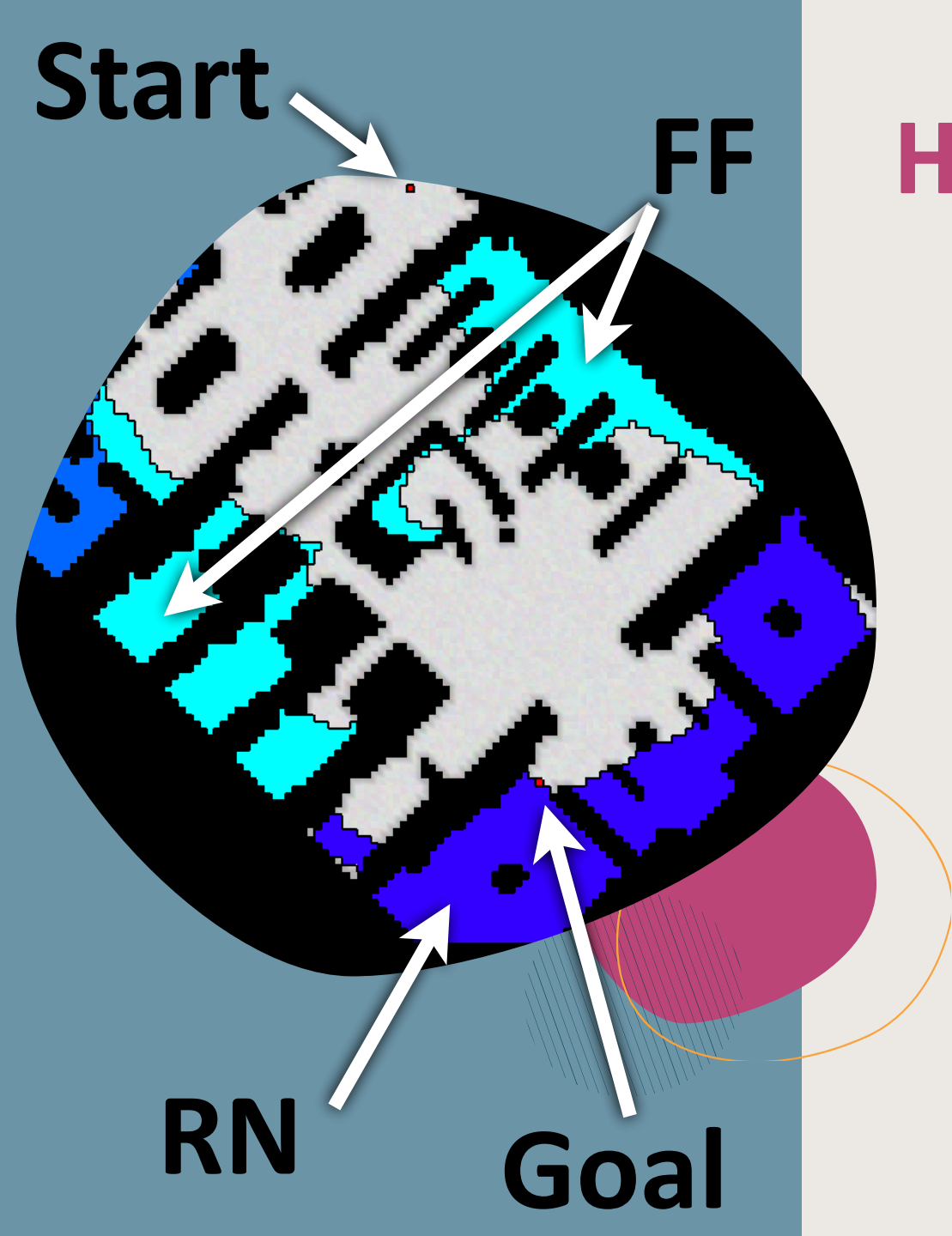
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# ECKERLE ET AL (2017)

## Front-to-end bidirectional search

- admissible heuristic

- not necessarily consistent

Corresponds to finding a minimum vertex cover on a bipartite graph

**Theorem 6.** Let  $I = (G, h_F, h_B) \in I_{CON}$  have an optimal solution cost of  $C^*$ . If  $U$  is an optimal forward path and  $V$  is an optimal backward path such that  $U_0 = \text{start}$ ,  $V_0 = \text{goal}$ , and:

- (1)  $f_F(U) < C^*$
- (2)  $f_B(V) < C^*$
- (3)  $c(U) + c(V) < C^*$

then, in solving problem instance  $I$ , any admissible DXBB bidirectional front-to-end search algorithm must expand  $(\text{end}(U), \text{end}(V))$ .

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**Necessary expansions for a pair of states:**

$$f_f(a) < C^*$$

$$f_b(b) < C^*$$

$$g_f(a) + g_b(b) < C^*$$



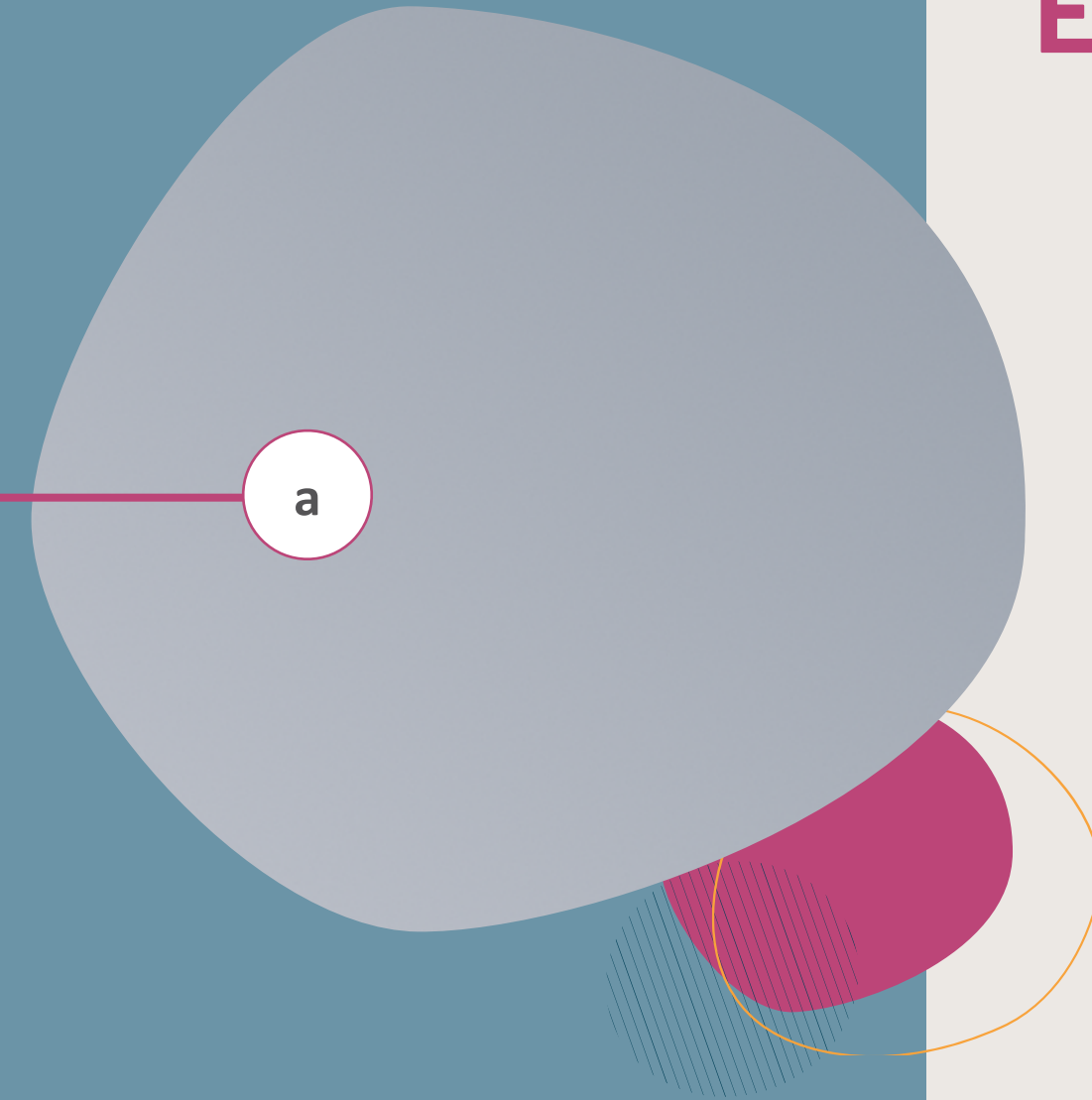
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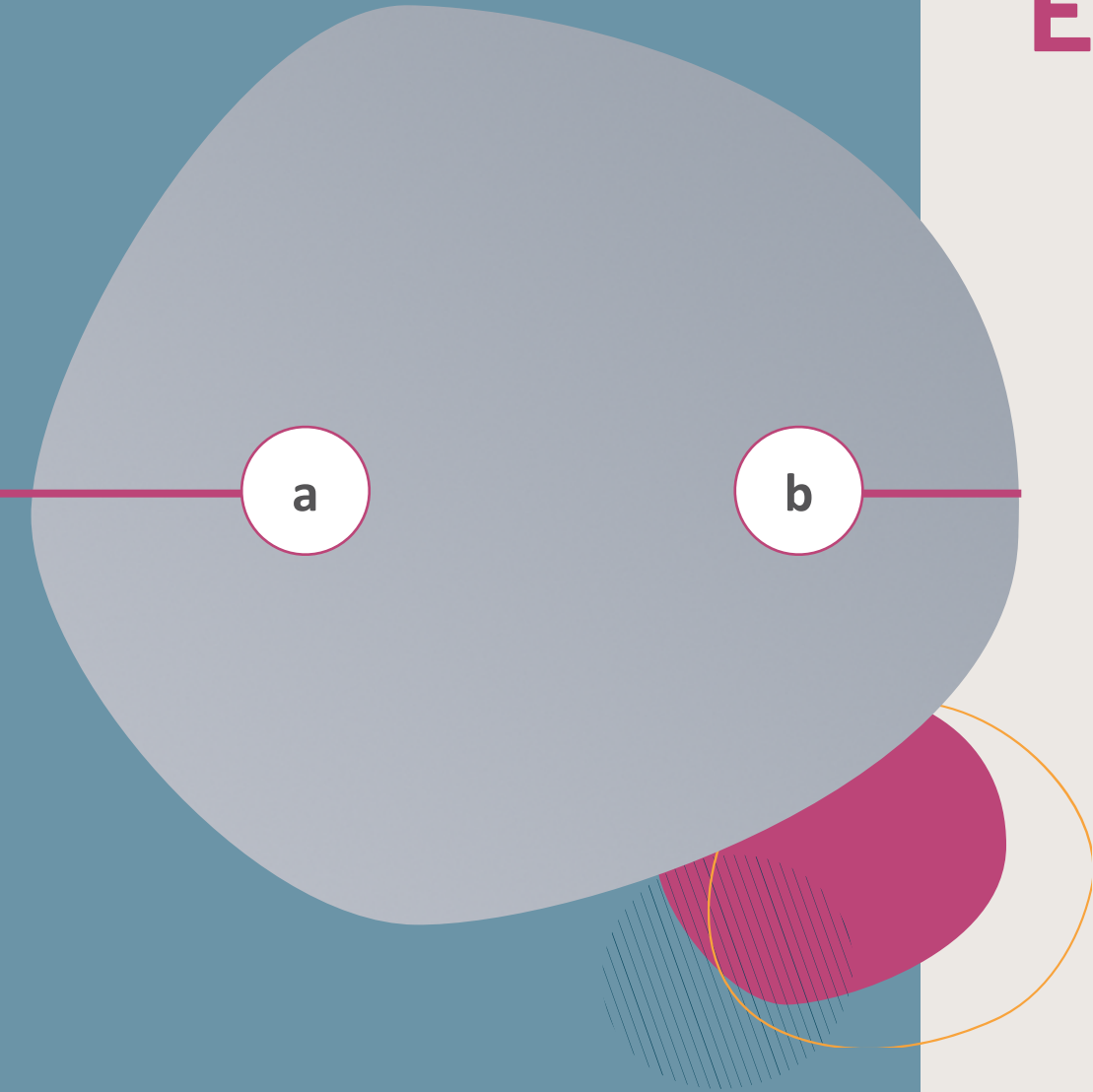
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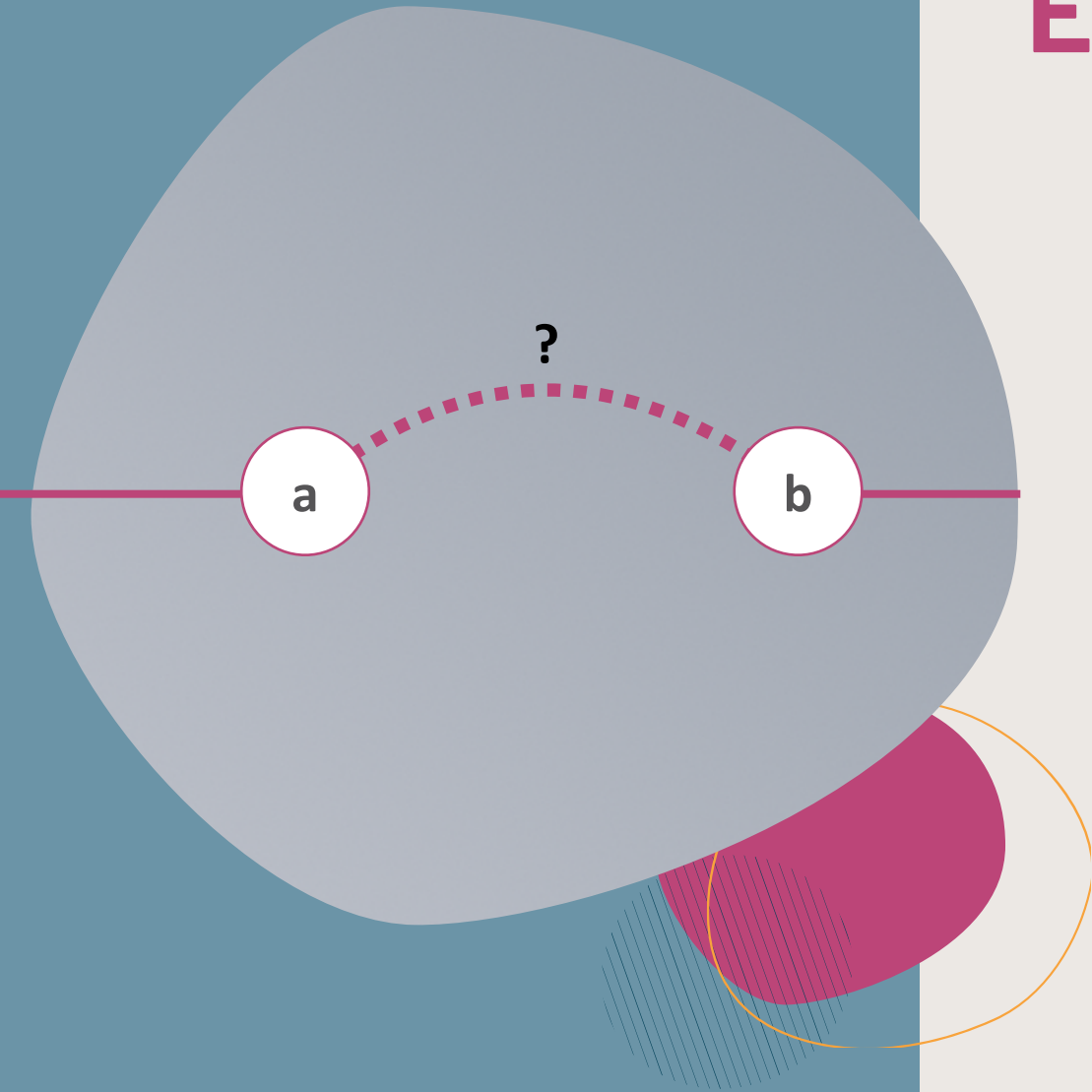
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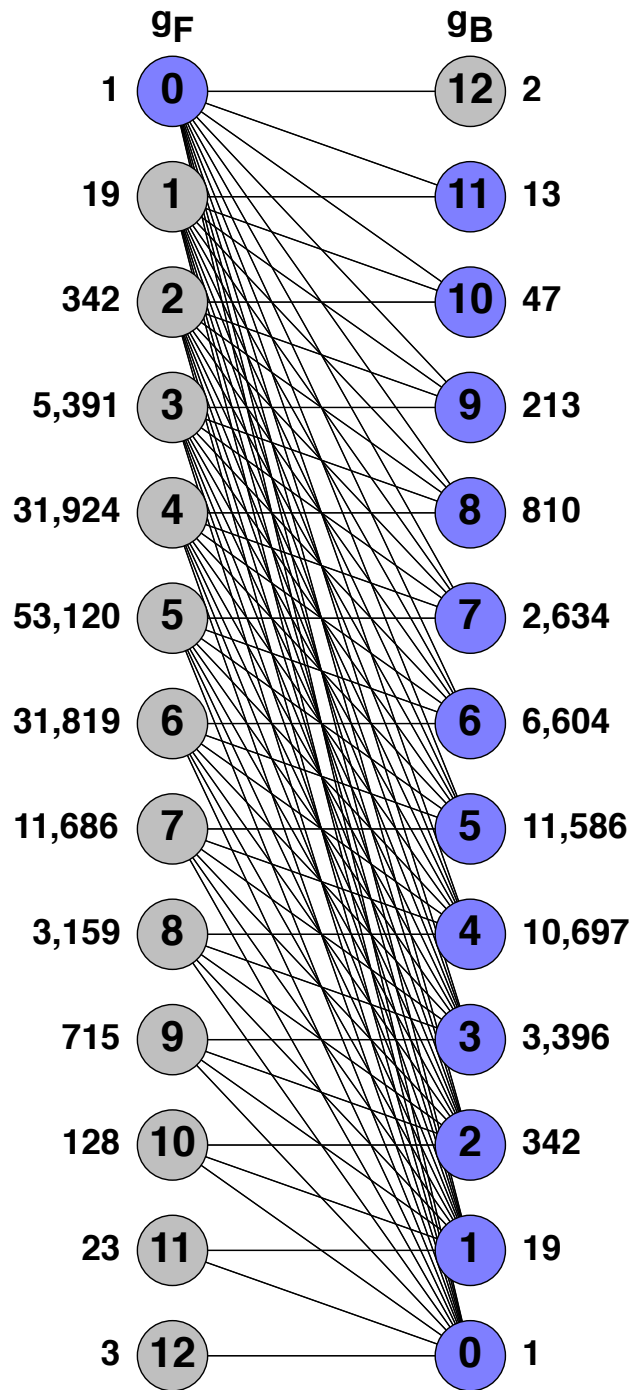
$$f_b(b) < C^*$$

$$g_f(a) + g_b(b) < C^*$$



$$C^* = 13$$

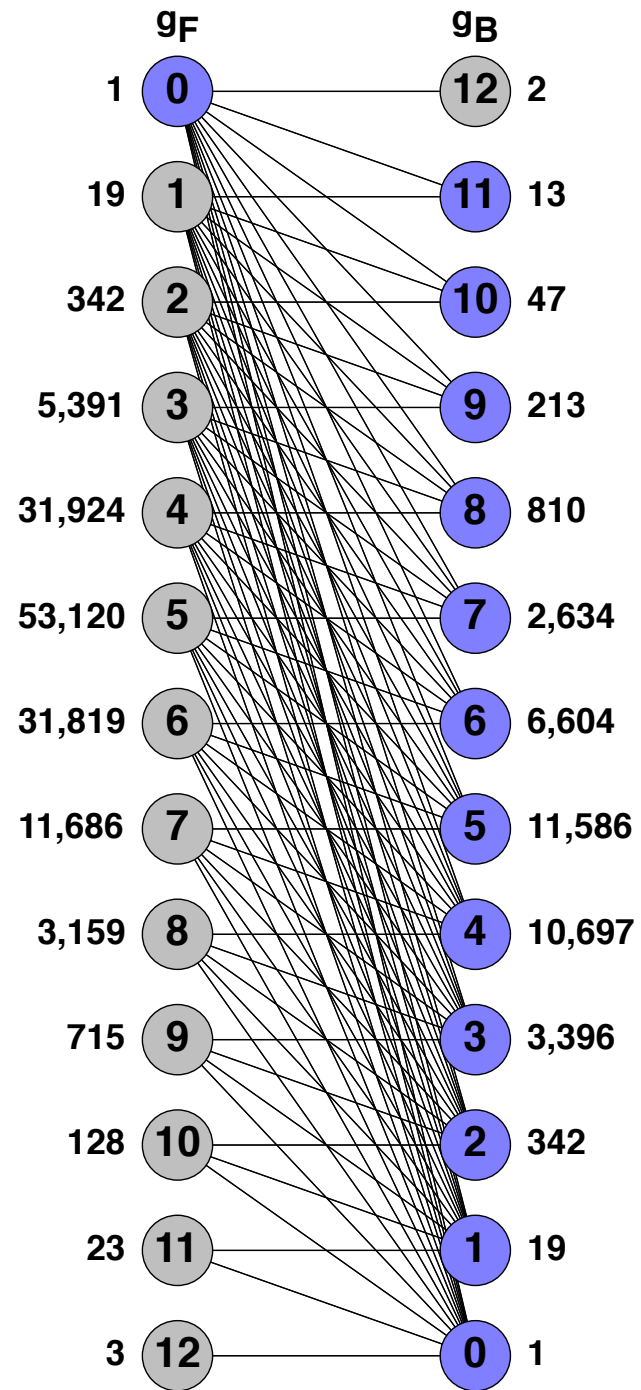
# EXPLANATION



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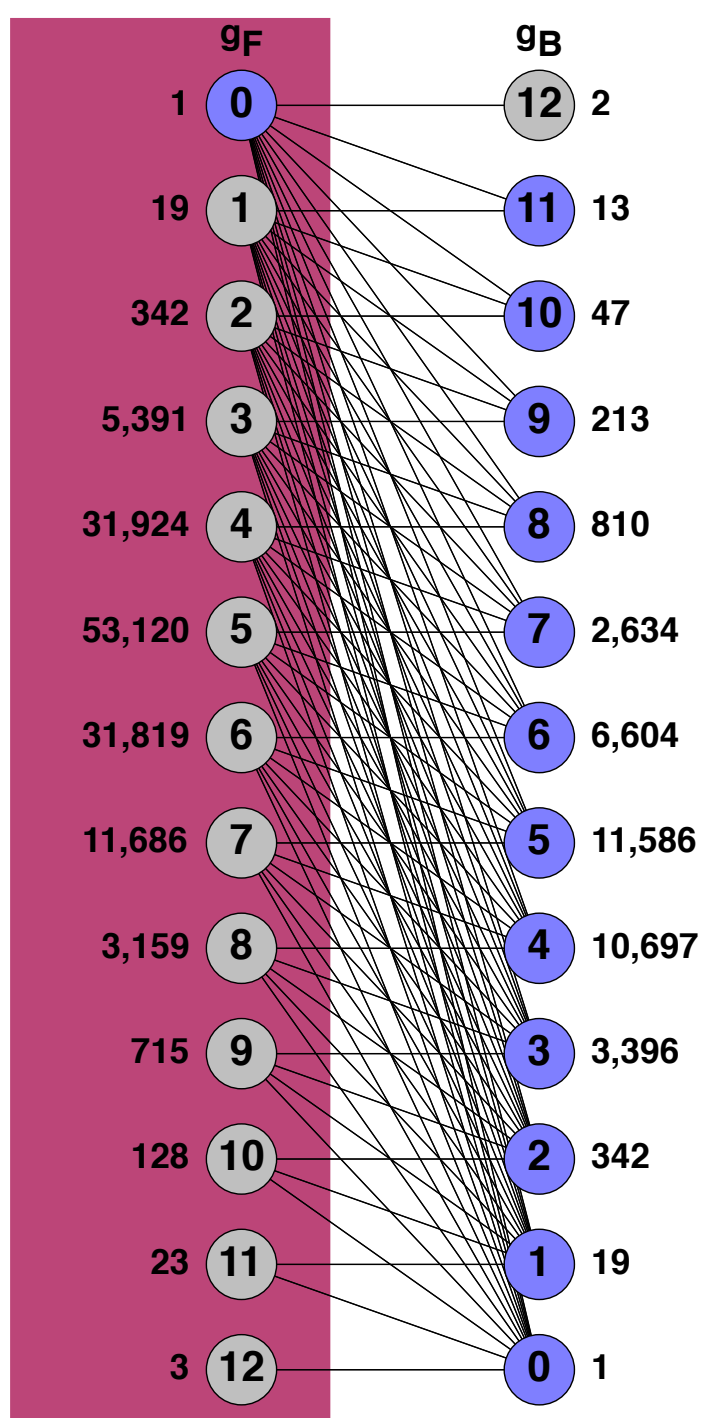
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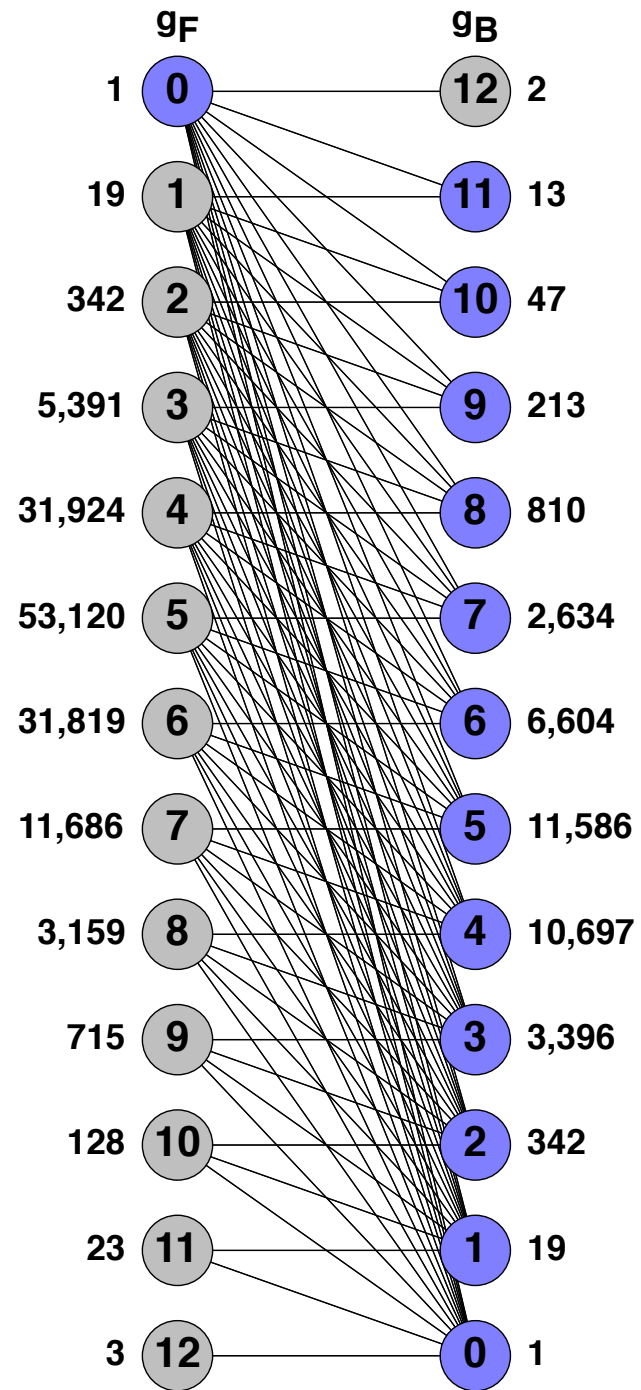
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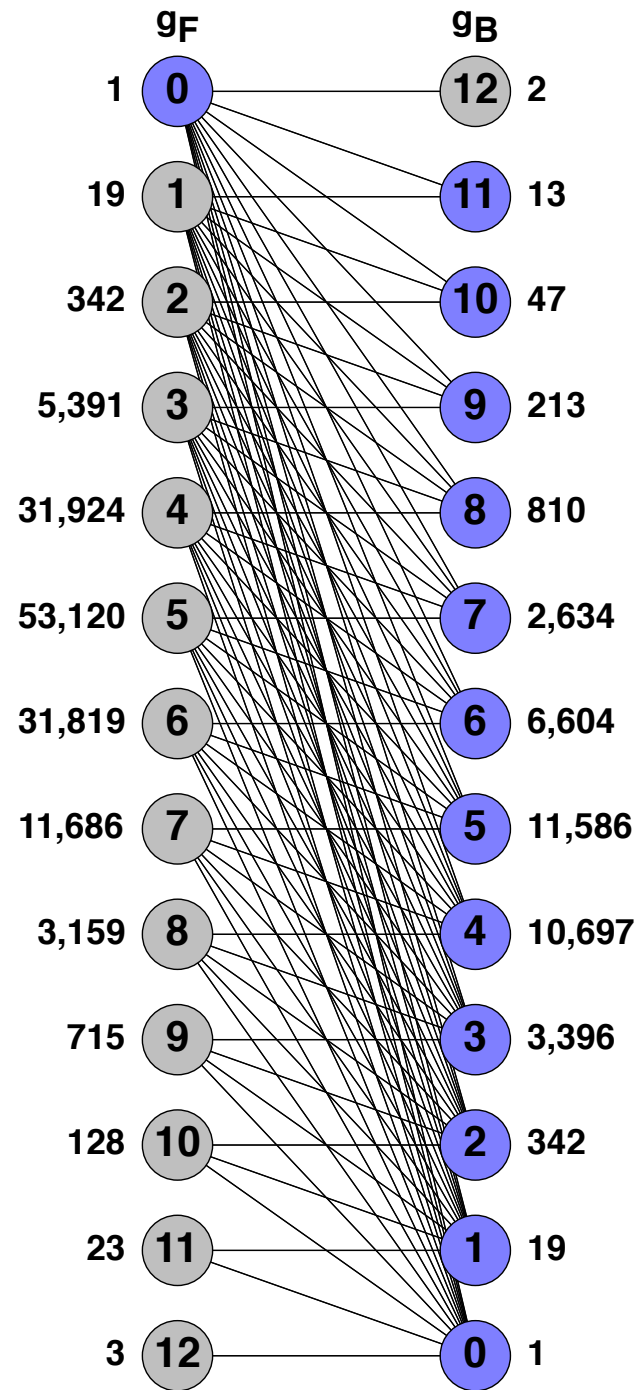


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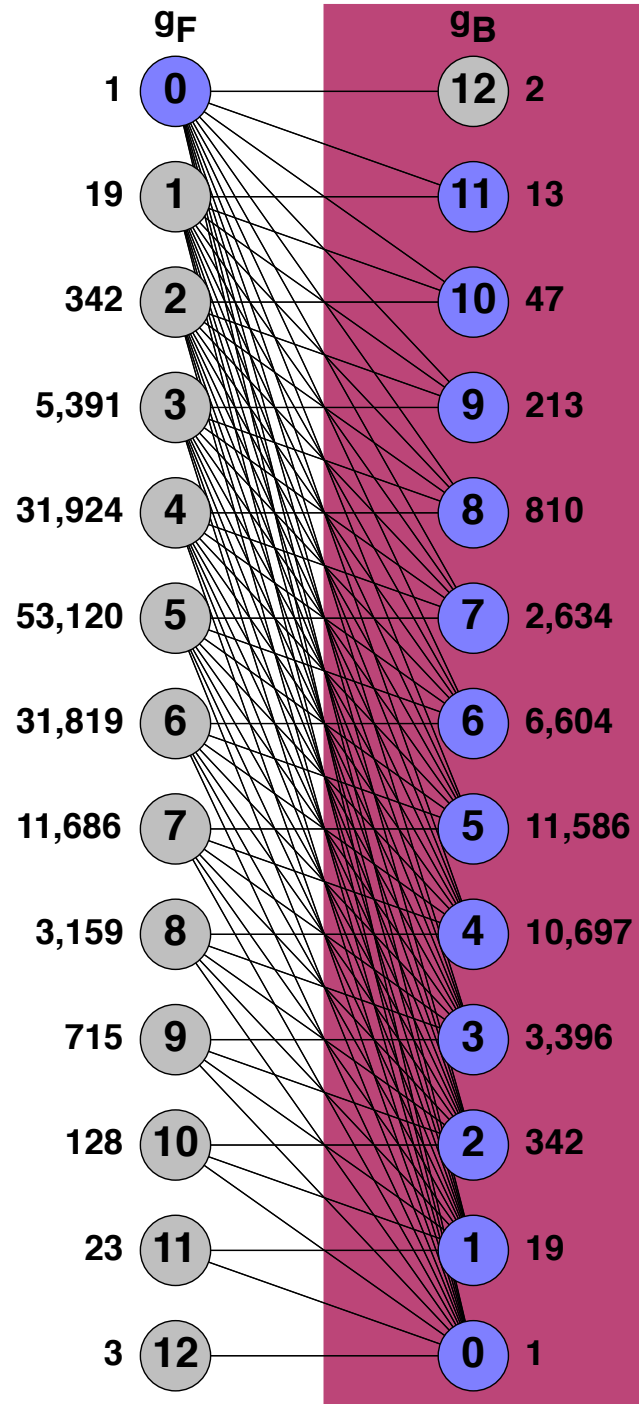


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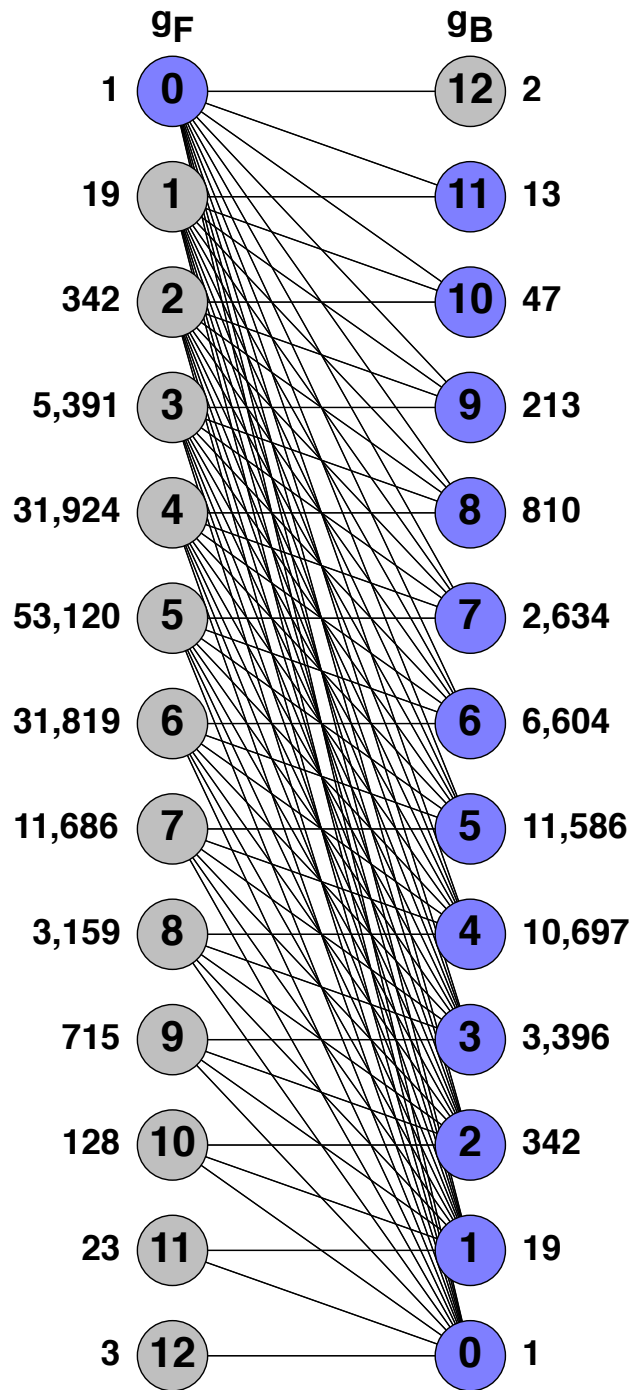


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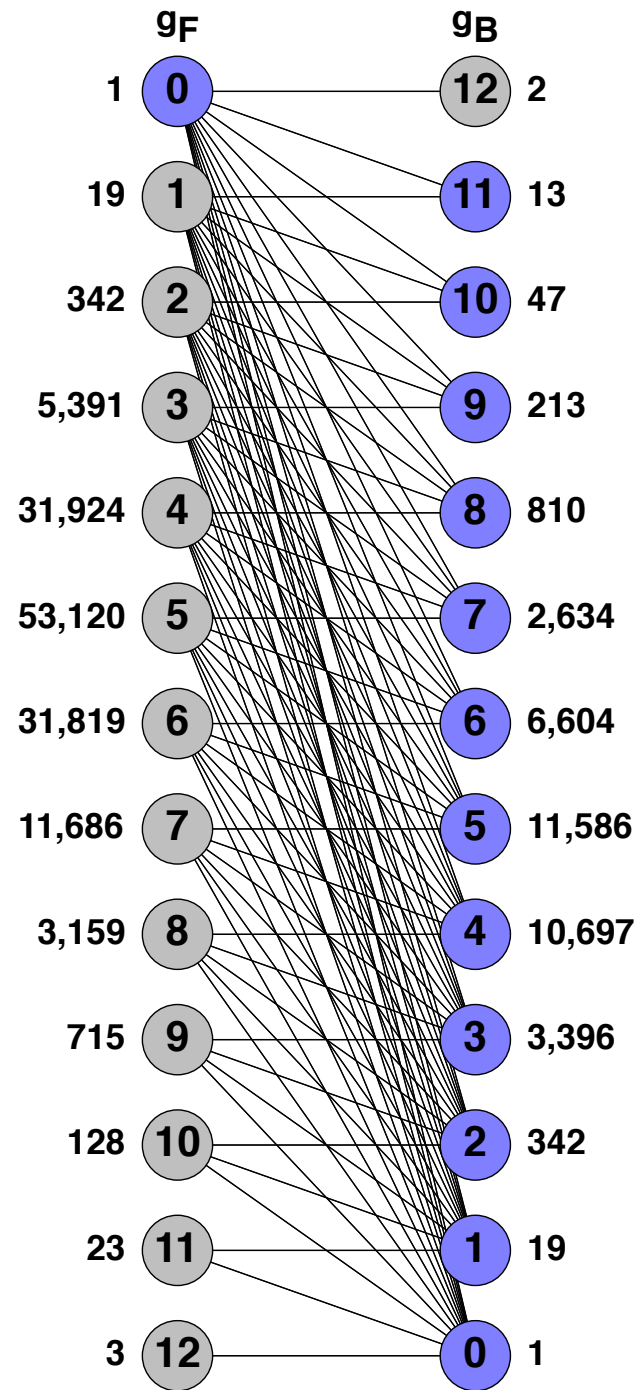
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$$f_f < C^*$$



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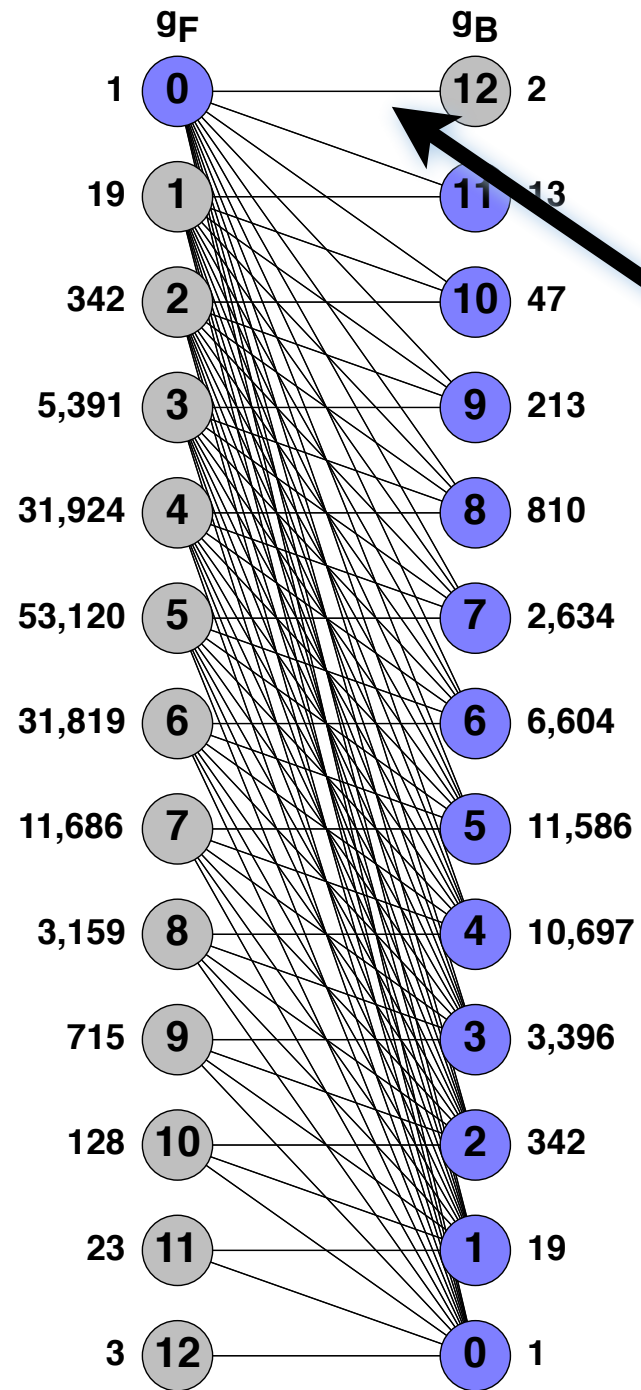
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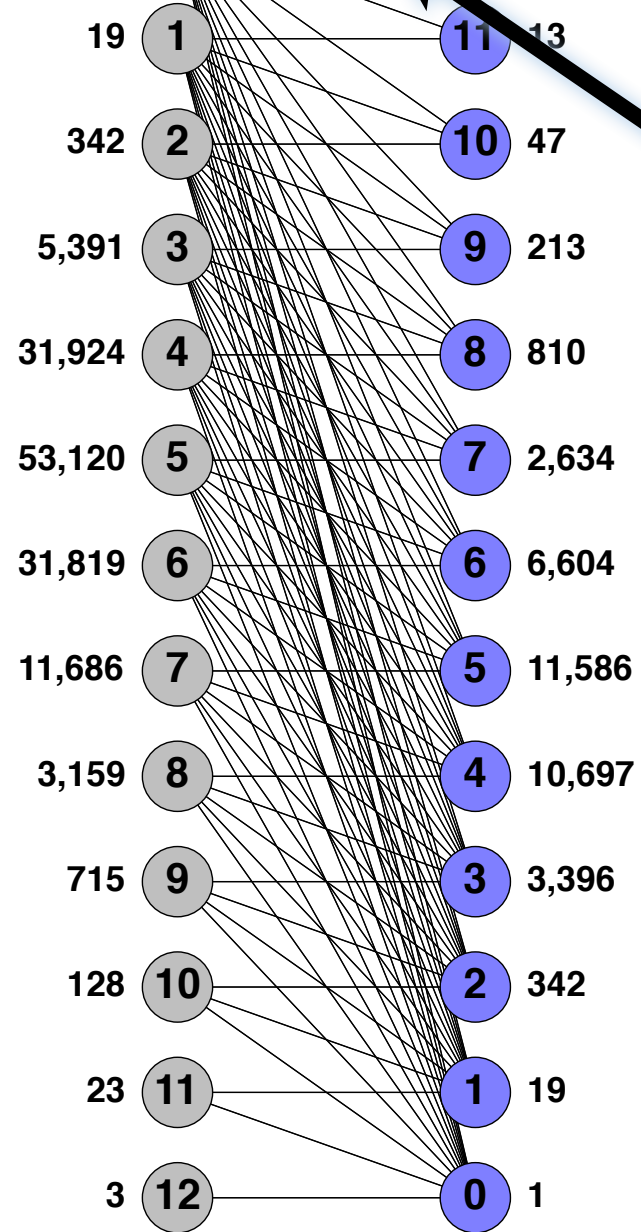
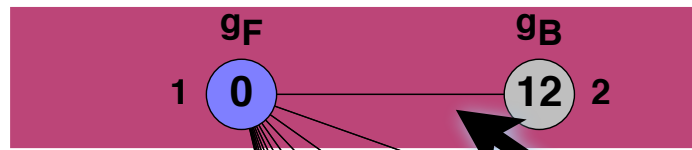
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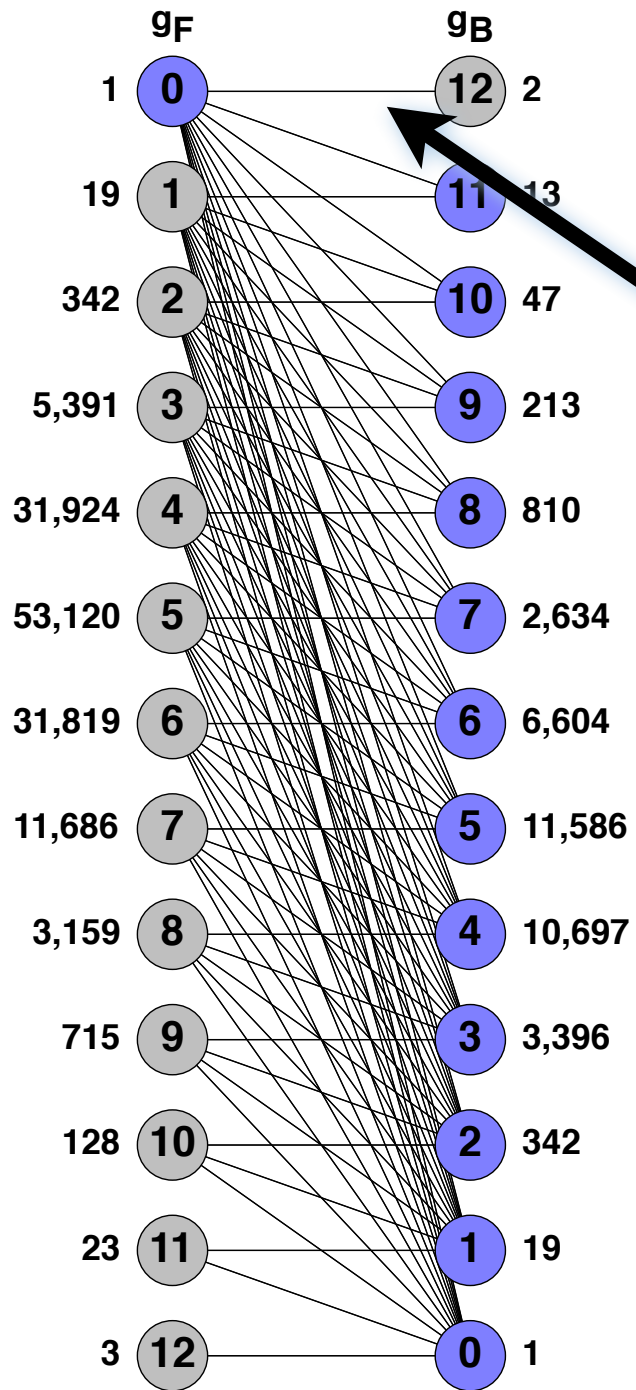
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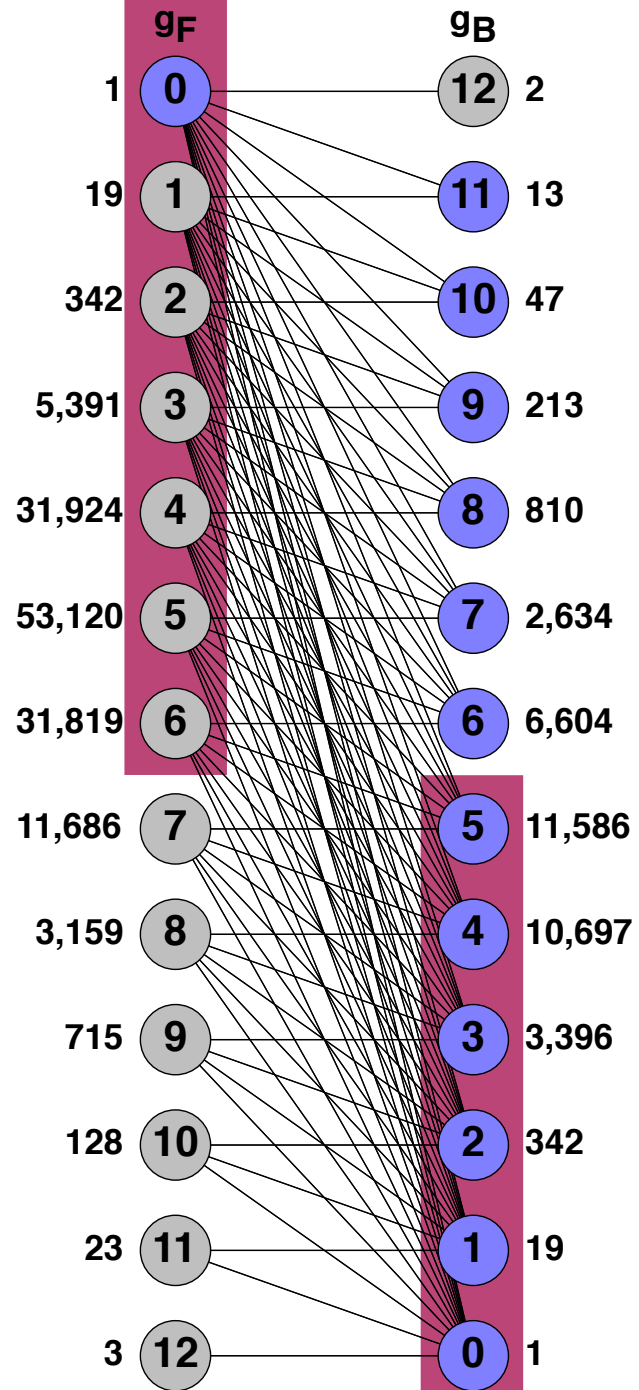
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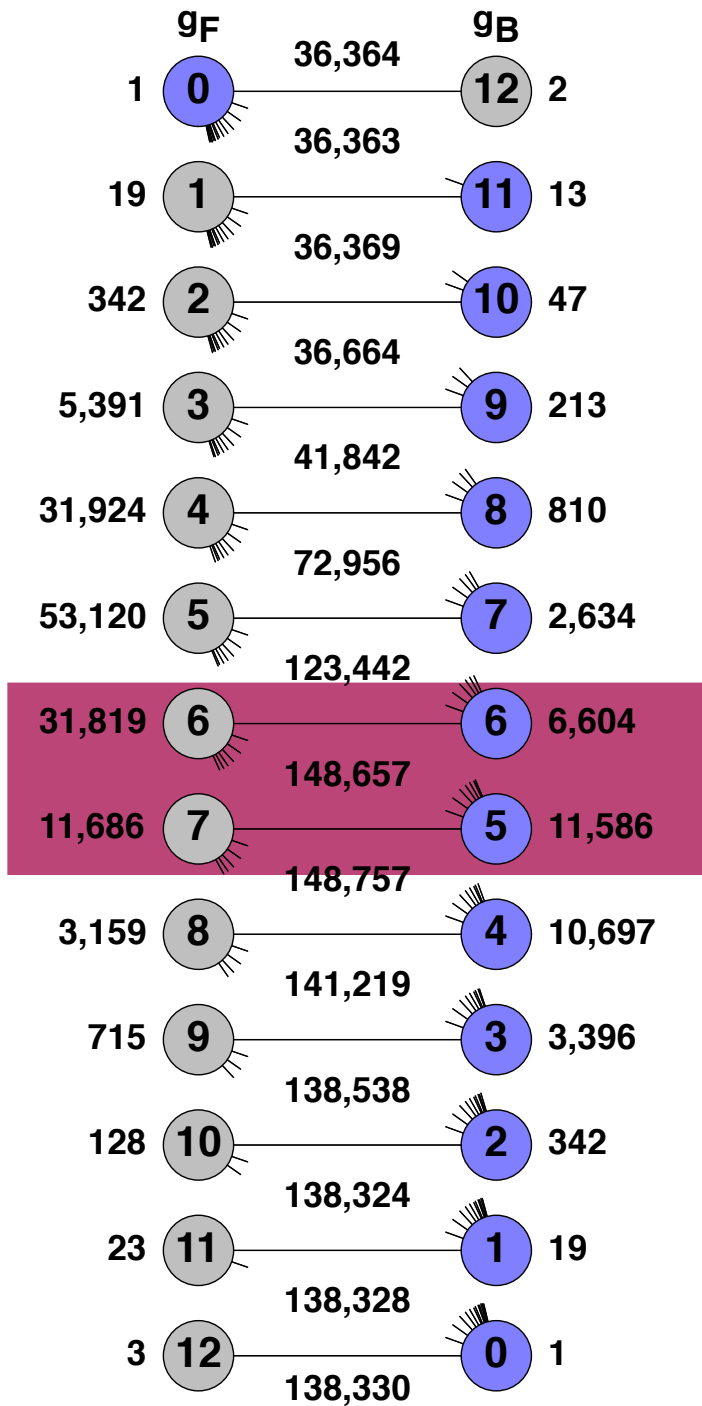
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# EXPLANATION





# RUBIK'S CUBE 7-TILE PDB

$C^* = 12$

	9F		9B	
1	0	—	11	3,749
18	1	—	10	4,371
237	2	—	9	4,273
1,201	3	—	8	4,156
1,981	4	—	7	3,904
2,670	5	—	6	3,615
3,291	6	—	5	3,123
3,567	7	—	4	2,438
3,638	8	—	3	1,330
3,647	9	—	2	237
3,740	10	—	1	18
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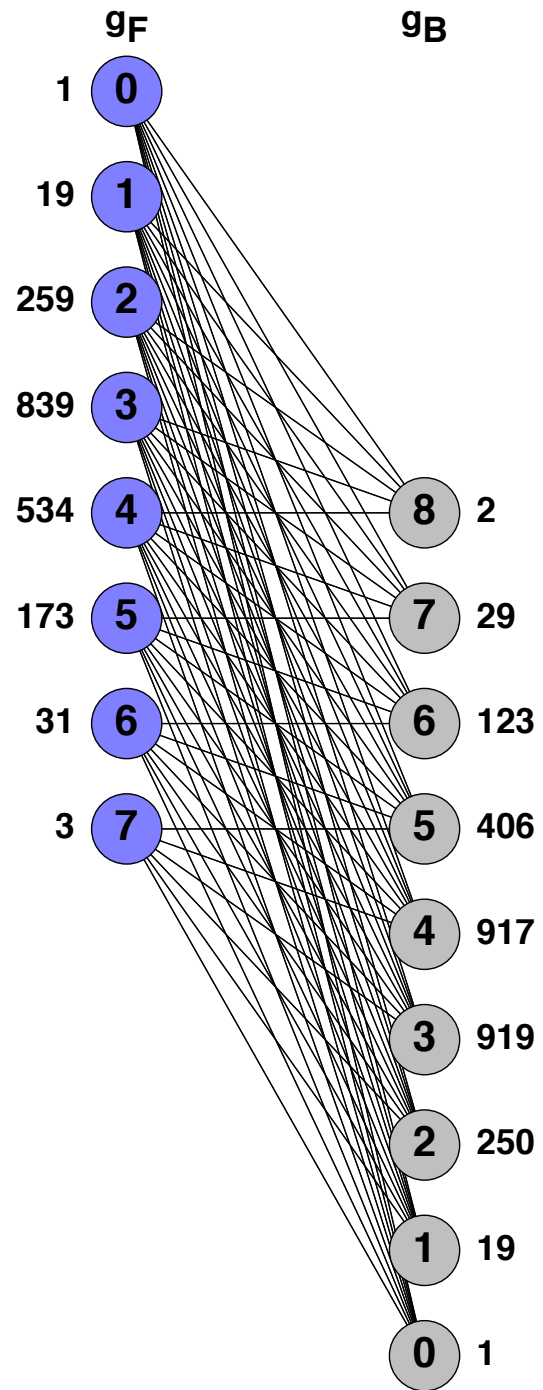
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# PANCAKE GAP HEURISTIC

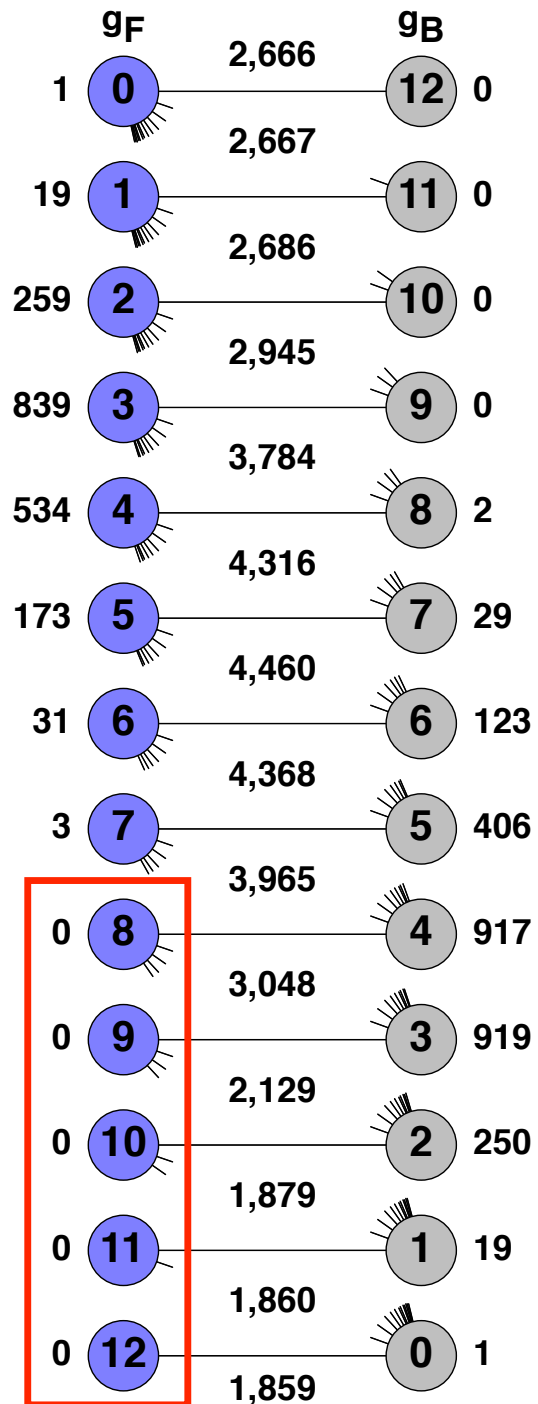
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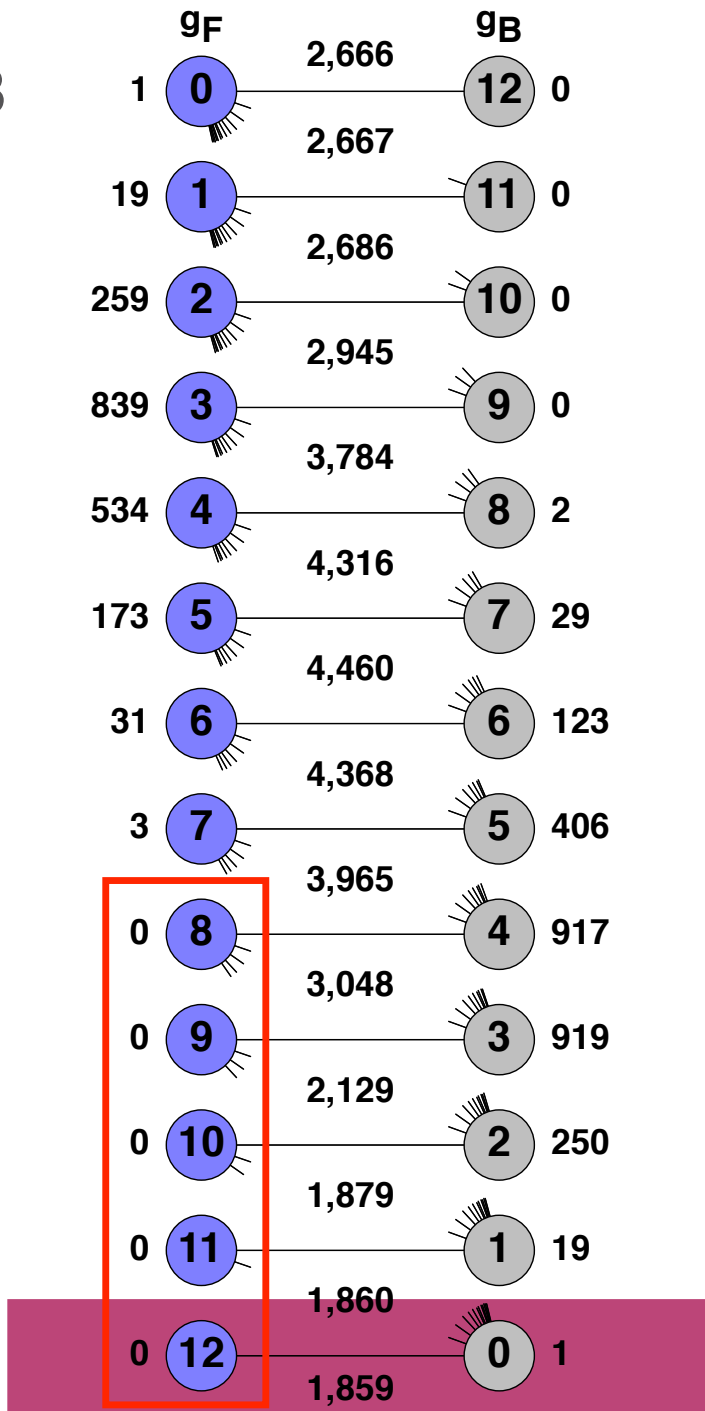
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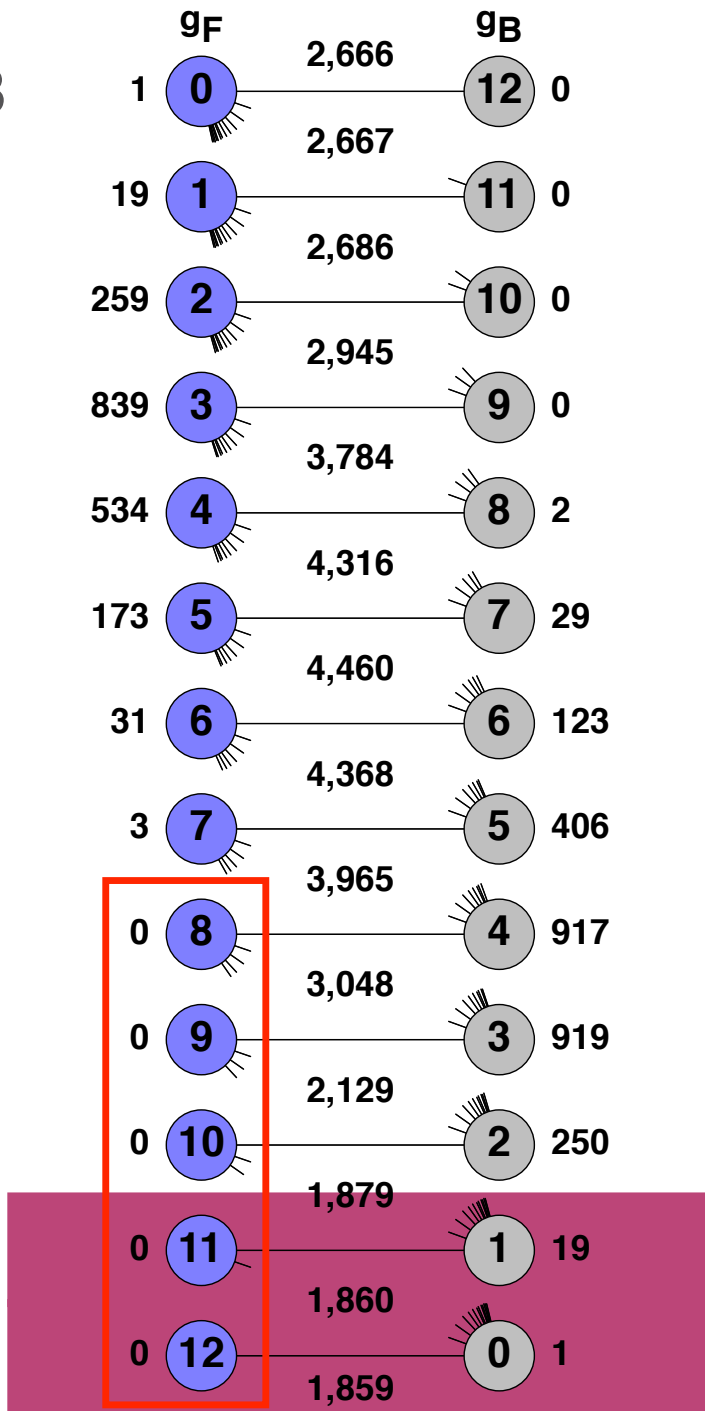
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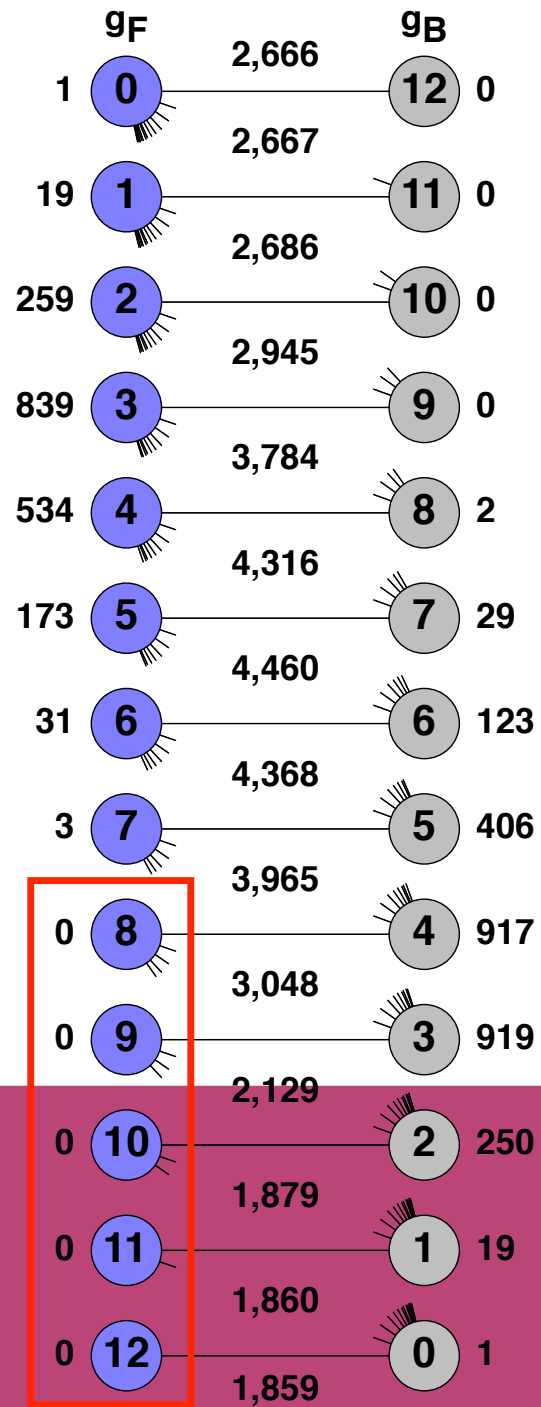
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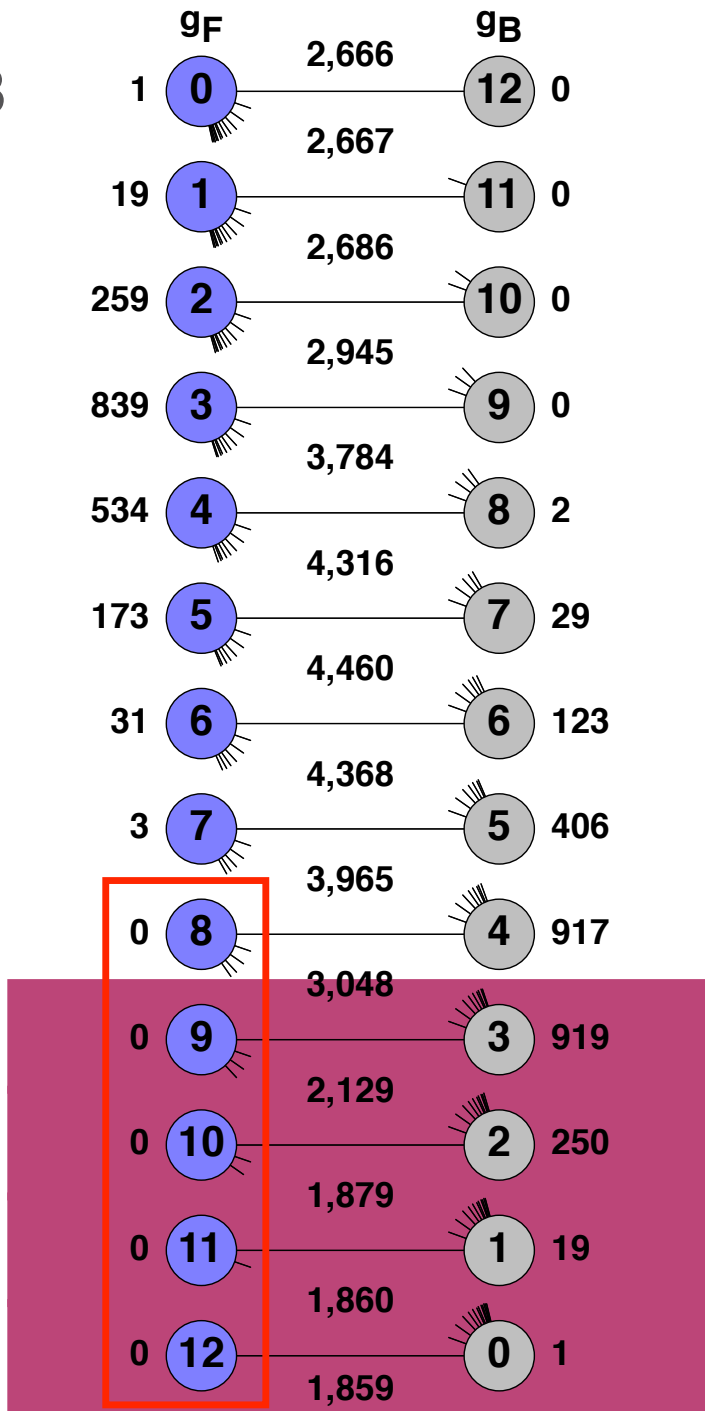
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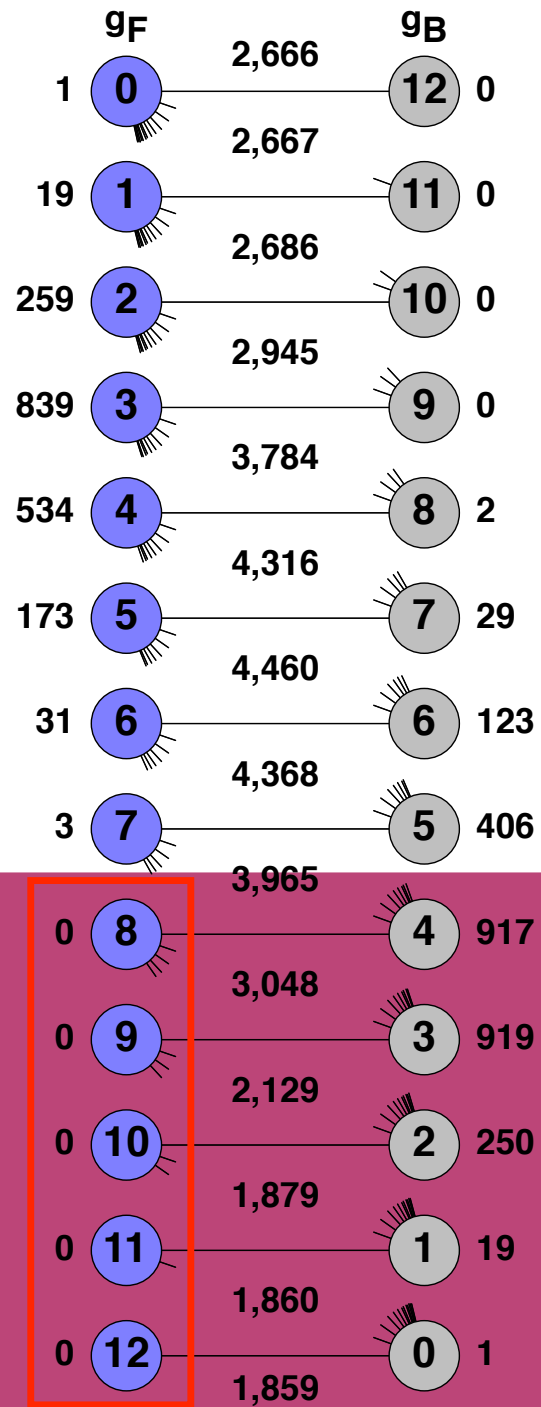
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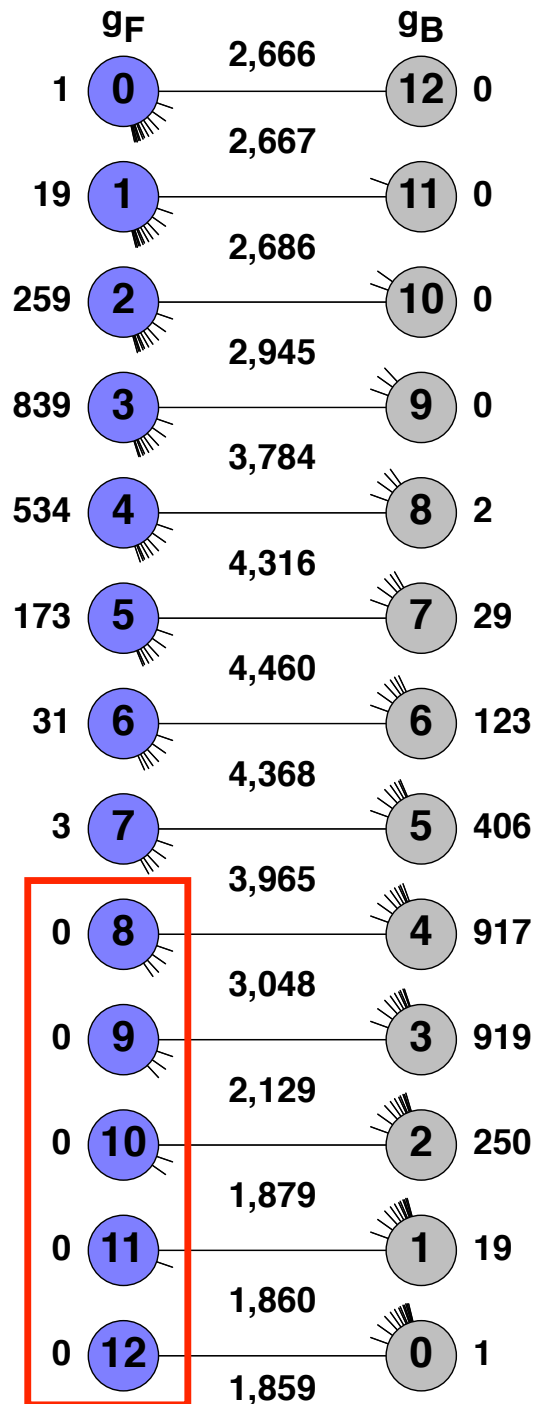
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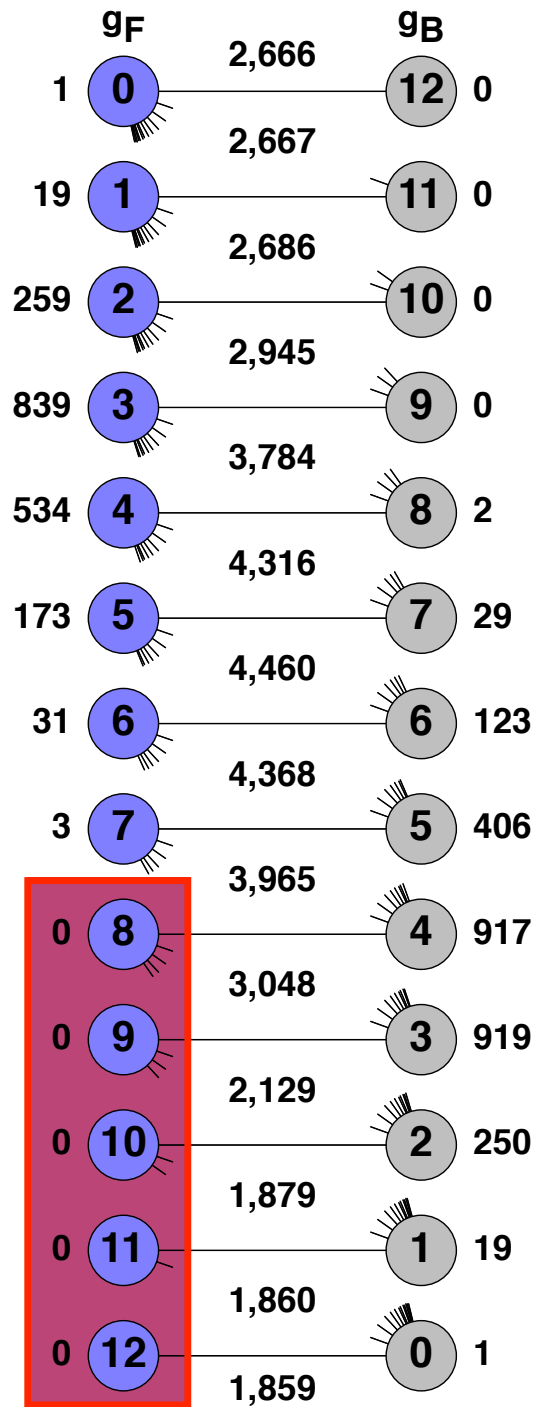
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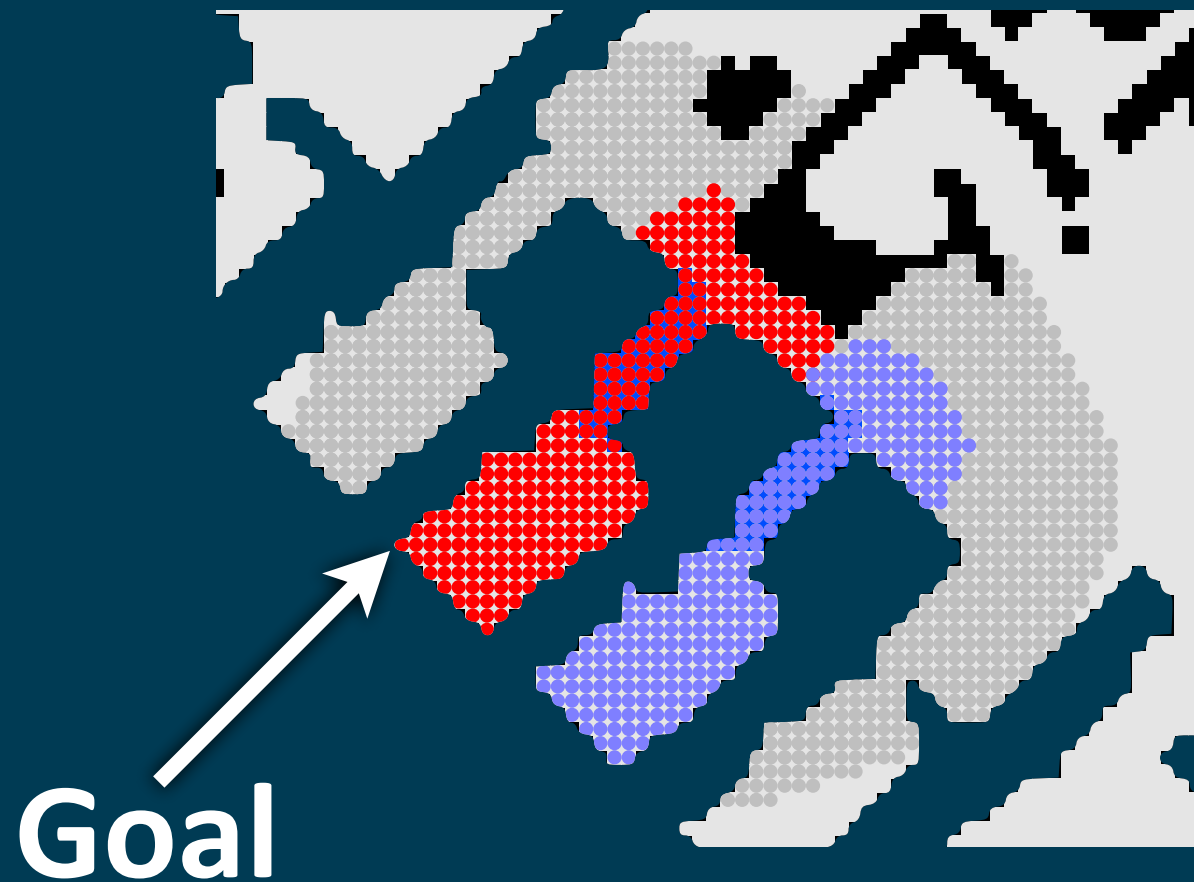
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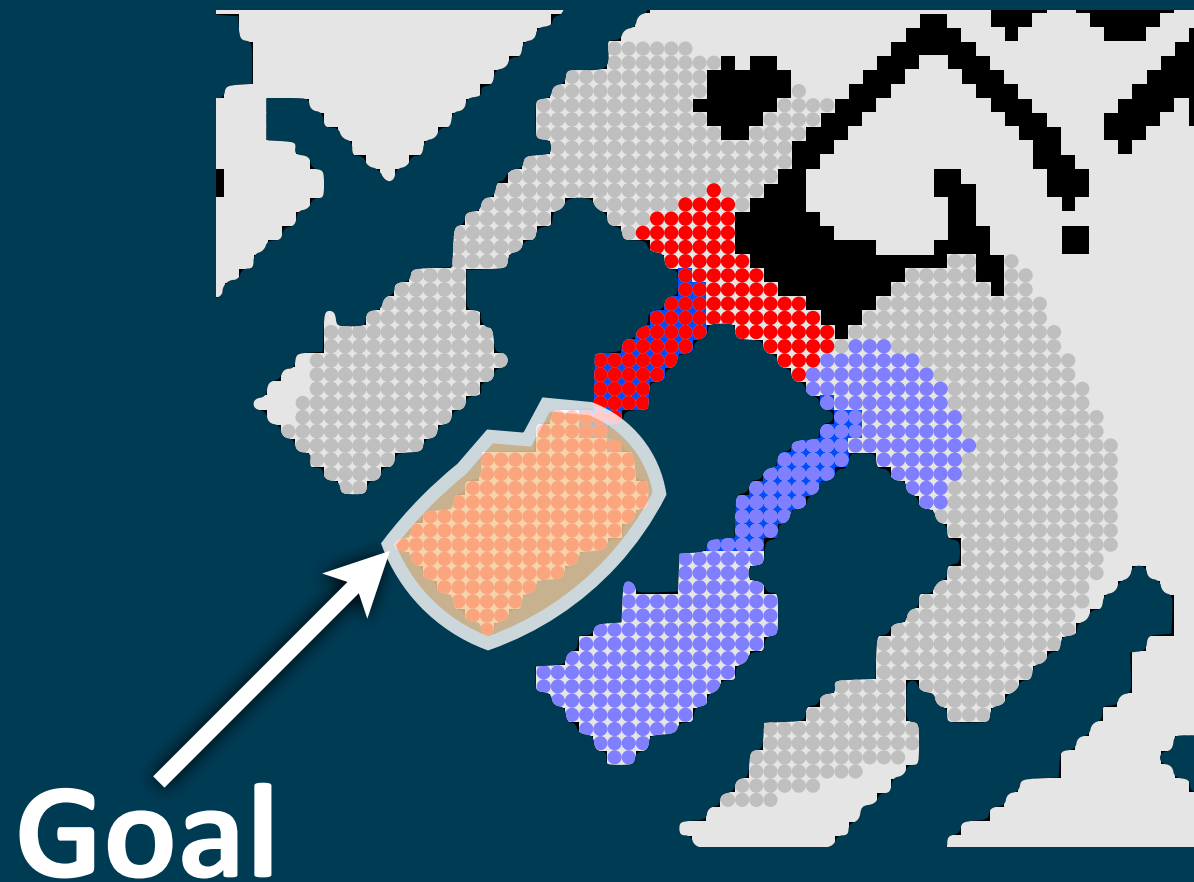
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- Sample states near goal and find heuristic inaccuracies



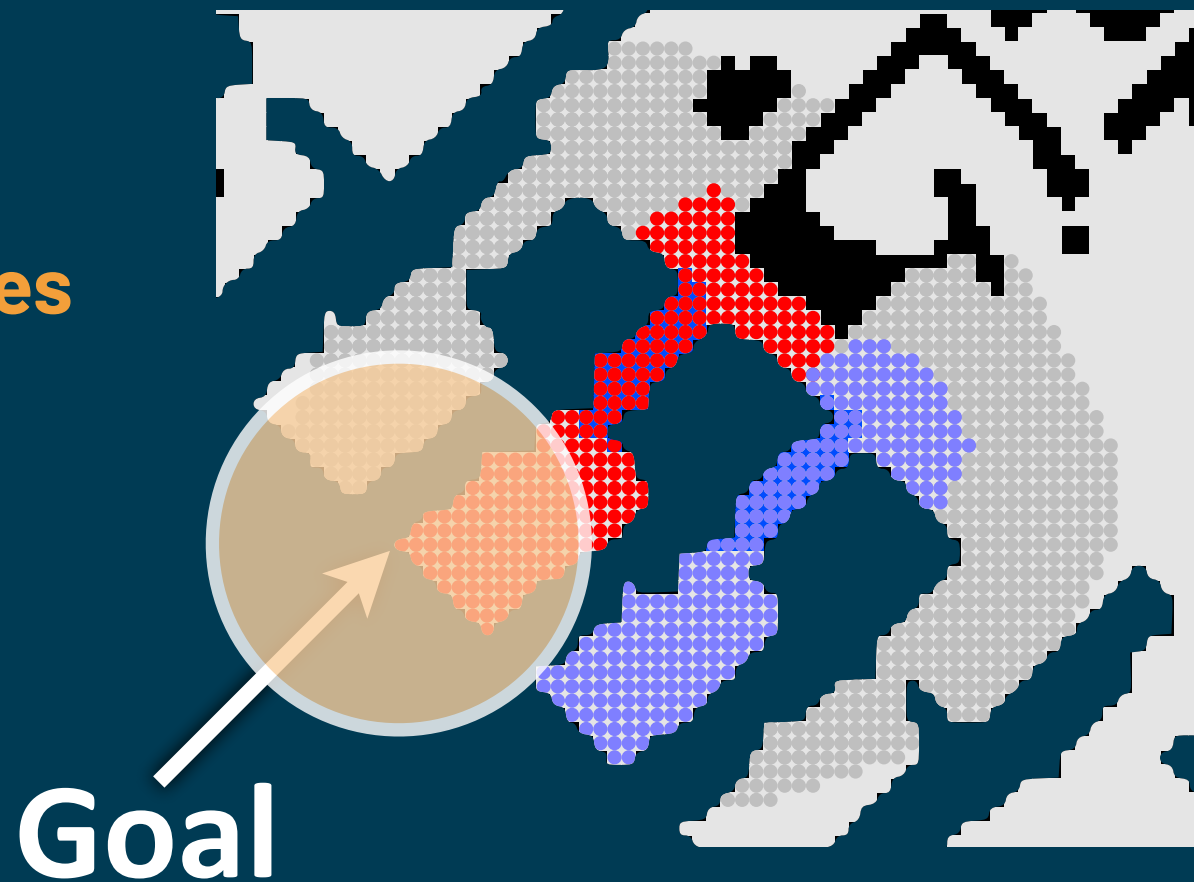
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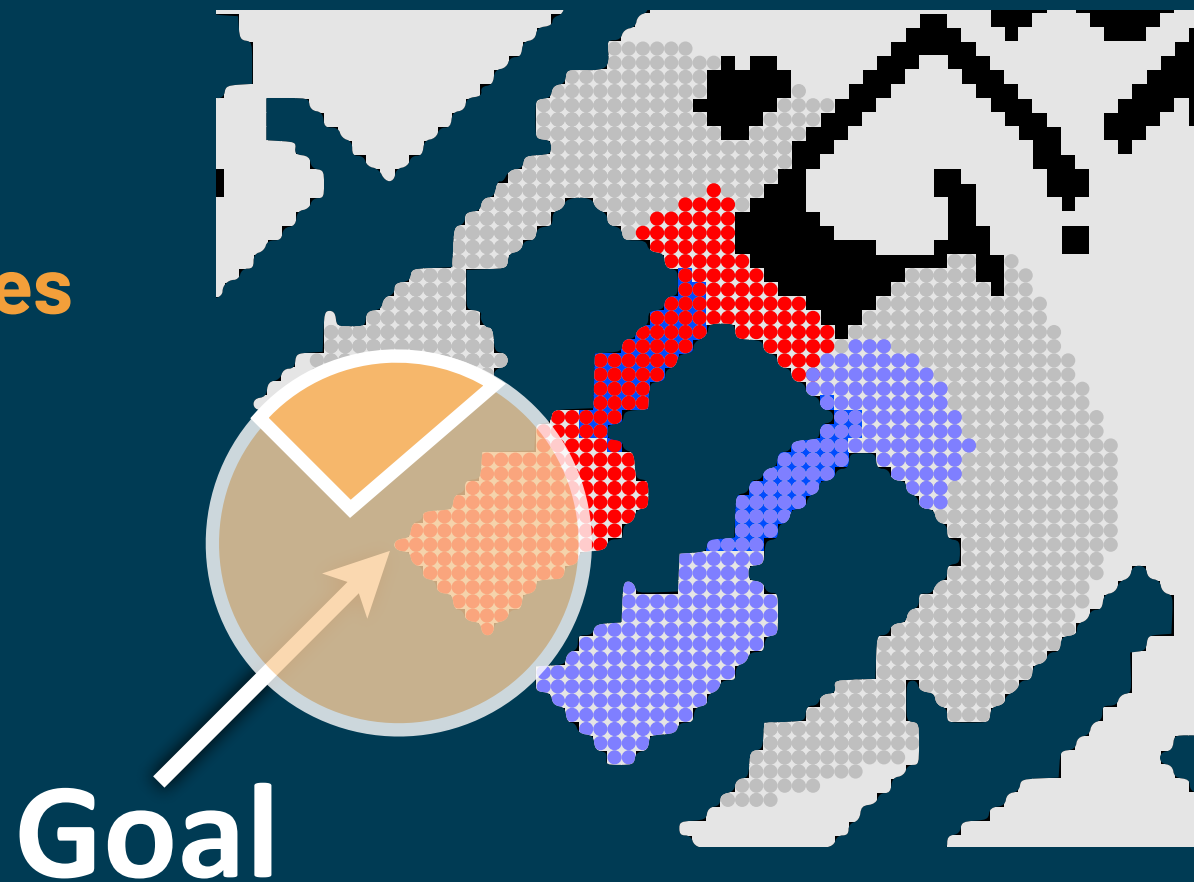
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# ASYMMETRY

- **Sample states near goal and find heuristic inaccuracies**
- **Sample the heuristic to find how many low heuristic values there are**
- **Look at the asymmetry of the state space**



# EXAMPLE: TOH WITH PDB HEURISTIC

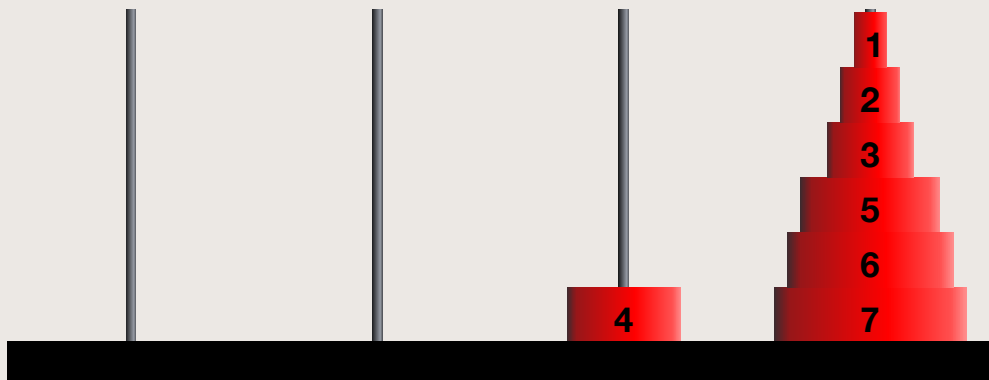
$\phi(s)$  is close to the goal and has a low heuristic value

$s$  is far from the goal

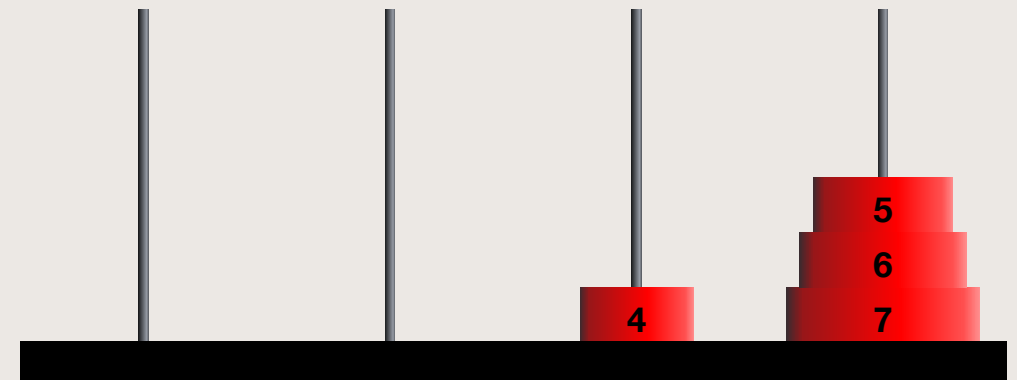
Low heuristics are inaccurate

**Bidirectional heuristic search outperforms unidirectional search**

Problem State  $s$

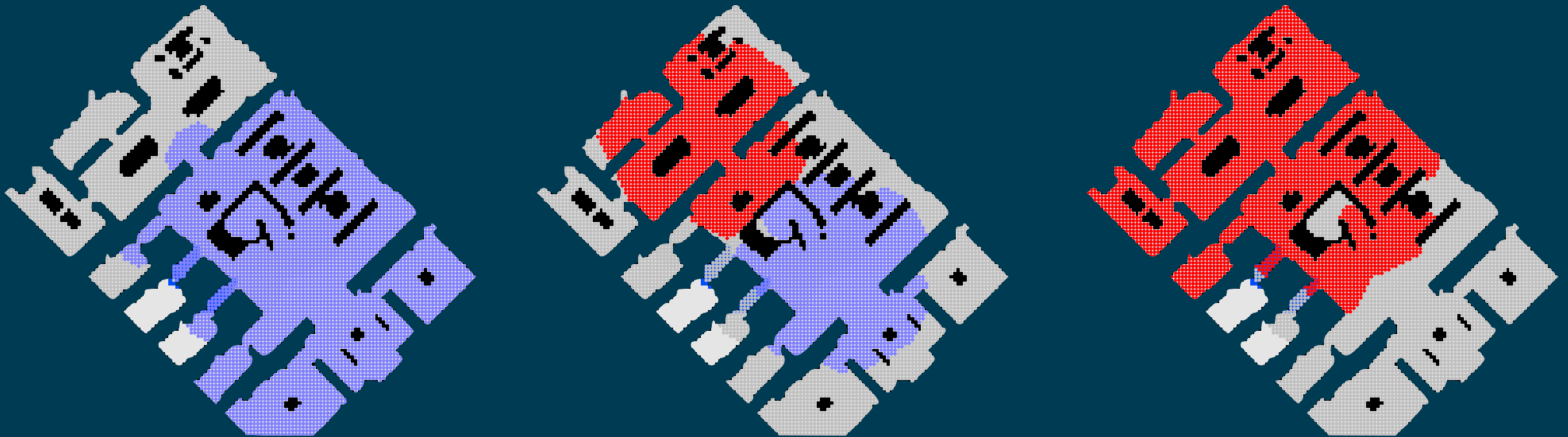


PDB Abstraction  $\phi(s)$



# EXAMPLE: UNINFORMED SEARCH

Low and inaccurate heuristics for almost all states



# CONCLUSION

Critical states have both low and inaccurate heuristics

Need critical states for bidirectional search to perform well

More critical states → bidirectional search will do better

<https://webdocs.cs.ualberta.ca/~nathanst/papers/sturtevant2020unibidi.pdf>